ON THE COMPARATIVE STUDY OF MATHEMATICAL MODELS FOR EARLIEST VISIBILITY OF THE CRESCENT MOON AND THEIR MODIFICATION

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CERTIFICATE

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In the name of Allah
The most merciful and the most beneficent

Dedicated to my late mother

Anwer Sultana

whose patient struggle in life and passion for mathematics motivated me to study

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MSQ

Abstract

The problem of determining the day when the new lunar crescent can be seen first at any site of observation has remained an open problem since antiquity. The phenomenon remains important for beginning a lunar month in a purely observational lunar calendar. In the first chapter the astronomical parameters related to the problem are reviewed along with a brief description of some rules of thumb attributed to the ancient and the medieval models. The affects of geographical location on the problem are also described. A brief review of calendars and associated celestial cycles is also presented with special emphasis on the rules enunciated for Islamic observational lunar calendar as early as 10^{th} century AD. In the end of the chapter review of the contribution of the astronomers of 20^{th} century is presented that begins with the empirical model of Fotheringham.

Solving the problem of the first visibility of the new lunar crescent involves lengthy calculations and the use of all elementary astronomical techniques. A review of these techniques and algorithms is presented in the second chapter. Special emphasis is given to the determination of time of conjunction of Moon with the Sun. i.e. birth of new Moon, the rising and setting of the Sun and the Moon, positions of the Sun and the Moon at any time on the day or the day after conjunction. The chapter ends with a brief description of the computer program Hilal01 developed for computations done in this work. A new program is developed in order to produce the data required for this work that is generally not available from other software that already exist.

Starting with simple Babylonian criterion, the third chapter explores the ancient and the medieval mathematical models followed by the description of mathematical exploration of medieval Muslim Astronomers. Geometrical considerations associated with the problem are explored in more detail in order to evaluate the Lunar Ripeness Law and its success, suggested in the medieval era. Some modifications to the law are also suggested. The shortcomings of the ripeness law are then discussed and light is shed on the reasons that lead to the development of ARCV-DAZ relation based models by the

early 20th century astronomers. The significant finding of the chapter is that both the simple Babylonian criterion and the Lunar Ripeness law are more successful in terms of their consistency with the positive sightings records in comparison to the models developed in the first half of 20th century. A set of 463 observations is used for testing all models in this work, that include observations collected since the later half of 19th century till recent times.

In comparison to the empirical models used to predict the visibility of the new lunar crescent till the first half of the twentieth century, the models developed on the basis of physical theories of sky brightness and extinction are explored in the fourth chapter. These models include those developed by Bruin and Schaefer separately. Bruin based his model on the average brightness of the full Moon and the twilight sky. Schaefer's model calculates the actual limiting magnitude of the sky and the magnitude of the crescent and tests a visibility claim on the basis of magnitude contrast and is different in nature from all other models. After a brief description of Bruin's model the semi-empirical model of Yallop is discussed in detail, which is considered to be the most comprehensive and authentic model. Yallop deduced his basic data from Bruin's visibility curves. On the basis of Schaefer's techniques we have reconstructed Bruin's model and produced new visibility curves and a new semi-empirical model for the visibility of new lunar crescent. The development of this new model is one of the major achievements of this work. All the models are tested ón the same data set as is used in the previous chapter. The new model developed in this work is found to be the best amongst the modern day models in terms of its consistency with the number of positive sightings in the data set used. A comparison of success of each model is also discussed in this chapter.

At the end of the fourth chapter a strategy is framed to verify the authenticity of a claim of sighting of new crescent on the basis of a semi-empirical model and the magnitude contrast model. The significance of such a strategy has been highlighted as there are found a number of authentic new crescent visibility claims that are not consistent with a semi-empirical model. In these cases a semi-empirical model does not

allow visibility of the crescent without optical aid but the magnitude contrast is in favour of visibility. This happens as a semi-empirical model does not take into consideration the elevation of the site of observation above sea level and the weather conditions. The magnitude contrast model considers all these factors.

Beyond theoretical considerations a mathematical model should possess power of applicability. The prime application of the mathematical models explored, analysed and developed in this work is to determine the earliest visibility of a new lunar crescent at any location of the world and to verify a claim of crescent visibility. Apart from this prime application in the fifth chapter the semi-empirical models are applied to develop a technique for calculating the length of new observed lunar crescent. The phenomenon of shortening of crescent length is known for centuries and during 20^{th} century a number of reasons have been suggested for the same. However, our suggested technique is the first of its nature that provide a simple computational tool for calculating length of observed crescent. Moreover, the semi-empirical models are also applied to verify the actual practised observational lunar calendar in Pakistan for the last seven years. The models and the practised calendar are found to be in agreement in 95% of the new moons during the period of study. Motivated by this high rate of consistency we have presented a "Predicted Observational Lunar Calendar" for Pakistan.

A summary of this whole effort is presented in the last chapter. Various important issues are highlighted with a discussion on the future scope of research in the area explored.

خلاصہ

زمانۂ قدیم ہی سے کسی بھی مقامِ مشاہدہ سے نئے قمری ہلال کے پہلی مرتبہ نظر آنے کے دن کا تعین ایک کھلا مسئلہ رہا ہے. ایک خالص مشاہداتی قمری تقویم میں قمری مہینہ کے آغاز کیلئے یہ مظہر اہم مسئلہ ہے. پہلے باب میں اس مسئلے سے متعلق فلکیاتی نقاط کا احاطہ کرنے کے ساتھ ساتھ زمانۂ قدیم اور قرونِ وسطیٰ سے منسلک بنیادی اصولوں پر مبنی نمونوں کو اختصار سے بیان کیا گیا ہے. تقویمات اور ان سے متعلق آسمانی دورانیوں کا ایک مختصر خاکہ بھی پیش کیا گیا ہے. جس میں خصوصی اہمیت ان اصولوں کو دی گئی ہے. جو اسلامی مشاہداتی قمری تقویم کیلئے دسویں صدی عیسوی تک وضع کرلئے گئے تھے. باب کے آخر میں بیسویں صدی کے فلکیات دانوں کی کاوشوں کا جائزہ پیش کیا گیا ہے جس کی ابتداً فوتھرنگھم (Fotheringham) کے ایمپیرکل نمونے سے ہوتی ہے.

نئے قمری ہلال کے اولین دیدار کے مسئلے کو حل کرنے کیلئے انتہائی طویل حساب کتاب اور بنیادی فلکیات کی تمام تیکنیکات کا استعمال ہوتا ہے. ان تمام تیکنیکات اور الگورتھم کا احاطہ باب دوم میں پیش کیا گیا ہے. خصوصی اہمیت سورج اور چاند کے کنجنکشن یعنی چاند کی پیدائش کی وقت کے تعین، سورج اور چاند کے طلوع وغروب اور کنجنکشن کے دن یا اس کے ایک روز بعد کسی بھی لمحے سورج اور چاند کے مقام کو دی گئی ہے. یہ تمام حسابات لگانے کیلئے اس کام کے دوران ایک کمپیوٹر پروگرام 'ہلال 01' تخلیق کیا گیا ہے. جس کا مختصر بیان اس باب کے آخر میں پیش کیا گیا ہے. ایک نئے پروگرام کی تخلیق اس لیئے کی گئی کہ موجودہ کام کے لیئے جن اعداد وشمار کی ضرورت تھی وہ پہلے سے موجود دوسرے سوفٹویئر سے حاصل نہیں ہوتے.

سادہ بابلی شرط سے ابتداً کرتے ہوئے تیسرے باب میں قدیم اور قرونِ وسطیٰ کے ریاضیاتی نمونوں کو کنگھالا گیا ہے. جس کے بعد قرونِ وسطیٰ کے مسلمان فلکیات دانوں کی ریاضیاتی کاوشوں کا جائزہ لیا گیا ہے. مسئلہ سے متعلق ہیئتی نقاط کا تجزیہ زیادہ تفصیل سے کیا گیا ہے تاکہ قمری اصولِ پختگی Lunar Ripeness Law اور اسکی کامیابی کا احاطہ کیا جا سکے جو اسی قرونِ وسطیٰ سے منسلک ہے. اس اصولِ پختگی کی حدود اور مشکلات پر بحث کرتے ہوئے ان وجوہات پر روشنی ڈالی گئی ہے جو بیسویں صدی کی ابتداً میں فرقِ عمودی وافقی کے درمیان رشتوں کی بنیاد پر بننے والے نمونوں کی بنیاد بنے. اس باب کا سب سے اہم نتیجہ یہ ہے کہ سادہ بابلی شرط اور اصولِ پختگیٔ قمر دونوں بیسویں صدی کے ابتداً میں تیار کردہ نمونوں کے مقابلے میں مثبت مشاہداتی ریکارڈز سے مطابقت کے معاملے میں زیادہ کامیاب ہیں. اس کام کے دوران تمام نمونوں کو جانچنے کیلئے 463 مشاہداتی ریکارڈز استعمال کیئے گئے ہیں جو انیسویں صدی کے اواخر سے ماضی قریب تک جمع کیئے گئے.

نئے قمری ہلال کے دیدار کی پیشنگوئی کیلئے بیسویں صدی کے پہلے نصف تک زیرِ استعمال ایمپیریکل نمونوں کے مقابلے میں آسمانی روشنی (شفق) اور اس میں ہونے والی کمی سے متعلق طبیعاتی نظریات کی بنیاد پر تیار کردہ نمونوں کا جائزہ باب چہارم میں لیا گیا ہے. یہ نمونے برونن (Bruin) اور شیفر (Schaefer) نے علیحدہ علیحدہ تیار کیئے. برونن کے نمونے کی بنیاد مکمل چاند اور غروبِ آفتاب کے بعد آسمان کی اوسط روشنی ہے. شیفر کے نمونے آسمان کی چمک کی اصل حد اور ہلال کی اصل چمک کا حساب لگاتے ہیں اور دیدارِ ہلال کے دعوے کو دونوں کی چمک کے فرق کی بنیاد پر جانچتے ہیں اور بقیہ تمام نمونوں سے مختلف ہیں. برونن کے نمونے کے مختصر بیان کے بعد یالپ (Yallop) کے نیم

ایمپیریکل نمونے پر تفصیلی بحث کی گئی ہے جو کہ سب سے زیادہ مکمل اور قابل بھروسہ تصوّر کیا جاتا ہے. یالپ نے اپنے نمونے کیلئے بنیادی اعدادوشمار برو-ئن کے دیداری خطوط سے اخذ کیئے تھے. ہم نے شیفر کی تکنیک کی بنیاد پر برو-ئن کے نمونے کی از سرِنو تعمیر کر کے نئے دیداری خطوط وضع کیئے ہیں اور نئے قمری ہلال کے دیدار کیلئے ایک نئے نیم ایمپیریکل نمونہ کی بنیاد ڈالی ہے. اس نئے نمونے کی تعمیر اس کام کا ایک اہم حاصل ہے. تمام نمونوں کو گزشتہ باب میں استعمال کیئے جانے والے مشاہداتی ریکارڈز کی بنیاد پر جانچا گیا. اس کام کے دوران حاصل کیئے جانے والے نئے نمونے کو جدید دور کے تمام نمونوں سے مشاہداتی ریکارڈز میں مثبت دیداری مشاہدات کی تعداد سے مطابقت کے معاملے میں بہتر پایا گیا. ہر نمونے کی کامیابی کے تقابلی جائزہ پر بحث بھی اسی باب میں شامل ہے.

باب چہارم کے اختتام میں ایک لائحہ عمل پیش کیا گیا ہے جو نئے ہلال کے دیدار کے دعووں کی صحت کو ایک نیم ایمپیریکل نمونے اور چمک کے فرق کے نمونے کی بنیاد پر پرکھنے کیلئے استعمال کیا جاسکتا ہے. ایسے لائحہ عمل کی وضاحت اس لیئے کی گئی ہے کہ نئے ہلال کے دیدار کے ایسے دعوے بھی ریکارڈز میں ملتے ہیں جو ایک نیم ایمپیریکل نمونے سے مطابقت نہیں رکھتے. ایسے دعووں میں ایک نیم ایمپیریکل نمونہ بغیر کسی آپٹیکل مدد کے دیدارِ ہلال کی اجازت نہیں دیتا مگر چمک کا فرق دیدار کے حق میں ہے. اس کی وجہ یہ ہے کہ ایک نیم ایمپیریکل نمونہ مقامِ مشاہدہ کی سطحِ سمندر سے بلندی اور موسمی حالات کا احاطہ نہیں کرتا جبکہ چمک کے فرق کا نمونہ ان عوامل کا احاطہ کرتا ہے.

نظریاتی عوامل کے علاوہ ایک ریاضیاتی نمونہ کو اطلاقی قوت کا حامل بھی ہونا چاہیئے. جن ریاضیاتی نمونوں کا احاطہ اس کام میں کیا گیا ہے یا جو نیا نمونہ متعارف کرایا گیا ہے ان کا اوّلین اور بنیادی اطلاق تو

یہ ہے کہ نئے قمری ہلال کے اوّلین دیدار کے حالات معلوم کیئے جا سکیں اور دیدارِ ہلالِ قمر کے دعووں کو پرکھا جا سکے. اس اوّلین اطلاق کے علاوہ باب پنجم میں نیم ایمپیریکل نمونوں کی بنیاد پر ایک ایسی تکنیک وضع کی گئی ہے جس کی مدد سے نئے ہلالِ قمر کی طوالت معلوم کی جا سکے. نیا ہلالِ قمر ہمیشہ 180 درجے سے کم طویل ہوتا ہے. یہ حقیقت صدیوں سے زیرِ مشاہدہ رہی ہے اور بیسویں صدی میں اس کی وجوہات تجویز کی گئیں ہیں. بہرحال ہماری مجوزہ تکنیک اپنی نوع کی پہلی تکنیک ہے جو مشاہدہ میں آنے والے ہلالِ قمری کی طوالت کا حساب لگانے کیلیئے ایک سادہ اصول فراہم کرتی ہے. مزید برآں نیم ایمپیریکل نمونوں کا اطلاق گزشتہ سات برسوں میں پاکستان میں عملی مشاہداتی قمری تقویم کو پرکھنے کیلئے بھی کیا گیا ہے. مطالعاتی دور میں نمونوں اور عملی تقویم میں مطابقت 95% رہی. مطابقت کی اتنی بلند شرح سے حوصلہ پاتے ہوئے پاکستان کیلیئے 'پیشنگو مشاہداتی قمری تقویم' پیش کیا گیا ہے.

آخری باب میں پوری کاوش کا ایک خلاصہ اور گفتگو پیش کی گئی ہے. اہم معاملات کو خصوصی اہمیت دیتے ہوئے زیرِ نظر میدان میں مستقبل کیلئے تحقیق کی راہوں پر نظر ڈالی گئی ہے.

CONTENTS

Chapter No. 1

INTRODUCTION

Since the ancient times the appearance of new lunar crescent marked the beginning of a new month. With the development of civilizations organizing time for extended periods into weeks, months and years, the lunar phase cycles lead to the evolution of calendars. Therefore the problem of determining the day of first sighting of new crescent moon attracted human beings. It involves consideration of a number of astronomical as well as other factors. On the other hand the problem of observing the new lunar crescent at an earliest possible moment is challenging for both amateurs and professional astronomers. The astronomical parameters on which the solution of this problem is based are briefly discussed in the first article of this chapter.

The circumstances and parameters associated with the problem of sighting new lunar crescent greatly vary with the varying position of observer on the globe. The affects of the geographical location on the problem is briefly discussed in the next article. Attempts to determine criteria for the determining the first visibility of new lunar crescent at any place appeared as early as the Babylonian era (Fatoohi et al, 1999, Bruin, 1977, Ilyas, 1994a). Significant advances were made by Muslims and Arabs during medieval period. A brief account of these ancient and medieval efforts is discussed in third article of this chapter.

Since antiquity changing phases of the Moon and a complete cycle of these variations has been used as means of keeping account of calendars. Ilyas has given a detailed account of the history of the science of lunar crescent visibility and the Islamic Calendar (Ilyas, 1994a). Dogget has discussed the history and the development of Calendars (Dogget, 1992). Reingold & Dershowitz have presented a comparative study of more than twenty calendars of various types both ancient and

modern (Reingold & Dershowitz, 2001). A number of other authors have contributed on related issues (Birashk, 1993, Ilyas, 1997, Odeh, 2004 etc.). As right from the beginning the calendars are associated with the cycles of the heavens, the same are reviewed along with the associated calendars in the next article of this chapter with special emphasis on the Islamic Lunar calendar.

During the modern times it was only in the last quarter of the 19th century that western astronomers started exploring the problem of earliest sighting of new lunar crescent. The last article is a brief survey of literature that described and developed various criteria or models for the solution of problem of determining the day of the first visibility of lunar crescent at any place on the globe during 20th century.

1.1 PARAMETERS FOR VISIBILITY OF NEW LUNAR CRESCENT

The Moon, the only natural satellite of the Earth, is going round the Earth in an orbit that is a highly irregular ellipse. The irregularities in the path of the Moon are due to the fact that its motion is governed by not only the gravitational pull of the Earth, but is also affected by many of the neighbouring celestial objects (Danby, 1992). With the Earth's varying distance from the Sun, the effect of the Sun's gravitational pull varies substantially with the relative positions of the Earth and the Moon. Moreover, the affects of the configuration of the whole Solar System (positions of all the major planets and the major asteroids) on the motion of the Moon is not negligible. Thus, according to the "Ephemirides Lunaires Parisiennes" popularly known as Chapront's lunar theory ELP-2000/82 (Chapront-Touze & Chapront, 1983, Chapront-Touze & Chapront, 1991), for the best possible precision in the longitude, latitude and the distance (between the Moon and the Earth), there are required as many as 35,227 periodic terms. On the other hand, for determining the position of the Sun to a similar degree of accuracy one requires 2,425 periodic terms in view of the 'Variations Seculaires des Orbites Planetaires', the French planetary theory known as VSOP87 (Bretagnon & Francou, 1988, Meeus, 1998). The

accurate determination of the positions of the Sun and the Moon is the first step towards exploring the circumstance of the visibility of the new lunar crescent. Handling such a large number of periodic terms makes this first step very crucial. The rest of the study depends on the relative positions of the Sun and the Moon near the horizon at and after the sunset for any place on the Earth.

These theories determine the celestial ecliptic coordinates of the solar system objects that are based on spherical polar coordinates. The fig 1.1.1 shows the spherical polar coordinate system. The origin of the system O is either the centre of the Earth or that of the Sun. The xy-plane is the plane of ecliptic, the plane in which the Earth orbits round the Sun. The x-axis points in the direction of the Vernal Equinox γ which is the point of intersection of the Ecliptic (path of the Earth around the Sun) and the celestial equator (whose plane coincides with the plane of the terrestrial equator). P is the object (the Moon in geocentric system or the Earth in Heliocentric system) whose spherical polar coordinates (ρ, θ, φ) are $\rho = |OP|$, $\theta = \angle XOP'$ and $\varphi = \angle ZOP$, where P' is the projection of the position P of the object on to the xy-plane. In the celestial ecliptic coordinates the celestial longitude $\lambda = \theta$ and the celestial latitude $\beta = 90^0 - \varphi$ ($= \angle POP'$). When the heliocentric ecliptic coordinates of the Earth are evaluated using VSOP they are then transformed into the geocentric ecliptic coordinates of the Sun. The conjunction or the Birth of New Moon occurs when $\lambda_M = \lambda_S$.

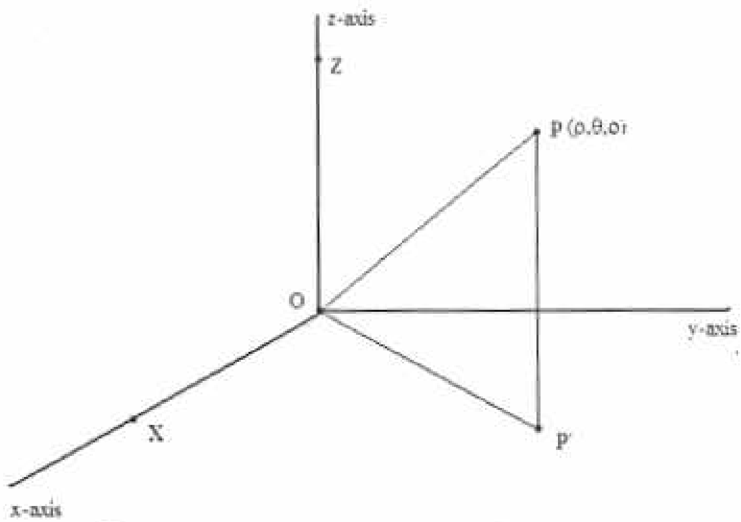


Fig. No. 1.1.1

Suppose $(r_m, \lambda_m, \beta_m)$ and $(r_s, \lambda_s, \beta_s)$ are the precise distances, ecliptic longitude and the latitude of the Moon and the Sun, respectively, referred to the mean equinox of the day of conjunction. For any location on the Earth with terrestrial coordinates, l the terrestrial longitude of the place and ϕ is the terrestrial latitude, the first step in the determination of the visibility of the new lunar crescent, is to determine the actual dynamical time (TD or TT) T_c, of the conjunction. Next, one requires considering the local times of setting of the Sun and the Moon. If T_s and T_m (Coordinated Universal Time TUC) be the times of the local sunset and the moonset, then the new crescent may be visible only if $T_s < T_m$. This leads to the parameter LAG = $T_m - T_s$.

Using the ecliptic coordinates of the sun and the Moon recalculated for the T_s or any other moment of time, the equatorial coordinates of the two bodies (α_m, δ_m) and (α_s, δ_s) can be obtained. Where α, the right ascension is the displacement of the object from vernal equinox along the celestial equator (in the same quadrant as λ), and δ is the declination or the displacement of the object from the plane of the equator. Local Hour Angle H is then obtained from the difference of the local Sidereal Time (LST) and the right ascension. This finally gives the local horizontal coordinates, azimuth (A), the displacement of the object from the direction of the North towards East along the horizontal in local sky and the altitude (h), the height of the object above horizon. After adjusting for the refraction and the height of the observer's location above see level the topocentric coordinates (A_m, h_m) and (A_s, h_s) of the Moon and the Sun, respectively are obtained.

In almost all the models for earliest moon-sighting, the ancient as well as the modern, the difference of azimuths ($DAZ = |A_s - A_m|$, called relative azimuth) and that of altitudes (ARCV = $h_m - h_s$, called arc of vision) as shown in the fig 1.1.2, at the time of local sunset T_s. The arc of vision is also termed as arc of depression. The fig. 1.1.2 also shows the separation between the Sun and the Moon that is known as the arc of light abbreviated as ARCL and is also known as elongation.

Apart from the relative azimuth, the arc of vision and the arc of light, the criteria for earliest visibility of lunar crescent requires to take into consideration a number of other parameters. One such parameter is the Age of the Moon (AGE) = $T_s - T_c$, defined as the time elapsed since the last conjunction till the time of (the sunset or the any other relevant time) observation. Another important factor is the Width of Crescent (W) that depends on the distance of the Moon. As the distance of the Moon from the earth varies from around 0.34 million km to around 0.4 million km the semi-diameter of the lunar disc varies from 15 arc minutes to 16.5 arc minutes.

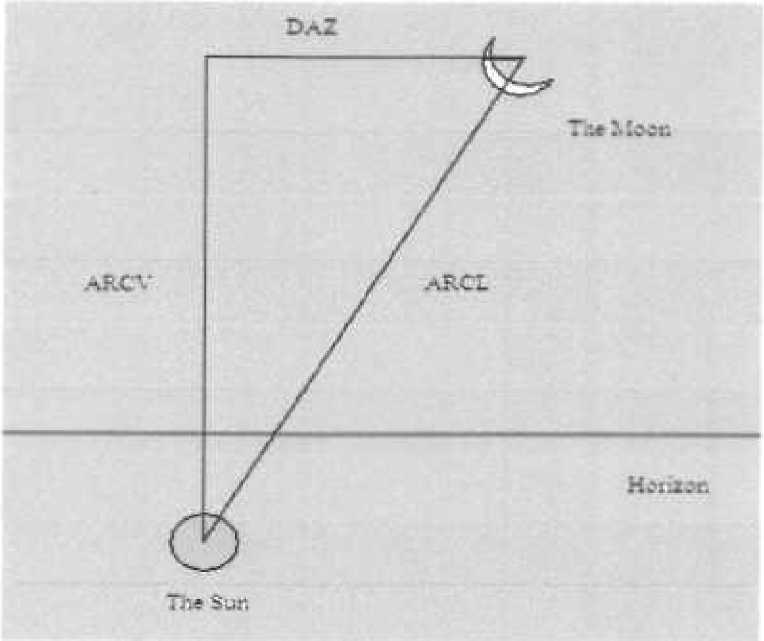


Fig. 1.1.2

Thus, if the Moon is closest to the earth at the time of observation the crescent would be widest and thus brightest. Width of the crescent is directly proportional to the Phase (P) of the Moon that is a function of the ARCL. A complete list of all these parameters is as follows:

1.	Time of conjunction	T_c
2.	Time of Sunset	T_s
2.	Time of Moonset	T_m
3.	LAG	$T_m - T_s$

4.	Best Time of Visibility	T_b
5.	Age of the Moon at T_b	AGE
6.	Arc of Vision	ARCV
7.	Relative Azimuth	DAZ
8.	Arc of Light (Elongation)	ARCL
9.	Phase of Crescent	P
10.	Width of Crescent	W

A computational strategy may be to start by determining the time of Birth of the New Moon or conjunction of the Moon with the sun. For any place on the globe determine the time of sunset that follows the time of birth of new Moon. For this moment compute the geocentric ecliptic coordinates of both the Sun and the Moon. Transform these coordinates to the local horizontal coordinates. Thereby compute the LAG, the Age, ARCL, ARCV, DAZ and the width W. The details of all these computations shall be discussed in chapter 2.

1.2 SIGNIFICANCE OF GEOGRAPHICAL LOCATION

For the problem of the earliest visibility of the New Crescent Moon the orientation of the paths of the Sun and the Moon relative to each other and relative to the horizon is significant. They change season to season as well as year to year and also depend on the latitude of the place. Due to the axial rotation of the Earth every object in the sky appear to travel along a circular path (The Diurnal Path) extending from eastern horizon to the western horizon. This path lies in a plane parallel to the plane of the equator. The objects in our sky whose declination is constant (stars and other extra-solar-system objects) always remain in a fixed small circle in our sky with North Celestial Pole being their Pole i.e. their diurnal paths are not only fixed but lie in planes parallel to each other. The plane of the orbit of the earth around the Sun is inclined to the plane of the Equator at an angle of around 23^0 26' 26" therefore the declination of the Sun in our sky varies from 23^0 26' 26" south to 23^0 26' 26" north over a year. The declination of the Moon varies from around 28^0 35' south to 28^0 35' north during a lunation period. Thus

during a day neither the path of the Sun nor the path of the Moon can be considered a small circle that have their poles at the North celestial Poles i.e. the diurnal paths of the Sun and the Moon do not lie in planes parallel to the planes of the diurnal paths of stars. For places on Earth with latitudes greater than 66^0 34' (north or south) there are days during every year, when Sun remains below the horizon all day (or above the horizon all day). Similarly for places with latitudes greater than 61^0 25' north or south, there are days during every lunar month when the Moon is below the horizon all day (or above the horizon all day).

Apart from the places close to the equator both the Sun and the Moon may travel very close to the horizon. For places close to the equator the Celestial Equator passes close to the zenith and therefore the paths of the Sun and the Moon remain high in the sky. But in places with higher latitudes the Celestial Equator is closer to the Horizon so that the paths of the Sun and the Moon may be close or even below the horizon. The figure no. 1.2.1 shows the relative orientation of the ecliptic and celestial equator in comparison to the horizon for a place with high latitude at the time of local sunset at around vernal equinox (ecliptic in blue) and around autumnal equinox (ecliptic in pink) .

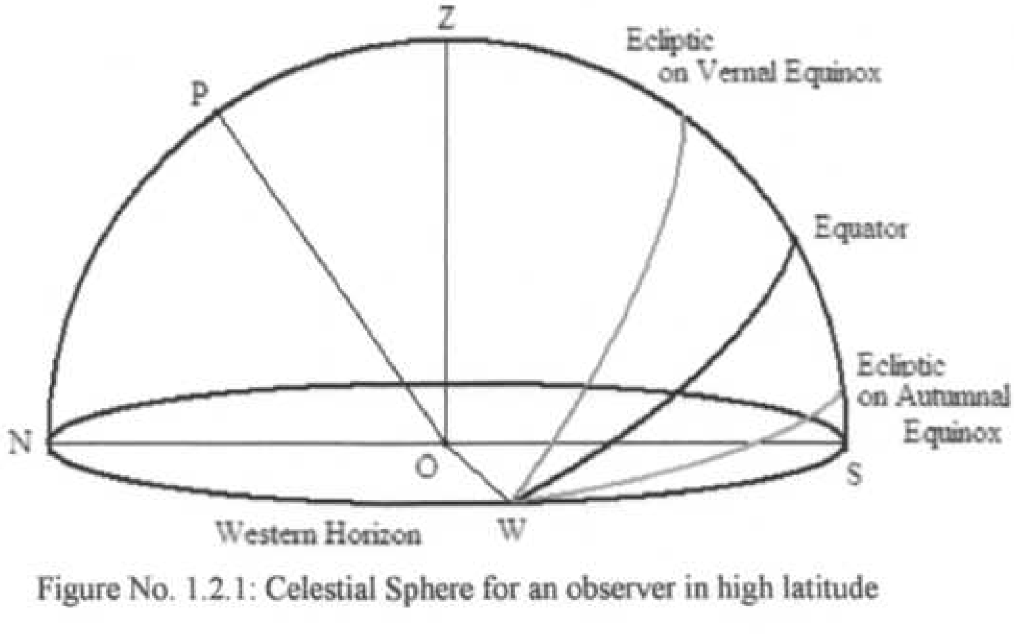


Figure No. 1.2.1: Celestial Sphere for an observer in high latitude

This figure clearly shows that for high latitudes when the declination of the Sun is south (winter in the northern hemisphere between Summer solstice and the Autumnal Equinox or between autumnal equinox and the winter solstice) the path of the Sun and consequently that of the Moon remain very close to the horizon. In these conditions the new Moon after conjunction either remain below horizon or very close to the horizon making it impossible to see even after two or three days from conjunction. The situation becomes worse if during this part of the year the declination of the Moon is south of the declination of the Sun, particularly close to Autumnal Equinox (around September and October). Between vernal equinox and the Summer Solstice (around June) the ecliptic is relatively high and the paths of the Sun and the Moon are higher too, making it easier to see relatively younger crescents. The situation is reversed seasonally for the southern hemisphere.

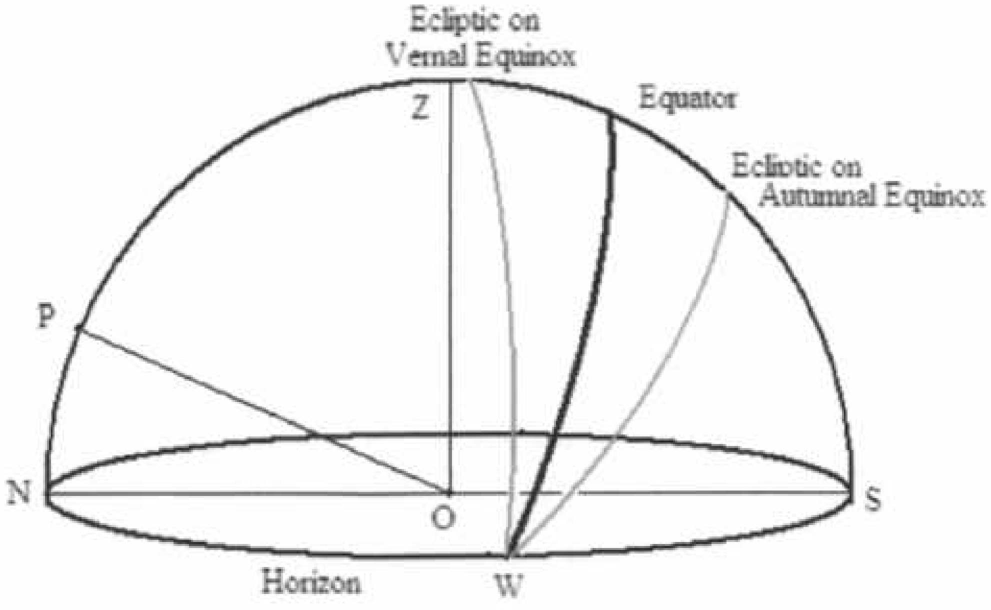


Figure No. 1.2.2: Celestial Sphere for an observer in low latitude

Similarly the figure no. 1.2.2 shows the relative orientation of the ecliptic and celestial equation in comparison to the horizon for a place with lower latitude at around vernal equinox (ecliptic in light blue) and around autumnal equinox (ecliptic in light pink). This figure also show that close to autumnal equinox the paths of the Sun and the Moon are relatively close to the horizon and if the Declination of the Moon is south of that of the Sun the condition are not very good for sighting of a relatively young crescent.

This is the main reason behind considering Age of Moon as not a very good indicator for the visibility of crescent. In spring and summer very young age crescent can be seen and during the autumn and winters very old crescent may escape sighting in the northern hemisphere. The situation is reversed seasonally for the southern hemisphere.

When the Moon is west of the Sun (celestial longitude of the Moon is more than that of the Sun) in our sky it is Old Moon catching up with the Sun. The Old Crescent can be seen in the mornings before the sunrise. After the birth of New Moon (celestial longitude of the Moon becomes just greater than that of the Sun) the Moon reaches east of the Sun and can now be seen just after sunset. However depending on the declinations of the Sun and the Moon and the location of the observer the new crescent may set well before the sunset in which case it is impossible to see the new crescent. Thus the first and the most important criterion for the visibility of the new crescent Moon is that the Moon sets after the sunset. Alternately the criterion for the visibility of the last crescent is that the Moon rises before the sunrise.

When the crescent is very close to the sun it remains invisible due to the fact that the atmosphere close to the sun remains highly illuminated even after the sunset. Therefore there has to be a minimum threshold separation or Elongation between the Sun and the Moon below which the crescent cannot be seen. This minimum or threshold elongation varies month to month as the distance between the Earth and the Moon keeps varying. When closest (at apogee) the Moon is around 350000 km from the Earth and at the farthest (perigee) it is around 400000 km. Thus a New Moon at apogee may be seen when its elongation from the Sun is small and a New Moon at perigee may not be seen at much larger elongation. This is due to the fact that closer is the Moon the larger and brighter it appears in our sky and when it is farther it appears smaller and less bright at the same elongation. Thus another important criterion is that the New Moon can be seen for a combination of certain optimum values of elongation and the distance of the Moon from the Earth. This combination results into the optimum value of the Width of crescent. A crescent with small elongation that appears larger and brighter at apogee may be seen

and a crescent with large elongation that appears smaller and fainter at perigee may not be seen.

1.3 RULES OF THUMB: THE ANCIENT & THE MEDIEVAL:

The earliest reference for any criterion for the visibility of new lunar crescent is attributed to the Babylonians (Fotheringham, 1910, Bruin, 1977, Schaefer, 1988a, Ilyas, 1994, Fatoohi et al, 1999 etc.). Most of the explorers attribute the following rule of thumb to the Babylonians:

"The new lunar crescent is seen when its age is more than 24 hours and the moonset lags 48 minutes behind the sunset".

It has been pointed out that the actual Babylonian criterion was much more sophisticated as compared to this simple rule (Fatoohi et at, 1999). It is either our lack of knowledge of their era or the missing historical records that has restricted our comprehension of their efforts. Authentic records of sighting of new crescent as young as 5 hours during Babylonian era exists (Fatoohi et at, 1999, Anderlic & Firneis, 2006) in literature. Similarly, in modern time crescents lagging only 35 minutes behind the sunset have been seen with age less than 20 hours. The data set of 463 observations considered during this work (to be discussed in detail in chapters 3 and 4) includes 31 cases when the crescent with LAG less than 48 minutes was reported to have been seen without any optical aid, whereas, 26 cases are there in which the age of Moon was less than 20 hours. Recent re-evaluation of records of the sightings of crescents in Babylon and Nineveh also show that crescents much younger than 20 hours and those lagging behind sunset much less than 40 minutes were seen (Anderlic & Firneis, 2006).

These recent computational efforts for the recorded observations of the Babylonian era clearly indicate that the rule of thumb associated with this era is an over simplification. It has been recently claimed that Babylonians had formulated a truly mathematical lunar theory which they used for predicting various parameters of the lunar

motion as founded in the lunar ephemeris they prepared (Fatoohi et al, 1999). According to these studies it is pointed out that the moonset-sunset lag alone could not have been used as the visibility criterion by the Babylonians. Babylonians system had the following criterion

Elongation (L) + moonset lagtime (in degrees) (S) > constant

In various studies the value of the constant is deduce from 17 degrees to anywhere around 23 degrees. However on the basis of the confirmed 209 positive sightings from year -567 to year -77 the value of the constant is deduced to be 21 degrees (Fatoohi et al, 1999).

Similarly most of the modern authors attribute rules of the thumb of the following type to the Muslims and Arabs of medieval time:

"*The relative altitude (ARCV) > 8^0 and moonset-sunset lag time > 42 minutes*".

It has been indicated that Muslims' realization that the Earth-Moon distance varies during one complete cycle of lunar phases and the minimum moonset lag time considered by Arabs varied from 42 minutes for Moon at perigee and 48 minutes for Moon at apogee (Bruin 1977).

Though this "rule of thumb" is more sophisticated as compared to the one attributed to the Babylonians still it does not provide the complete picture of the efforts of Arabs and Muslims. In the medieval times extensive use of Ptolemaic system and the spherical trigonometry developed by Arabs lead to the Lunar Ripeness function (that depends on local latitude of the place of observation and the celestial longitude of the sun and the Moon) as early as 10 century AD (Bruin 1977).

Babylonian criterion is indeed very simple for practical purposes and is supposed to have been empirical in nature. There has been no significant change in this till

relatively recent times, however, the earliest Hindu texts like *Panch Sidhantika* (AD 500) hints towards the importance of the Width of the lunar crescent (Bruin, 1977). Thus an elaborate system of calculations involved in determining the time of earliest visibility appears to have developed only around AD 500. More explicit mentions of these detailed calculations are found at various places in the early Islamic literature (Bruin, 1977).

One of the earliest Muslim Astronomer who developed the tables for ascertaining the lunar crescent's visibility was Yaqub Ibn Tariq (Kennedy, 1968). It has been reported in the literature (Bruin, 1977) that Ibn Tariq had recognized the importance of the Width (W) of the crescent. This not only shows that at that time the varying distance of the Moon had been realized but it also made it possible to improve upon the new crescent visibility criterion. Bruin has reported that Al-Biruni had the realization of the long and difficult calculations involved in the determination for the new lunar crescent visibility and in his "*Chronology*" recommended the work of Muhammad Ibn Gabir Al-Battani ("*Handbook of Astronomy*" translated to Latin by Nallino, 1903).

The simple criterion for earliest visibility of lunar crescent evolved since the times of Babylonians was passed onto the Muslims through Hindus with very little improvement. Motivated by the Quranic injunctions and the sayings of their Prophet Muhammad (PBUH) the problem of earliest sighting of lunar crescent was thoroughly investigated by the early Muslim astronomers of 8^{th} to 10^{th} century AD.

On the basis of realization of the importance of width various Arab astronomers concluded that for earliest visibility of crescent the minimum equatorial separation of the Sun and the Moon varies from 10^0 when the crescent is widest up to 12^0 when the crescent is narrowest. Such detailed calculations were worked out as early as 9^{th} century AD by Muslim astronomers associated with the Abbasid court of Al-Mamun. From amongst these astronomers Al-Batani knew that the criteria that age of moon should be more than 24 hours (or arc separation between the Sun and the Moon) is a good starting point but it is only an approximation. He believed that the ancient astronomers did not understand the phenomenon completely. According to Bruin, Al-Batani's computational work is a very elaborate system of mathematical calculations (Bruin, 1977).

This work is not intended to explore the history of Astronomy related to the earliest sighting of the new lunar crescent. The Babylonian and the medieval efforts shall not be explored from a historical perspective. We shall restrict our exploration only on the comparison of these efforts with those of the modern ones. However the medieval mathematical ideas shall be explored in more detail in chapter 3.

1.4 CALENDARS AND THE CELESTIAL CYCLES

Dogget defines calendar as "a system of organizing *time* for the purpose of reckoning time over extended periods" (Doget, 1992). It is a scheme for keeping an account of "days", "weeks", "months", "years", "centuries" and "millennia". The basic notion behind a calendar is organizing ***time*** out of its continuous flow that is independent of any scheme of its organization. Amongst the divisions of time in days, weeks etc. some are directly associated with the celestial cycles, the ***Diurnal,*** the ***Annual*** and the ***Lunar***. A complete daily revolution of the sky, the Diurnal motion, is referred to as a day. Technically an apparent ***solar day*** is the interval between two successive transits of the Sun at any place. Due to the motion of the Earth around the Sun the sky appears to revolve round the Earth very slowly (less than a degree per day) and one complete revolution of sky in this way is referred to as a ***year***. Technically a ***Tropical Year*** is the time interval between two successive passages of the Sun through vernal equinox, one of the points of intersection of the celestial equator and the ecliptic.

Each calendar is divided into years, years into months, months into weeks and days. Most of the known calendars that men devised had seven days in a week. In different regions and eras 4 to 10-day weeks have also been considered. However the number of days in a month has remained variable in different calendars and within a particular calendar. The scheme, if there is any, of different number of days in a month of a calendar is based on the type of calendar. Most of the known calendars are classified into three major types. Solar Calendars, Strictly Lunar Calendars and Luni-Solar Calendars. A brief description is reviewed in the following:

Solar Calendars are based on the annual motion of the Earth around the Sun. A "year" in such calendars is the "Tropical Year", defined above. Then the length of a "tropical year" (Dogget, 1992) is given by:

$$365.2421896698 - 0.00000615359T - 7.29\times10^{-10}T^2 + 2.64\times10^{-10}T^3 \quad (1.4.1)$$

T is the time in Julian centuries since the epoch J2000.0 given by:

$$T = (JD - 2451545.0)/36525 \quad (1.4.2)$$

where JD is the Julian Date which is the time in number of days elapsed since mean noon at Greenwich on January 1, -4712 in Julian Calendar. Currently the length of tropical year is 365.2421898 days or 365 days, 5 hours, 48 minutes and 45.2 sends. As it is not a whole number one year of a solar calendar consists of either 365 days or 366. In old Julian calendar every fourth year contained 366 days (i.e. a leap year) all other years contained 365 days. In the currently used Gregorian calendar the leap year rule is modified. A century year Y (like 1700, 1800 etc) which is a leap year in Julian calendar but is not completely divisible by 400 is not a leap year in Gregorian calendar. Thus in every 400 years there are 100 leap years in Julian Calendar whereas in Gregorian calendar there are only 97. The seasons and many other natural phenomena follow this solar cycle, specially harvesting times, the length of days, the times of transit, sunset, sunrise, etc. The Julian calendar was instituted on January 1, 45 BC by Julius Caesar, with the help of Alexandrian astronomer Sosigenes and was a modification of the Roman Republican and the ancient Egyptian calendars (Michels, 1967). Whereas the Gregorian calendar was necessitated due to the fact that during one and a half millennia the Julian calendar was displaced from the seasonal variations by as much as 10 days. Thus Pope Gregory XIII constituted a commission in 16th century AD for the calendar reforms. The main author of the new system was astronomer Aloysius Lilius of Naples (Coyne et al, 1983, Dutka, 1988, Moyer, 1982 and Michels 1967). When it was implemented officially, the date October 4, 1582 (Thursday) in the Julian calendar was followed by the date October 15,

1582 (Friday) in the Gregorian calendar. Thereby all date conversion algorithms have to keep account of this skipping of days in the solar calendar. Different countries, cultures and religious communities adapted to this modification at different time. It is therefore a high task for historians to keep track of the appropriate dates.

In every solar calendar there are twelve months. For simple arithmetic reasons there could have been seven months of 30 days and five of 31 days (in a normal year and six of 30 days plus six of 31 days in a leap year). However in practice since the time of Greeks it was known that the winter half of a normal year has 181 days whereas the summer half contains 184 days. The reason for the same is the faster motion of the Earth the perihelion, the Earth closest to the Sun that occurs on around January 4th. Starting with January every alternate month is of 31 days till July, February is of 28 days (or 29 days for a leap year) and the rest are of 30 days each. The alternate months of 31 days and 30 days continues then from August to December.

In a Strictly Lunar calendar, that is based on the lunation period in one year there are either 354 days or 355 days. The current average of the lunation period is 29.530589 days or 29 days, 12 hours, 44 minutes and 2.9 seconds (Astronomical Almanac, 2007). However this average is changing. According to the lunar theory of Chapront-Touze' and Chapront these variations are accounted for by the following expression (Chapront-Touze' and Chapront , 1988):

$$29.5305888531 + 0.00000021621 * T - 3.64\times10^{-10} * T^2 \text{ Days} \qquad (1.4.3)$$

where T is given by (1.4.2). Any particular phase cycle may vary from the mean by up to seven hours. Thus this period varies from around 29.2 days to more than 29.8 days from month to month. Therefore in all lunar calendars the number of days in a month is either 29 or 30. In Arithmetic Lunar calendar there are alternate months of 29 and 30 days. In observational lunar calendars there can be as many as three consecutive months of 29 days each and as many as four consecutive months of 30 days each (Ilays, 1994). In an

Arithmetic calendar there are either 6 months of 29 days and 6 months of 30 days, (a normal year) or 5 months of 29 days and 7 months of 30 days (a leap year).

There is no fixed rule for leap years in an observational lunar calendar, there can not be any. However in the Arithmetic Lunar calendar out of 30 years' cycle 11 years are leap years (355 days) (Ilyas 1994, Reigold & Dershowitz 2001, Tsybulsky, 1979). The rule is that the year number Y is a leap year if:

$$((14 + 11 * Y) \bmod 30) < 11 \qquad (1.4.4)$$

Otherwise the year is not a leap year. In such an arithmetic lunar calendar all the odd numbered months contain 30 days and the even numbered months contain 29 days each. In the leap year a day is added to the twelfth month. In general, a strictly lunar calendar advances by 11 days against the solar calendars. Therefore the seasons and all other phenomena that depend on the solar cycle do not follow a strictly lunar calendar. The month number 9 (Ramadan) in the Islamic calendar may fall in winter, summer, autumn etc.

The Luni-Solar calendars are basically lunar but to keep track of the seasons in place of adding single days in a leap year a whole month is added (intercalation) to follow the solar cycle. The Hebrew and the Hindu calendars fall into this class (Reingold & Dershowitz 2001, Bushwick 1989, Sewell & Dikshit 1896, al-Biruni 1000, al-Beruni 1030). In case of Hindu calendars (they have both the solar and the luni-solar calendars) a lunar month is "intercalated" whenever it fits in to a complete solar month. In case of the Hebrew luni-solar calendar an additional month of 30 days is intercalated before the usual 12th month of the year. The rest of the details of these calendars is more of the social and religious nature and is not in line with the present work.

Calendrical calculations for each calendar have their own sophistications, but being phenomenological the observational lunar calendar is most challenging. A lunar month has to begin with the actual sighting of the new lunar crescent and the conditions

of its sighting greatly vary not only longitudinally on the globe but depend on the latitude of places. Thus an observational lunar calendar may vary along the same longitude. Apart from calendrical aspects the problem of sighting very young crescent is one of the most exciting and challenging observations for both the amateur and the professional astronomers. Besides, the prediction of the visibility of a particular new crescent at a particular place is a long and interesting computational exercise. The same was realized as early as the medieval times by the great astronomers Al-Khwarizmi, Al-Battani, Al-Farghani etc (Bruin 1977). Moreover, the prediction for naked eye observation has two more complicated issues under scrutiny in the latest research. One of these is biological, the ability of human eye to contrast the dimly illuminated crescent in the bright twilight. The other aspect is of physical nature of atmospheric conditions that can badly affect the visibility and the contrast. In this work emphasis is more on the astronomical aspects of the earliest sighting of the new lunar crescent and atmospheric condition are only partly considered.

Opinions are divided as to the origin of the custom of counting years in any form in the Arabian Peninsula. According to Al Hazwi it started as soon as the children of Prophet Adam multiplied and spread around the world (Rosenthal 1952, Faruqi 1979). A calendar was originated when the Himyarites adopted one with an epoch that marked the beginning of the reigns of Tubba. Generally it is believed that the practice of 12 lunar months to a year existed in pre-Islamic Arab calendars since the time of construction of Kaaba by the prophet Abraham and continued in Islam (Ilyas, 1994). The names of the months and their sequence were the same as those used in the current Islamic lunar calendar followed by more than one fifth of the total population of the world.

From historic perspective the importance of the lunar calendar in the pre-Islamic Arabia was the pilgrimage to Kaaba (Haj) that falls in the month of Zul-hajjah, the 12th month of the Islamic and pre-Islamic lunar calendar. Although this event was a purely religious event, it was also important for trade and business with lots of goods exchanging hands. It was this economic activity that was badly affected as the lunar year advanced through seasons. Procurement of crop and the availability of sacrificial animals

greatly varied season to season. The reason that the intercalation was introduced in the Arabian Peninsula was this economic activity rather than any astronomical reason. "Qalmas" a native of Mecca is reputed to be the first person assigned to determine the dates for the coming years' pilgrimage and whether the intercalation was due or not (Hashim, 1987, Ahmed, 1991).

Since the early days of the inception of lunar calendar in the Arabian Peninsula four months including the Zil Hajjah were considered sacred and wars were prohibited during these sacred months. The custom carried over to the post Islamic era in the Islamic culture. As with the Roman calendar the intercalation was abused in Arabia in order to change the sacred months into non-sacred months and vice-versa. At the same time the lunar calendar used in Madina remained in its original 12 months a year form.

Muslims followed the calendar of Mecca in the beginning. But after the Prophet Muhammad migrated to Madina along with his companions, Muslims adopted the calendar used in Madina. After the conquest of Mecca by Prophet Muhammad, Muslims continued to use the calendar of Madina but the calendar of Mecca ran in parallel. With the last pilgrimage to Mecca of Prophet Muhammad in the 10^{th} year after migration to Madina (AD 632) frequently abused practice of intercalation was abolished through a Quranic injunction. The practice of starting a lunar month with the first sighting of new crescent Moon was authenticated by Quranic injunction and the saying of Prophet Muhammad, with particular emphasis on beginning and the ending of the month of fasting and the month of the pilgrimage.

With adoption of purely lunar calendar with a lunar month beginning with the first sighting of the new lunar crescent through Quranic injunction and the sayings of Prophet Muhammad the evolution of Islamic lunar calendar began. As for any calendar one requires a starting point of time (epoch) or beginning of an era, for counting years, the people of Madina are believed to have used an epoch as some times a month or two after Prophet Muhammad migrated to Madina in AD 632 (Ilyas, 1994). However a more widely accepted time of official adoption of Hijra as the beginning of the Islamic era is

AD 637 during the caliphate of Umer bin Khattab. Whatever be the time of adoption, the Islamic calendar, or the Hijri calendar starts with Friday 16^{th} July 622 AD on Julian calendar which according to arithmetic lunar or Istalahi calendar is 1^{st} day of Muharram (1^{st} month of an Islamic year) of the Islamic year 1 (Reingold & Dershowitz, 2001). The official date of adoption of this era and calendar is 1^{st} Muharram 11 AH (after Hijra) (Ilyas, 1994a).

Simple schemes or zeroth order approximations based on long term averages had been devised to prepare long term calendar in view of inter-conversion of Islamic dates and the dates in other luni-solar and solar calendar dates.

Islamic legal system called Sharia', is the source of Islamic time-keeping system, while legal precepts are safeguarded by seeking assistance from the scientific knowledge. Till the time of sacking of Baghdad by Halagu Khan in AD 1258, Islamic law had evolved clear guidelines for calendrical considerations (Ilyas, 1994). The calendrical guidelines evolved under the Islamic law can be outlined as follows:

i) Length of a lunar month is either 29 days or 30 days.

ii) Length of a lunar year is either 354 days or 355 days.

iii) There can be a maximum of 4 consecutive months of 30 days each or 3 consecutive months of 29 days each.

iv) Each new month begins with first sighting of new lunar crescent over the western horizon after the local sunset.

v) Attempts should be made on 29^{th} of each month for sighting of new crescent. If it is not seen on the 29^{th} due to any reason (astronomical conditions or weather constraints) the month should be completed as of 30 days.

vi) Visual sighting report must be supported through a witness report.

vii) The person involved in reporting must be reliable, adult, truthful, sane and with good eyesight. If it is proved that the person providing witness has purposely misled the person must be punished.

viii) The visual sighting report should not conflict with basic scientific knowledge. Testing of evidence of sighting on scientific grounds include checking of the shape of crescent, its inclination, position in sky, altitude, time of observation and sky conditions.

ix) Sighting should be carried out in an organized way for each month.

x) Accumulation of errors (particularly in view of considering a month to be of 30 days due to invisibility of the crescent on the 29th of consecutive months) has to be avoided whenever the new crescent is sighted on 28th of a month. In such cases corrections are made to beginning of that month.

As the Islamic law and the Quranic injunctions depend heavily on the first sighting of new lunar crescent the early Islamic state placed special emphasis on the research in the field of Astronomy. Consequently enormous contributions were made in the development of science of the earliest visibility of new crescent and prediction criterion for the same.

1.5 CONTRIBUTION OF 20TH CENTURY ASTRONOMERS

The modern development of the science of earliest moon-sighting begins with the observational work of Schmidt who recorded a large number of new and old crescents from Athens in the later half of the 19th century. On the basis of this observational data Fotheringham (Fotheringham, 1910) and Maunder (Maunder, 1911) developed the observational criteria of earliest visibility of new lunar crescent. A similar work is

reported in the Explanation to the *Indian Astronomical Ephemeris* that is based on Schoch (Schoch, 1930). In all these works for determining the day of the first visibility of new crescent, ARCV (arc of vision) is shown to be a second degree polynomial function of DAZ (relative azimuth). These early efforts are only empirical in nature as the criteria developed are based on fitting the data so that most of the observations are consistent with the criteria.

These methods do not take into consideration the width of crescent. Bruin (Bruin, 1977) considers the importance of crescent width and his model describes ARCV (in terms of altitude of crescent above horizon plus the solar depression below the horizon) as a function of the crescent width. The model of Bruin, however, takes the width as a function of the ARCV and DAZ and a fixed Earth-Moon distance (in terms of fixed semi-diameter of the lunar disc). He was also the first who considered physical aspects associated with the problem like brightness of sky and that of the Moon. Thus the model due to Bruin is the first theoretical model of the modern times. Using the models for the brightness of the full Moon as a function of altitude (Bemporad, 1904) and the sky brightness during twilight (Koomen et. al., 1952, Siedentopf, 1940) Bruin develop the visibility curves relating the altitude of crescent with the solar depression. Along these curves the brightness of crescent (modelled on the basis of full Moon) is at least as much as that of the twilight sky. He also developed curves showing relation between ARCV and solar depression that later proved to be crucial for further modelling of earliest visibility criteria.

Yallop has gone for further improvement and considers crescent width as a function of ARCV, DAZ and the actual semi-diameter of the Moon (that depends on the Earth-Moon distance) at the time of observation (Yallop, 1998). With developing a model for the best time of visibility Yallop's model also considers ARCV as a third degree polynomial function of the actual crescent width at the best time of visibility. Such polynomial is obtained by applying least square approximation on a basic data set. This basic data set was deduced by Yallop from the limiting visibility curves of bruin by selecting ARCV for a given width from the minimum on the corresponding limiting

visibility curve. So far the model due to Yallop is the most authentic and dependable astronomical criteria for expected earliest visibility of new lunar crescent. The model due to Yallop is a semi-empirical model based on the theoretical considerations of Bruin and a criterion on the basis of a basic data deduced from Bruin's visibility curves.

The most extensive treatment of the physical aspects associated with the problem is due to Schaefer (Schaefer, 1988(a), 1988(b)) who has considered the problem of brightness contrast rigorously. He also realizes importance of the physiological aspects like the ability of human eye to sense the limiting contrast. Schaefer's model is a purely theoretical model. Recently Odeh (2004) has claimed to have developed a new criterion for the earliest visibility of new lunar crescent but his criterion is just another form of Yallop's criterion.

Other authors have contributed on related issues (Ashbrook, 1972a, 1972b, Caldwell and Laney, 2000, Fatoohi et. al., 1998, Ilyas, 1983a, 1983b, 1984a, 1984b, 1985, 1988, 1994a, 1994b, McNally, 1983, Schaeffer, 1988a, 1988b, 1996, Schaeffer et. al. 1993, Sultan, 2005, Qureshi & Khan, 2005, 2007 etc). In this work, each ancient and modern mathematical model for determining the day of the first sighting of new lunar crescent is explored. A comparative study of these models is carried out and each criterion is transformed into a one parameter visibility criterion like that due to Yallop. The resulting calendarical implications are also explored.

It has been pointed out in the beginning of the chapter that the problem of determining the day when the new lunar crescent would be visible has its calendarical implication. The earliest sighting of new lunar calendar is also a challenging task for both amateurs and professional astronomers. In view of this the chapter has highlighted:

- Parameters on which visibility of new lunar crescent depends.

- The efforts of the astronomers in the ancient and the medieval times.

- The dependence of the calendars on the cyclic motion of the Moon and the Sun.

- Lunar calendar of the Muslims especially as their believes strictly emphasise on the first sighting of the new lunar crescent for starting and ending lunar months.

- An account of the efforts of the astronomers of the modern times to address the problem of determining the first day of sighting of new lunar crescent.

In the background of these efforts we have explored all the old and the modern methods, mathematical model or criterion that require to be satisfied for the first visibility of the new lunar crescent. During this exploration the results of these models are compared and modifications have been suggested where ever possible. Further the most authentic of the models have been used to explore the observational lunar calendar followed in Pakistan. Based on the results of these explorations a future "observational lunar calendar" for Pakistan is computed.

Chapter No. 2

ASTRONOMICAL ALGORITHMS & TECHNIQUES

For the determination of the precise location of the objects in the Solar System, particularly the Sun and the Moon, the French planetary theory VSOP87, (Bretagnon & Francou, 1998) and the lunar theory ELP-2000 (Chapront-Touzé and Chapront, 1983, 1991) are well suited. A number of software have been developed for the simulation of celestial phenomena based on these theories and similar other works. In the current study the same theories have been used to follow the positions of the Sun and the Moon.

Moreover, to convert the theories into computational techniques, mathematical techniques, tools and algorithms have been explored. Much of the computational work is based on the algorithms developed by Meeus (Meeus, 1998) but a substantial amount of work on computational algorithms has been done independently. For a thorough understanding of the computational tools the problem of time is explored and discussed in detail. In this regard the contribution of a number of authors has been studied in as much detail as is required (Aoki et. al., 1988, Borkowski, 1988, Clemence 1948, 1957, de Jagger and Jappel (Eds.), 1971, Dick, 2000, Essen and Parry, 1995, Essen et. al., 1958, Guinot and Seidelmann, 1988, Markowitz et. al. 1958, Muller and Jappel, 1977, Munk and MacDonald, 1975, Nelson et. al., 2001, Newcomb, 1895, Sadler (Ed.), 1960, Seidelmann and Fukushima, 1992, Spencer, 1954, Stephenson and Morison, 1984, 1995, Stephenson, 1997, Wells, 1963, etc.).

The output of these efforts is a computer program for analysis of first visibility of lunar crescent named Hilal01 written in C-language discussed at the end of the chapter.

2.1 INTRODUCTION

For the determination of the visibility conditions of New lunar Crescent (or the oldest lunar crescent) over a local horizon, the first task is to determine the Universal Time (UT) and date of the geocentric conjunction of the Moon and the Sun or the Birth of New Moon. In its motion around the Earth the Moon travels around 12 degrees from west to east every day and takes over the Sun in around every 29.5 days on the average. When the Moon is very close to west of the Sun it appears before the sunrise and when it is east of the Sun it appears just after the sunset. The lunar crescent is very rarely visible on the day of the conjunction. Danjon Limit (Danjon 1932, 1936) has been interpreted as a limit on the minimum elongation of the visible lunar crescent. According to this limit the lunar crescent is not visible if the elongation is less than 7 degrees (Doggett & Schaefer 1994, Schaefer 1991, Yallop 1998). The maximum elongation of the Moon at the time of the geocentric conjunction is same as the inclination of the lunar orbit from the plane of ecliptic (5^0 9'). When the Moon takes over the Sun at its maximum elongation the minimum time it takes to move from being 7^0 from the sun (on the eastern side) to be 7^0 again (on the western side) is around three quarters of a day. Thus it is theoretically possible that the crescent is first seen on the day of conjunction or is last seen on the day of conjunction. Theoretically it is also possible that if the crescent is last seen on the day of conjunction and then the new crescent is seen on the day after, or the crescent is last seen on the day before the conjunction and then the new crescent is seen on the day of the conjunction. In these cases the crescent remains invisible for "one-a-half" day. However none of these theoretical possibilities are realized in practice too frequently. Mostly the crescent remains invisible for "two-and-a-half" days at least. These conditions depend on the observer's location.

Once the time of the geocentric birth of the New Moon is determined, the next task is to determine the local circumstances of the Sun and the Moon at the time of sunset on the day of the conjunction or a day after the conjunction (or at the time of sunrise on the day of conjunction or the day before). For this task one must determine the local times of the sunset and the moonset. In the morning in order to be visible the Sun should lag behind the Moon in order that the crescent is visible and in the evenings the Moon should be lagging behind the Sun. Classically the LAG of the Moon has remained an important consideration for the earliest visibility of the new lunar crescent. Since the times of the Babylonians, through middles ages and till the 20^{th} century it has been considered a decisive factor. Babylonians considered minimum LAG required for the visibility of new lunar crescent to be 48 minutes whereas the Muslim/Arabs considered it to be 42 to 48 minutes depending on the Earth-Moon distance. In modern times though the visibility conditions have been worsened mainly due to all kinds of artificial pollutions the new lunar crescent has been reported to be sighted when its LAG was much less than 42 minutes.

The determination of the local times of the sunset and the moonset, though stated to be second task in sequence, is dependent on the determination of the precise topocentric coordinates of the Sun and the Moon. It is therefore imperative that before the determination of the LAG one must find the geocentric coordinates and then the topocentric coordinates for the location on the globe from where observation is to be made, of these bodies. These are derived from the two theories, the VSOP87 and the ELP-2000 (discaussed later in the chapter). As both these theories describe the lunar and the solar coordinates as explicit time series, working out the precise "time argument" is essential for the application of these formulas. The "time" considered in these theories and the other theories, is a time independent of the rotations of the Earth and is generally termed as "Dynamical Time". However the "time" specified by our clocks is based on the average motion of the Earth and the Sun and is termed as the "Mean Solar Time". The time considered in the application of the theories is the Barycentric Dynamical Time (TBD) or the Terrestrial Time (TT) which is consistent with the General Theory of Relativity (Chapront-Touzé & Chapront, 1991, Nelson et. al., 2001, Guinot &

Seidelmann, 1988). The TT is defined in relation with the "International Atomic Time" (TAI) as:

$$TT = TAI + 32^{sec}.184 \qquad (2.1.1)$$

where TAI is regulated according to atomic time. In TAI the basic unit of time is the SI second (defined by Bureau International des Poids et Mesures, BIPM, in 1967, as duration of 9,192,631,770 periods of radiations corresponding to the transition between two hyperfine levels of the ground state of the Caesium 133 atom (Nelson et. al., 2001)). A day on this scale is 86400 SI seconds long (Astronomical Almanac, 2007). On the other hand the "clock time" is the Universal Time (UT) defined with reference to mean sun and associated with the Greenwich Mean Sidereal Time (GMST). UT is defined as the hour angle of the Mean Sun at Greenwich plus 12^{hrs}. Due to the irregularities in the rotations of the Earth there are discrepancies between the two times, the TT and the UT. This difference is referred to as the delta t (Δt):

$$\Delta t = TT - UT \qquad (2.1.2)$$

Therefore whenever we want to determine the position of the Sun and the Moon for a particular time on our clocks we have to formulate the time argument using these considerations otherwise the clock time of the phenomena shall not be appropriate. Finally, the time argument in the theories requires the determination of the Julian Date of the UT in question. The Julian Date is the system of continuous time scale that begins on Noon at Greenwich January 1, year -4712 (called the epoch of the "Julian date"). In this time scale the moment described by a date (Gregorian or Julian) and time (UT) is considered as the "number of days", denoted as JD, (consisting of a whole number indicating the number of days elapsed since the epoch of Julian Date and a fraction describing the fractions of a day after the whole number of days) since the epoch of Julian Date. Using this JD for any moment the Δt is then added to account for the irregularities in the rotation of the Earth. The theories use time argument t that is on the scale of "number of Julian centuries" elapsed since the epoch J2000.0. Thus using this

time argument and the explicit time series formulas of the ELP and the VSOP the coordinates of the Moon and the Sun are calculated for the same instant of the day or the day after conjunction. These are spherical polar coordinates r, the geocentric distance, λ, the ecliptic longitude and β, the ecliptic latitude, referred to as the Ecliptic Coordinates. These coordinates in turn are used recursively to obtain the times of the Sunset and the Moonset (or those of the sunrise and moonrise) for the day in question.

The lunar crescent (or the crescents of Mercury and Venus) is formed by the region of the lunar surface towards the Sun that falls between the two planes through the centre of the Moon, one perpendicular to the line of view of the observer and the other perpendicular to the direction of the Sun. The ratio of the area of this crescent and the total area of the Lunar disc is called the 'phase' of the Moon. The phase of the Moon is directly related to the separation between the Sun and the Moon or elongation.

The Astronomical Almanac published annually states that the new lunar crescent is generally not visible when its phase is less than 1% (Astronomical Almanac, 2007). This has proved to be misleading in view of the fact that the brightness of the crescent can greatly vary for the same value of the Phase owing to the varying distance of the Moon from the Earth. The Earth-Moon distance varies from 350 thousand kilometres to 400 thousand kilometres. Thus when closest to the Earth the lunar crescent may be visible with its phase much less than 1% and in case of farthest it may not be visible even with phase greater than 1%. Due to this varying distance the size of the lunar disc in fact changes. Closer the Moon the disc appears larger. The Muslims had noticed this variation in the size of the lunar disc around 1000 years ago. In the Modern times it was not before Bruin that the importance of the actual visible width of the lunar crescent was realized. Ultimately it was Yallop who used the width of lunar crescent in his one-parameter model of lunar crescent visibility relating it to the altitude of the crescent on the local horizon.

Once the geocentric coordinates of the Sun and the Moon are calculated the affects of Refraction, Aberration and the Parallax are calculated for the coordinates of

both the Sun and the Moon. These corrected ecliptic coordinates of the Sun and the Moon are then transformed into Equatorial coordinates α, the Right Ascension and δ, the Declination. In order to get the Local Horizontal coordinates; Altitude and Azimuth, of the Sun and the Moon, the observer's terrestrial coordinate and the Local Sidereal Time (defined as the Local Hour Angle of the Equinox) are required. A simple algorithm leads to the Greenwich Mean Sidereal Time (GMST) at the 0^{hr} Universal Time for any given date. Adding the local longitude to this GMST gives the Local Mean Sidereal Time for 0^{hr} Universal Time for any given date is obtained. Finally to get the Local Sidereal Time for any moment of the day may be obtained keeping in mind the faster pace of the "Sidereal Day".

Once the Local Sidereal Time of any moment is known the Hour Angle of the any object is obtained. The local Equatorial Coordinates H, the Hour Angle and δ, the declination, lead to the Altitude (height above the horizon) and Azimuth. Three points of time are important for this study, when an object is at the local meridian (i.e. Transit), when the object rises and when an object sets.

The time of transit may be calculated using the Hour Angle to be zero, or the considering the Local Sidereal time to be the Right Ascension of the object. The estimate of this point of time can be improved using an iterative process that involves readjusting the "time argument" to be this approximate time and recalculating the coordinates of the object at this time argument.

Considering Hour Angle H to be 90^0 or 6^{hrs} approximate time of the rising (negative H) or the setting (positive H) are obtained. Recalculating the time arguments for approximate times of these events the coordinates of the object are determined again and the better approximation of the times of these events are obtained. This gives the rising and the setting of the centres of the objects (the Sun and the Moon) that can be adjusted to get the actual rising (the first appearance of the western limb of the object) and actual setting (moment of disappearance of the eastern limb of the object).

Once the times of rising and/or setting of both the Sun and the Moon are obtained one can work out all the parameters of the importance for the first (or last) visibility of the new (or old) lunar crescent.

2.2 DYNAMICS OF THE MOON AND THE EARTH

The development of modern dynamical theories for the solar system began with the discovery of Laws of Planetary Motion by Johannes Kepler in the 16^{th} century. At about the same time Isaac Newton came up with his Laws of Motion and the Universal Law of Gravitation. What followed is a long history of development of mathematical techniques leading to the formulation of Celestial Dynamics. The efforts were directed to describe the motion of planets and their satellites, asteroids and comets in order to predict their positions in future with accuracy of greater and greater degree. Contributions of Euler, Laplace, Poisson, Gauss, Olber, Cowell, Encke, Clairaut, Hansen, Delaunay, Hill and Brown, besides many other mathematicians and astronomers, have been of great significance. A number of classical and modern books are now available that describe the details of these contributions (Smart, 1953, Danby, 1992, Plummer, 1966, Pollard, 1966, Woolard and Clemence 1966, Brouwer and Clemence, 1961 etc.). Contributions are also available that give details of the lunar dynamics (Chapront-Touzé and Chapront, 1983, 1991, 1988, Chapront et. al. 1998, Standish 1981, 1998 etc.). Einstein's theory of relativity succeeded in describing the motion of the perihelion.

The satellite to planet mass ratio in case of the Moon-Earth system is largest (1.23×10^{-2}) in comparison to any other satellite-planet pair (the next largest being that of Triton-Neptune mass ratio $= 2 \times 10^{-3}$). Therefore the Earth does not provide the dominant effective force acting on the Moon. It is not only the Sun but all major planets and larger of the asteroids that contribute to the effective force acting on the Moon. Thus an ephemeredes prepared without taking into account all these contributions is bound to be erroneous. Many of the ephemeredes of the Moon of early days, both before and after the formulation of the Newtonian mechanics, were based on observed and computed averages of various kinds related to the dynamics of the Moon.

Today it is known with a great degree of accuracy that the average synodic period (interval between two consecutive new Moons) is 29.530589 days (29 days, 12 hours 44 minutes and 2.9 seconds). The average anomalistic month (interval between two successive passages of the Moon through its perigee is 27.554550 days (27 days 13 hours 18 minutes and 32.1 seconds) and an average sidereal month (interval between two successive passages of the Moon through a fixed star) is 27.321662 days (27 days 7 hours 43 minutes and 11.6 seconds). Thus on the basis of a sidereal month the Moon travels around 12.176358 degrees per day on the average. Relative to the Sun the Moon travels 12.190749 degrees per day on average. However, the minimum rate can be 12.08 degrees per day and the maximum 12.43 degrees per day. This all happens because the orbit of the Moon around the Earth is a highly "irregular" ellipse whereas the deviations are caused by perturbations due to the Sun, the planets and other solar system objects.

The semi-major axis of the orbit of Moon is on average 384,400 km but has a small oscillation around this value whose period is half the synodic month. The eccentricity of the orbit is 0.549 but varies as much as ± 0.117. The inclination of the lunar orbit from the ecliptic is 50^0 9' but varies up to $\pm 9'$. Even the nodes (points of intersection of the lunar orbit and the ecliptic) of the orbit are not fixed and go round the ecliptic in 18.6 years with an oscillation about the secular motion that amounts to as much as 1.67 degrees. The line of apsides also go round the ecliptic completing one round in 8.85 years and oscillations with amplitude of 12.433 degrees. Thus all the "elements" of the orbit exhibit both secular as well as periodic variations. This makes the determination of lunar ephemeredes a daunting task.

The understanding of the dynamics of the Moon and that of the planets in the modern setup began with the revolutionary exploits of Johannes Kepler and Isaac Newton. Kepler empirically deduced his Laws of Planetary Motion on the basis of an extensive study of the observational data collected over centuries of the position of planets. These can be stated as follows:

1. Planetary orbits are elliptic with the Sun at one of the foci.
2. Radius vector of a planet (Vector drawn from the sun to the planet) sweeps equal areas in equal time intervals.
3. The square of the period of revolution of a planet around the Sun is proportional to the cube of its mean distance from the Sun.

Newton not only presented his Law of Universal Gravitation but verified Kepler's Laws of Planetary Motion using the Law of Gravitation. According to Newton's Law the force of attraction between two bodies with masses m_1 and m_2 placed at a distance r apart is given by:

$$\vec{F} = G\frac{m_1 m_2}{r^3}\vec{r} \tag{2.2.1}$$

The force is attractive and G is the proportionality constant called Universal Gravitational Constant ($6.672 \times 10^{-11} m^3 kg^{-1} s^{-2}$). $\vec{r}$ is either directed from m_1 to m_2 or from m_2 to m_1. From the time of publication of the "*Principia*" by Newton in 1687 a number of astronomers, physicists and mathematicians contributed significantly in the development of the understanding of Lunar and Planetary dynamics.

However at the time of Newton (and perhaps till today) the dynamics of the Moon presented great difficulty that forced Newton to state that "*the Lunar theory made his head ache and kept him awake so often that he would think of it no more*" (Danby 1992). He had difficulty in describing the motion of perigee (the point in lunar orbit closest to the Earth) and could explain it to only within an accuracy of 8 percent. Clairaut (1749) applied analytical methods and succeeded in explaining the motion of perigee by using second order approximation. He published his *Thèorie de la lune* and a set of numerical tables in 1752 for computation of the position of the Moon. The most significant contribution from Euler appeared in 1772 when he published his second lunar theory.

Laplace's theory of lunar motion, published in 1802, employed transforming the equation of motions so that the true longitude was an independent variable. His work also provided an explanation of the secular acceleration of the Moon. Laplace's methods were carried to a high degree of accuracy by several mathematicians. One of them was Damoisean, who published his theory and tables in 1827 that remained in wide use until Hansen's work appeared. P. A. Hansen's work extended for over forty years from 1829 and his tables were published in 1857. These tables remained in use for well over fifty years. Delaunay published his work in 1860 that was based on disturbing functions that included 320 terms. By analytical means he removed the terms of disturbing function one by one and gradually builds up the solution. Authors claim that Delaunay's work is the most perfect solution of the lunar problem yet found (Danby 1992).

The position of the Moon around the Earth is described by spherical polar coordinates $(r, \lambda, 90-\beta)$ with r being the heliocentric distance of the planet, λ the ecliptic longitude and β the ecliptic latitude. The most commonly used easy to handle lunar tables during the most part of 20th century were due to Brown (Brown 1960). This lunar theory was improved by Eckert and was known as ILE, short for *Improved Lunar Ephemeris*. The theory constructed by Chapront and Chapront-Touzé is known as ELP (Chapront et. al., 1983, 1988) short for *Ephémérides Lunaires Parisiennes*. In ELP simplified tables have been extracted from the theory to represent the lunar motion in the form of explicit time series formulae. These tables can be used to directly compute the lunar coordinates. ELP is not only more precise and complete in comparison to ILE it also provides more modern values of lunar parameters and other physical constants. For 6000 years on each side of J2000.0 ELP provides lunar coordinates that rarely have errors exceeding few arc seconds. Together with the development of VSOP (Variations Séculaires des Orbites Planétires) by Bretagnon and Francou (1988) the tables due to Chapront et. al. describe the motion of all major bodies (except Pluto) in the Solar System. Both the theories, ELP and the VSOP were developed at the Bureau des Longitude, Paris.

Basically a theory of planetary and lunar motion involves integration of a system of differential equation that constitutes the major part of the study of celestial mechanics. There are in general two approaches for solving such dynamical systems, analytical and numerical. Analytical methods are based on solution by Fourier series and the Poisson's Series. The ELP-2000-85 (Chapront et.al., 1988) is semi-analytic and has been obtained from a fit of ELP-2000-82 (Chapront et. al. 1983) to the numerical integration of the Jet Propulsion Laboratory DE200/LE200 (Standish 1981). Precession in this theory has been taken from Laskar (Laskar, 1986).

For the analytic part, ELP-2000 separates the main problem from the perturbations. The main problem takes into account the action of the Earth's centre of mass and the action of the Sun's orbit around the Earth-Moon barycentre such that the Sun's orbit is assumed to be Keplerian ellipse. This results into Fourier series with numerical coefficients and arguments that are sums of multiples of four fundamental parameters D (difference of the mean longitudes of the Sun and the Moon), l' (mean anomaly of the Moon), l (mean anomaly of the Sun) and F (Moon's argument of latitude). This main problem results into time series formulas for Moon's longitude, latitude and geocentric distance containing 2645 terms in all. Apart from these series actions of all the other significant objects in solar system are considered as 'perturbations' to the main problem that include:

1. Indirect planetary perturbations that are induced by the differences between the true orbit of the Sun around the Earth-Moon barycentre and assumed Keplerian Elliptical orbit of the Sun assumed in the main problem.

2. Direct planetary perturbations due to actions of other planets on the Moon. For both the direct and the indirect planetary perturbations the ELP-2000 considers the orbits of the planets given by Bretagnon's VSOP82 theory.

3. Perurbations due the figures of the Earth and the Moon (Moons, 1982).

4. Relativistic perturbations (Lestrade & Chapront-Touzé 1982).

5. Perturbations due to tidal affects (Williams et. at., 1978).

6. Motion of the reference frame considered with respect to an inertial frame of reference.

Consideration of all these perturbations' results into time series formulae for geocentric longitude, latitude and distance of the Moon extends the number of terms to 35,237.

An alternate to this theoretical approach is to represent the coordinates explicitly as time series formulae. This representation of the time series is developed by Chapront-Touzé and Chapront (Chapront-Touzé & Chapront, 1991, pp. 10) and has been used in this work. The major formulas used in this work due to these authors are listed below:

The geocentric longitude V is expressed as:

$$\begin{aligned} V = 218.31665436 + 481267.8813424 * t - 0.00013268 * t^2 + 0.000001856 * t^3 \\ - 0.00000001534 * t^4 + S_V + (S'_V + t * S''_V + t^2 * S'''_V / 10000) / 1000 \end{aligned} \tag{2.2.1}$$

where t = is in Julian centuries since J2000.0

$$S_V = \sum_{n=1}^{218} v_n Sin\left(\alpha_n^{(0)} + \alpha_n^{(1)} * t + \alpha_n^{(2)} * t^2 \times 10^{-4} + \alpha_n^{(3)} * t^3 \times 10^{-6} + \alpha_n^{(4)} * t^4 \times 10^{-8}\right) \tag{2.2.2}$$

$$S'_V = \sum_{n=1}^{244} v'_n Sin\left(\alpha_n^{\prime(0)} + \alpha_n^{\prime(1)} * t\right) \tag{2.2.3}$$

$$S''_V = \sum_{n=1}^{154} v''_n Sin\left(\alpha_n^{\prime\prime(0)} + \alpha_n^{\prime\prime(1)} * t\right) \tag{2.2.4}$$

$$S_V''' = \sum_{n=1}^{25} v_n''' Sin\left(\alpha_n'''^{(0)} + \alpha_n'''^{(1)} * t\right) \tag{2.2.5}$$

The values of the constants v_n, v'_n etc, along with those of α's are given in Chapront-Touzé and Chapront (Chapront-Touzé and Chapront , 1991, pp 43-56).

The geocentric latitude U is given by:

$$V = S_U + (S'_U + t * S''_U + t^2 * S'''_U / 10000) / 1000 \tag{2.2.6}$$

where

$$S_U = \sum_{n=1}^{188} u_n Sin\left(\beta_n^{(0)} + \beta_n^{(1)} * t + \beta_n^{(2)} * t^2 \times 10^{-4} + \beta_n^{(3)} * t^3 \times 10^{-6} + \beta_n^{(4)} * t^4 \times 10^{-8}\right)$$

(2.2.7)

$$S'_U = \sum_{n=1}^{64} u'_n Sin\left(\beta_n'^{(0)} + \beta_n'^{(1)} * t\right) \tag{2.2.8}$$

$$S''_U = \sum_{n=1}^{64} u''_n Sin\left(\beta_n''^{(0)} + \beta_n''^{(1)} * t\right) \tag{2.2.9}$$

$$S'''_U = \sum_{n=1}^{25} u'''_n Sin\left(\beta_n'''^{(0)} + \beta_n'''^{(1)} * t\right) \tag{2.2.10}$$

The values of the constants u_n, u'_n etc, along with those of β's are given in Chapront-Touzé and Chapront (Chapront-Touzé and Chapront , 1991, pp 57-64).

Finally the geocentric distance is given by:

$$R = 385000.57 + S_R + S'_R + t * S''_R + t^2 * S'''_R / 10000 \tag{2.2.11}$$

where

$$S_R = \sum_{n=1}^{154} r_n Cos\left(\delta_n^{(0)} + \delta_n^{(1)} * t + \delta_n^{(2)} * t^2 \times 10^{-4} + \delta_n^{(3)} * t^3 \times 10^{-6} + \delta_n^{(4)} * t^4 \times 10^{-8}\right)$$

(2.2.12)

$$S'_R = \sum_{n=1}^{114} r'_n Cos\left(\delta_n'^{(0)} + \delta_n'^{(1)} * t\right) \tag{2.2.13}$$

$$S''_R = \sum_{n=1}^{68} r''_n Cos\left(\delta_n''^{(0)} + \delta_n''^{(1)} * t\right) \tag{2.2.14}$$

$$S'''_R = \sum_{n=1}^{19} r'''_n Cos\left(\delta_n'''^{(0)} + \delta_n'''^{(1)} * t\right) \tag{2.2.15}$$

The values of the constants r_n, r'_n etc, along with those of δ's are given in Chapront-Touzé and Chapront (Chapront-Touzé and Chapront , 1991, pp 65-73).

All $\alpha^{(0)}$s, $\beta^{(0)}$s and $\delta^{(0)}$s, V, S_V, S'_V, U, S_U, S'_U, v_n, v'_n, u_n, and u'_n are in degrees, $\alpha^{(1)}$s, $\beta^{(1)}$s, $\delta^{(1)}$s, S''_V, S''_U, v''_n and u''_n are in degrees/century, $\alpha^{(2)}$s, $\beta^{(2)}$s, $\delta^{(2)}$s, S'''_V, S'''_U, v'''_n and u'''_n are in degrees/century^2, $\alpha^{(3)}$s, $\beta^{(3)}$s, and $\delta^{(3)}$s are in degrees/century^3 and $\alpha^{(4)}$s, $\beta^{(4)}$s and $\delta^{(4)}$s are in degrees/century^4. R, S_R and S'_R are in kilometres, S''_R and r''_n are in kilometres/century, S'''_R and r'''_n are in kilometres/century^2.

For The determination of planetary coordinates the complete n-body problem is required to be solved. An analytic solution of planetary motion was presented by Bretagnon (Bretagnon, 1982) of Bureau des Longitudes of France that described only the elliptic coordinates of the planets. The solution is popularly known as VSOP82 (Variations Séculaires des Orbites Planètaires). Later, Bretagnon and Francou of the same Bureau modified VSOP82 into VSOP87 (Bretagnon & Francou, 1988) in such a way that their solution provides both the Cartesian (or rectangular) coordinates as well as the spherical polar coordinates of the planets in a heliocentric system. Their solution VSOP87 describes the elements of the osculating or instantaneous orbit in terms of:

a = semi-major axis of the orbit

λ = mean longitude of the planet

$h = e.\sin\pi$

$k = e.\cos\pi$

$p = \sin(\frac{1}{2} i) \sin\Omega$

$q = \sin(\frac{1}{2} i) \cos\Omega$

where e is the eccentricity of the orbit, π is the longitude of perihelion, i is the inclination of the orbit from the plane of ecliptic and Ω is the longitude of the ascending node of the orbit. Each of the rectangular coordinate (X, Y, Z) or the spherical polar coordinates (L, B, R) is an explicit function of time and is in the form of periodic series and Poisson series. Every term of these series is in the form of:

$$T^{\alpha} (S \sin\varphi + K \cos\varphi) \text{ or } T^{\alpha} A \cos(B + CT) \qquad (2.2.16)$$

where $\alpha = 0, 1, 2, 3, 4, 5$, T is the time in thousands of Julian years from J2000.0, i.e.

$$T = \frac{\textit{Julian Date} - 2451545}{365250}$$

$\varphi = \sum_{i=1}^{12} a_i \lambda_i$, i = 1 to 8, λ_i represent the mean longitudes of the planets Mercury to Neptune. For i = 9, 10 and 11 λ_i represent the Delauney arguments of the Moon D, F and L respectively. The last of λ_i is the mean longitude of the Moon given with respect to the equinox of the day. In the alternate expression,

$$A = \sqrt{S^2 + K^2} \qquad B = \sum_{i=1}^{12} a_i \lambda_i^0 + \beta \qquad C = \sum_{i=1}^{12} a_i N_i \quad (2.2.17)$$

B is defined by $\quad S = -A\sin\beta, \quad K = A\cos\beta \qquad (2.2.18)$

λ_i^0 and N_i are given in the table 2 of (Bretagnon & Francou 1988).

These data series are available on CD's and tapes. For rectangular coordinates of the planets the data files VSOP87A, VSOP87C and VSOP87E are used and for the spherical polar coordinates VSOP87B and VSOP87D are used. A shorter version of these data series is given by Meeus (Meeus, 1998) and the same are used in this work. In these tables first column gives *A*s, the second *B*s and the third gives *C*s. The data file has 6 series for each of the coordinates L (the heliocentric Longitude) and R (the heliocentric distance of the Earth) and 5 for the coordinate B (the heliocentric Latitude).

Each series for L, B and R are used as follows to obtain the heliocentric polar coordinates of the Earth:

$$V^k{}_i = \sum_{j=0}^{N} A_j^k \cos(B_j^k + C_j^k T^j), \qquad (2.2.19)$$

i = 0, 1, 2, 3, 4, and 5 for L and R and 0, 1, 2, 3, 4 for B. The superscript k stand for 1 (L), 2(B) and 3(R). N runs through 0 to different integers for different coordinates and their associated series. For k = 0, $V^k{}_i$ corresponds to longitude series, k = 1, $V^k{}_i$ corresponds to latitude series and k = 3, $V^k{}_i$ corresponds to distance series. Each coordinate is then evaluated as:

$$L = \left(\sum_{i=0}^{5} C^0{}_i T^i\right),\ B = \left(\sum_{i=0}^{5} V^1{}_i T^i\right) \text{ and } R = \left(\sum_{i=0}^{5} V^2{}_i T^i\right) \qquad (2.2.20)$$

L and B are in radian measures and R is in astronomical units. These are as mentioned earlier the coordinates of the Earth in heliocentric coordinate system whereas for the problem of determining position of the Sun in our sky we actually require the geocentric coordinates of the Sun instead. In case of the Earth this transformation is simple:

$$\lambda_S = L + 180^0 \qquad \& \qquad \beta_S = -B \tag{2.2.21}$$

and the heliocentric distance of the Earth is same as the geocentric distance of the Sun.

2.3 BIRTH OF NEW MOON

As mentioned earlier the Moon in its journey around the Earth travels around 12 degrees every day in our sky and takes over the Sun in around every 29.5 days. When the Geocentric Longitude of the Sun and the Moon are same the moment is known as the Time of Birth of New Moon. The duration between two successive Births of New Moons is called the Lunation Period. However the Lunation period is not constant and varies from 29.2 days to 29.8 days. This is the reason behind consecutive lunar months of 29 days each or the consecutive lunar months of 30 days each. For the time of Birth of New Moon one requires to find the moment when the geocentric longitudes of the Moon and the Sun coincide. Thus one needs to track the longitudes of each of them in time. Considering the major terms of the time series formulae for the longitudes of the Sun and the Moon in the planetary theory VSOP-2000-87 and the lunar theory ELP-2000-87 the moment when the two longitudes are same can be evaluated. An algorithm due to Meeus (Meeus, 1998) for the determination of the time of Birth of New Moon is as follows:

On average the Tropical Year (duration between two consecutive passages of the Sun through equinox) is currently 365.24219 days (from (1.3.1)). The average Synodic Month (the interval between two consecutive New Moons) taken over a century is 29.530589 days (from (1.3.3)). Thus in one tropical year there are on average 12.1682664 Synodic months. Therefore since the start of the year 2000, i.e. J2000.0 the number of synodic months elapsed are given by:

$$k = (year - 2000) \times 12.3682664 \tag{2.3.1}$$

and the time in tropical centuries elapsed since J2000.0 is given by:

$$\tau = \frac{k}{1236.82664} \tag{2.3.2}$$

An approximate value of the Julian Date of the New Moon is then given by:

$$JDE = 2451550.09766 + 29.530588861 * k + 0.00015437 * \tau^2 - 0.00000015 * \tau^3 + 0.00000000073 * \tau^4 \tag{2.3.3}$$

where k is an integer. Thus according to this formula the for $k = 0$ the Julian Date of the first crescent of year 2000 is 2451550.09766 that is January 6, 2000 at 18^{hrs} 14^{min} and $41^{sec}.1$ of dynamical time. For a more accurate value of the Julian Date of the New Moon the perturbation terms due to the Sun and the planets are added. The perturbations terms due to the Sun are given by:

$$\begin{aligned} X = \ & -0.4072*SIN(M')+0.17241*E*SIN(M)+0.01608*SIN(2*M')+0.01039*SIN(2*F) \\ & +0.00739*E*SIN(M'-M)-0.00514*E*SIN(M'+M)+0.00208*E^{\wedge}2*SIN(2*M) \\ & -0.00111*SIN(M'-2*F)-0.00057*SIN(M'+2*F)+0.00056*E*SIN(2*M'+M) \\ & -0.00042*SIN(3*M')+0.00042*E*SIN(M+2*F)+0.00038*E*SIN(M-2*F) \\ & -0.00024*E*SIN(2*M'-M)-0.00017*SIN(\Omega)-0.00007*SIN(M'+2*M) \\ & +0.00004*SIN(2*M'-2*F)+0.00004*SIN(3*M)+0.00003*SIN(M'+M-2*F) \\ & +0.00003*SIN(2*M'+2*F)-0.00003*SIN(M'+M+2*F) \\ & +0.00003*SIN(M'-M+2*F)-0.00002*SIN(M'-M-2*F)-0.00002*SIN(3*M'+M) \\ & +0.00002*SIN(4*M') \end{aligned} \tag{2.3.4}$$

where M = the mean anomaly of the Sun at the JDE

$$= 2.1534+29.1053567*k-0.0000014*\tau^2-0.00000011*\tau^3 \tag{2.3.5}$$

M' = the mean anomaly of the Moon at the JDE

$$= 201.5643+385.81693528*k+0.0107582*\tau^2+0.00001238*\tau^3 -0.000000058*\tau^4 \tag{2.3.5}$$

F = Moon's argument of latitude

$$= 160.7108+390.67050284*k-0.0016118*\tau^2 - 0.00000227*\tau^3 + 0.000000011*\tau^4 \qquad (2.3.7)$$

Ω = Longitude of ascending node of the lunar orbit

$$= 124.7746-1.56375588*k+0.0020672*\tau^2 + 0.00000215*\tau^3 \qquad (2.3.8)$$

E = Eccentricity of the orbit of Earth

$$= 1-0.002516*T-0.0000074*\tau^2 \qquad (2.3.9)$$

The perturbation terms due to planets are:

$$\begin{aligned} Y = \; & 0.000325*SIN(A1)+0.000165*SIN(A2)+0.000164*SIN(A3)+0.000126*SIN(A4) \\ & +0.00011*SIN(A5)+0.000062*SIN(A6)+0.00006*SIN(A7)+0.000056*SIN(A8) \\ & +0.000047*SIN(A9)+0.000042*SIN(A10)+0.00004*SIN(A11) \\ & +0.000037*SIN(A12)+0.000035*SIN(A13)+0.000023*SIN(A14) \qquad (2.3.10) \end{aligned}$$

where

$$A1=299.77+0.107408*k-0.009173*\tau^2 \qquad (2.3.11)$$

$$A2=251.88+0.016321*k \qquad (2.3.12)$$

$$A3=251.83+26.651886*k \qquad (2.3.13)$$

$$A4=349.42+36.412478*k \qquad (2.3.14)$$

$$A5=84.66+18.206239*k \qquad (2.3.15)$$

$$A6=141.74+53.303771*k \qquad (2.3.16)$$

$$A7=207.14+2.153732*k \qquad (2.3.17)$$

$$A8=154.84+7.30686*k \qquad (2.3.18)$$

$$A9=34.52+27.261239*k \qquad (2.3.19)$$

$$A10=207.19+0.121824*k \qquad (2.3.20)$$

$$A11=291.34+1.844379*k \qquad (2.3.21)$$

$$A12=161.72+24.198154*k \quad (2.3.22)$$

$$A13=239.56+25.513099*k \quad (2.3.23)$$

$$A14=331.55+2.792518*k \quad (2.3.24)$$

Thus the Julian Date of the New Moon is given by

$$JD = JDE + X + Y \quad (2.3.25)$$

The time described by this date is the Dynamical Time and the corrections for Δt must be made to get the Universal Time (discussed in the next article). For any local computation, the Local Zone time and date must be calculated from the Universal Time and date obtained above on the basis of the longitude of any place on the Earth. The date is then the day of Conjunction for the place and the time of conjunction (the birth of New Moon) can be any time from 0^{hrs} to $23^{hrs}\ 59^{min}\ 59^{sec}$ on that day.

2.4 THE TIME ARGUMENT

It was mentioned earlier that the dynamics of all solar system objects is described by formulas based on theories of Classical Mechanics and the Relativistic Dynamics in terms of time series. In order to effectively use these formulas an appropriate *time argument* corresponding to the moment of observing the lunar crescent at any place on the surface of the Earth has to be evaluated. Such a time argument has to be continuous and must have a clearly defined point of its beginning (the zero time), called epoch. Various theories and problems use different epochs depending on the context. For a general consideration in planetary and lunar dynamics there are two important epochs.

The first of these epochs is a moment in remote past corresponding to the Noon at Greenwich on January 1, 4712 B.C.E. on the Julian calendar (or November 24, -4713 on Gregorian calendar) (Reingold & Dershowitz, 2001). From this point of time the time elapsed till any later point of time in number of days and a possible fraction of a day is

called the Julian Date abbreviated as JD. So the JD corresponding to the 6^{hrs} 30^{min} on October 6, 2004 at Greenwich is 2453284.27083333.. Thus the Julian Date is a measure of time elapsed since this epoch and is expressed in number of mean solar days.

The other epoch of importance to the current work is the moment of time called J2000.0 and it represents the 12^{hrs} TDT on January 1, 2000 i.e. (Astronomical Almanac, 2007). The JD corresponding to this moment at Greenwich is 2451545 days. This is the epoch or zero time for both the Lunar Theory ELP-2000 (Chapront-Touzé & Chapront, 1991) and the planetary theory VSOP-87 (Bretagnon & Francou, 1988). In both these theories time measured from J2000.0 both forward and backwards. In ELP this time is in is considered in Julian Centuries (i.e. 36525 mean solar days) and in VSOP it is in Julian Millennia (365250 mean solar days).

Both these epochs are based on the interval of time called "mean solar day" which is defined as the interval between two successive transits (passage through the local meridian) of the fictitious body known as the mean Sun. This fictitious body moves with uniform speed along the celestial equator and is considered in place of the actual Sun that moves with non-uniform speed (due to the elliptic orbit of the Earth) along the Ecliptic. The transit of the actual Sun over a local meridian varies up to 31 minutes over a period of one tropical year (Astronomical Almanac, 2007). Thereby all civil time reckoning have been associated with the mean Sun that consists of 24 mean solar hours. The beginning of a civil day, i.e. zero hours on civil clocks occurs at midnight when the hour angle of the Mean Sun is twelve hours according to the local or standard meridian.

The time described by the clocks showing the mean solar time is not without its discrepancies. In fact it is the Earth, the globe, itself that is our clock and the mean solar time is supposed to be based on the average rate at which the Earth is spinning around its axis. However this consideration is only with respect to the Mean Sun. Due to the orbital motion of the Earth being in the same direction as its axis of rotation (from west to East) one axial rotation completes in less than this mean solar day. So the actual rate of axial rotation is better realized by the two successive transits of a star. This interval is termed

as a Sidereal Day and the time measured according to this scale is the Sidereal Time. Again due to the elliptical orbit of the Earth this period is also not uniform so we have to consider "Mean Sidereal Day" and accordingly Mean Sidereal Time. One mean solar day equals 1.00273790935 mean sidereal days (or 24^{hrs} 03^{min} 56.55537^{sec} on mean sidereal time). Alternately one mean sidereal day equals 0.99726956633 mean solar days (or 23^{hrs} 56^{min} 04.09053^{sec} on mean solar time).

In general the mean solar time is the time taken into account in both the civil time reckoning as well as the astronomical. Whereas the dynamical theories describe the motions of the objects in solar system on the basis of the continuously flowing time described as Dynamical Time. The Universal Time and the Dynamical Time are not consistent and the difference between the two is not a known function of time and could be found only by high precision observations of the skies. The difference of the two ΔT is tabulated in Astronomical Almanacs for the telescopic era (AD 1620 till todate). For the era prior to the telescopic era the values of ΔT are calculated on the basis of the calculations of eclipses, occultation and the times of these events recorded in the known history. The values of ΔT are given for only the start of each calendar year and those for other times of year can be interpolated. Various authors have given various techniques for obtaining these values (Dickey 1995, Meeus, 1998, Morrison & Stephenson, 2004, Morrison & Ward, 1975, Stephenson & Morrison, 1984 and 1995, Stephenson, 1997, Islam et al, 2007).

Whenever a calculation for an event is to be performed we work out everything based on local civil time and date. For instance when the time and date of the birth of new Moon is computed the result comes out to be in dynamical time. This dynamical time should be converted to the Universal time by adding the current value of Δt and then to the local civil time by adding the requisite zone time (considered positive for east longitudes and negative for west longitudes) for the location of observer. Similarly if one wants to calculate to the position of the Moon for any location of observer, the local zone time has to be converted to universal time by subtracting the zone time and then the current value of Δt should be subtracted from it to get the Dynamical Time.

However a more conventional approach is to convert local date and zone time to universal date and time which is then converted to the Julian date and finally the affect of Δt is taken into account. The same approach is used in the following algorithm for calculation of the time argument:

Step-1: *Insert Local Date from Gregorian Calendar LYY, LMM, LDD*

Step-2: *Insert Local Time & Zone Time LHH, LMIN, LSEC, ZON*

Set UYY=LYY, UMM=LMM, UDD=LDD,

UHH=LHH+ZON, UMIN=LMIN, USEC=LSEC

In general the only difference between local zone time and date and the Universal time and date is the difference between hours that equals zone time (considered in integral hours here). Due to this difference the local date and the Universal date may differ by one and needs adjustment.

```
If (UHH>=24)          ...... increase day
{     UDD = LDD+1
      If (UDD>days(LMM))  // days is number of days in a month array
      {     LDD=1;
            UMM=LMM+1;
            If (UMM>12)
            {     UMM=1;
                  UYY=LYY+1;
            }}}
If (UHH<0)            ...... decrease day

{     UDD=LDD-1;
      {     If (UDD=0)
            {     UMM=LMM-1
                  If (UMM=0)
```

```
{       UYY=LYY-1;
        UMM=12;
}
UDD=days(UMM)
}}
```

Once the Universal date and time is appropriately adjusted one proceeds to calculate the Julian Date for the date and the time. For this calculation initially if the month is January or February, it is considered month number 13 or 14, respectively, of the previous year and the year is also decreased by 1.

```
Step-3:     if UMM<3    {     UYY=UYY-1
                              UMM=UMM+12          }
```

The number of century years (like year 1100, 1700 etc) till the year UYY is required to account for number of normal Leap years.

```
Step-4:     A=INT(UYY/100)
```

In most of the astronomical calculations a date on and after Friday, October 15, 1582 is considered to be a date of Gregorian calendar and a date from Thursday, October 4, 1582 and prior to this date is considered as a date in Julian calendar. Thus if a date is from Gregorian calendar it further requires an account of non-Leap years from amongst the normal leap years due to the modified rule of Lead year in the Gregorian calendar (years divisible by 100 but not divisible by 400 are not Leap years).

```
Step-5:     if "calendar is Gregorian"
                {     B=2-A+INT(A/4)     }
            elseif "calendar is Julian"
                {     B=0   }
```

Now one needs to count the number of days elapsed since the Julian Date epoch (January 1, -4712) till the end of the previous year and the number of days elapsed from the first day of the previous year till the end of the current month. Meeus consider this account starting from year -4716 that adds additional days that are balanced by subtraction of the constant 1524.5.

Step-6: $$JD=INT(365.25(UYY+4716))+INT(30.6001(UMM+1))+UDD+B-1524.5 \\ +(UHH+(UMIN+USEC/60)/60)/24 \tag{2.4.1}$$

In both the theories VSOP87 and ELP2000 the epoch is the J2000.0 corresponding to the Julian Date 2451545 therefore one finally gets the time argument as:

Step-7: $$\tau = \frac{JD-2451545}{36525} + \frac{\Delta t}{3155760000} \tag{2.4.2}$$

Note that in the last step the first term on the right hand side is the number of Julian centuries elapsed since J2000.0 and the second term is for Δt which is normally given in number of seconds and here we need to convert it into number of centuries (fro ELP and Millennia for VSOP) due to which it has to be divided by the number of seconds in a Julian century (for ELP and Julian Millennia for VSOP).

If and when the dynamical time of an event is known we need to convert it to Universal and then into Local Zone time. The dynamical time is obtained as the number of Julian centuries since J2000.0 so that the Julian date of the event can be calculated as:

Step-1: *Input time argument* τ
Calculate Julian Date

$$JD = 365250 * \left(\tau - \frac{\Delta t}{31557600000} \right) + 2451545 \qquad (2.4.3)$$

The integral part of the Julian Date will be converted to Calendar Date and the fractional part to the Universal Time:

Step-2: $Z=INT(JD)$

Sted-3: $F=JD-Z$

For dates prior to October 15, 1582 (JD = 2299161) the integer part of JD is retained as it is otherwise adjusted for the conditions of the Gregorian calendar Leap year.

Step-4: *if* $Z<2299161$ { $A=Z$ }
Else { $\alpha=INT((Z-1867216.25)/36524.25)$
$A=Z+1+\alpha-INT(\alpha/4)$ }

Step-5: $B=A+1524$

Step-6: $C=INT((B-122.1)/365.25)$

Step-7: $D=INT(365.25*C)$

Step-8: $E=INT((B-D)/30.6001)$

Step-9: $day=B-D-INT(30.6001*E)+F$

Step-10: $UDD=INT(day)$ $LDD=UDD$

Step-11: *if* $E<14$ { $UMM=E-1$ }
Else { $UMM=E-13$ }

```
        LMM=UMM

Step-12:  if MM>2      {    UYY=C-4716 }
          Else         {    UYY=C-4715 }
          LYY=UYY

Step-13:  hour(=day-INT(day))*24

Step-14:  UHR=INT(hour)       LHR=UHR-ZON
          If (LHR<0)          ...... decrease day
          {     LHR=LHR+24;       LDD=UDD-1;
                If (LDD=0)
                {     LMM=UMM-1;       LDD=days(LMM);
                      If (LMM=0)
                      {     LMM=12;
                          . LYY=UYY-1;
                      }}}
          If (LHR>=24         ...... increase day)
          {     LHR=LHR-24;       LDD= UDD+1;
                If (LDD>days(UMM)
               ·{     LDD=1;       LMM=UMM+1;
                      If (LMM>12)
                      {     LYY=UMM+1;       LMM=1;
                      }}}

Step-15:  minute=(hour-LHR)*60

Step-16:  UMIN=INT(minute)  LMN=UMN

Step-17:  second=(minute-LMIN)*60
```

Step-18: *USEC=INT(second) LSEC=USEC*

Step-19: *Output UYY, UMM, UDD, UHR, UMIN, USEC*
And LYY, LMM, LDD, LHR, LMIN, LSEC

In the New-Moon Algorithm the out put is the dynamical time and this algorithm is particularly useful in converting this time to Universal and any local zone time.

2.5 COORDINATES OF THE MOON

For the determination of the coordinates of the Moon at any given local time and date first step is to formulate the time argument as discussed in article 2.4. So the process begins by selecting place of observer (Longitude and Latitude), local date and time that leads to the time argument τ according to the algorithm described above, as:

$$\tau = \frac{Julian\ Date - 2451545}{36525} + \frac{\Delta t}{3155760000} \tag{2.5.1}$$

Using this time argument the construction of the time series describing the lunar coordinates is done (Chapront-Touzė and Chapront 1991) as discussed in article 2.2 above.

Step-1: For ecliptic longitude of the Moon use 2.2.2 to 2.2.5 and substitute their results in 2.2.1.

Step-2: For ecliptic latitude of the Moon use 2.2.7 to 2.2.10 and substitute their results in 2.2.6.

Step-3: For geocentric distance of the Moon use 2.2.12 to 2.2.15 and substitute their results in 2.2.11.

Due to the motion of the observer, the diurnal and the annual motion of the Earth, position of every object in the sky is affected by the phenomenon of Aberration. The following consideration is only for the Earth-Moon planetary Aberration and does not include the diurnal motion of the observer (Woolard & Clemence, 1966).

Step-4: *CORRECTION FOR ABERRATION*

$$V=V-0.00019524-0.00001059*Sin(225+477198.9*\tau) \quad (2.5.2)$$

$$U=U-0.00001754*Sin(183.3+483202*\tau) \quad (2.5.3)$$

$$R=R+0.0708*Cos(225+477198.9*\tau) \quad (2.5.4)$$

Finally as the true equinox of the day and the mean equinox of the day are different due to the phenomenon of Nutation the precise coordinates can not be obtained without the nutation in longitude $\Delta\psi$ and the nutation in obliquity $\Delta\varepsilon$.

Step-5: *CORRECTION FOR NUTATION*

$$\Delta\psi = 10^{-3} * \sum_{n=1}^{13} (\psi_n + \psi'_n * \tau) * Sin(\mu_n^{(0)} + \mu_n^{(1)} * \tau + \mu_n^{(2)} * \tau^2 * 10^{-4})$$

(2.5.5)

$$V=V+\Delta\psi \quad (2.5.6)$$

$$\varepsilon = 22.63928-0.013*\tau+0.555*10^{-6}*\tau^3 - 0.0141*10^{-8}*\tau^4$$

(2.5.7)

$$\Delta\varepsilon = 10^{-3} * \sum_{n=1}^{13} (\varepsilon_n + \varepsilon'_n * \tau) * Cos(\mu_n^{(0)} + \mu_n^{(1)} * \tau + \mu_n^{(2)} * \tau^2 * 10^{-4})$$

(2.5.8)

$$\varepsilon = \varepsilon + \Delta\varepsilon \quad (2.5.9)$$

The value of μ's, ψ's and ε's used in the expressions above are given in the Table 9 in Chapront-Touzé and Chapront (Chapront-touzė and Chapront, 1991, pp. 19):

Once the correction due to nutation is done one may go to find the Equatorial coordinates, the Right Ascension α and the declination δ:

Step-6: *EQUATORIAL COORDIANTES*

$$\alpha = Tan^{-1}\left(\frac{Cos(Epsilon)Sin(V)Cos(U) - Sin(Epsilon)Sin(U)}{Cos(V)Cos(U)}\right) \qquad (2.5.10)$$

$$\delta = Sin^{-1}\left(Sin(Epsilon)Sin(V)Cos(U) + Cos(Epsilon)Sin(U)\right) \qquad (2.5.11)$$

These are the true equatorial coordinates of the Moon with reference to the true equator of the date and the true dynamical equinox of the date. These are still the geocentric coordinates and the affect of the position of the observer on the globe is yet to be taken into account so that the "topocentric" coordinate, (coordinates relative to the position of the observer) may be obtained. In other words the affects of the "Parallax" are to be taken into account. The Parallax π is given by:

$$\sin\pi = \frac{\sin 8''.794}{R} \qquad (2.5.12)$$

where R is the geocentric distance of the Moon. However this quantity parallax depends on the Hour Angle (time since the object crossed the local meridian) for which we need the local sidereal time LST.

Step-7: *Formulate time argument t for 0^{hr} UT for the date under consideration, then*

$$T_0 = 6^h41^m50^s.54841 + 8640184^s.812866 * t + 0^s.093104 * t^2 - 0^s.0000062 * t^3$$

(2.5.13)

gives the Greenwich Mean Sidereal Time at 0^{hr} UT of the date. Then the Greenwich Sidereal Time for the time argument for the time of observation is:

$$T = T_0 + (UHR + (UMIN + USEC/60)/60)*0.99726956633 \quad (2.5.14)$$

Then the Hour angle at this moment of the Moon is:

$$H = T - \alpha \quad (2.5.15)$$

Step-8: *Affects of Parallax*

If ρ is the geocentric radius of the Earth, φ' is the geocentric latitude of the observer then the right ascension α' and the declination δ' after correction for parallax are obtained as:

$$\tan \Delta\alpha = \frac{-\rho \cos\varphi' \sin\pi \sin H}{\cos\delta - \rho\cos\varphi' \sin\pi \cos H} \quad (2.5.16)$$

$$\alpha' = \alpha + \Delta\alpha \quad (2.5.17)$$

$$\delta' = \tan^{-1}\left(\frac{(\sin\delta - \rho\sin\varphi' \sin\pi)\cos\Delta\alpha}{\cos\delta - \rho\cos\varphi' \sin\pi\cos H}\right) \quad (2.5.18)$$

Finally to obtain the Local Horizontal Coordinates, Azimuth and the Altitude we have the following transformations:

Step-9: Azimuth $A = \frac{\sin H}{\cos H \sin\varphi' - \tan\delta' \cos\varphi'}$ (2.5.19)

Altitude $\qquad h = \sin\varphi'\sin\delta' + \cos\varphi'\cos\delta'\cos H \qquad (2.5.20)$

This completes the determination of ecliptic, equatorial and the horizontal coordinates of the Moon.

2.6 COORDINATES OF THE SUN

For the determination of the coordinates of the Sun one proceeds in exactly the same way as for the coordinates of the Moon.

Step-1: Select place of observer (Longitude and Latitude), local date and time that leads to the time argument τ according to the algorithm described above, as:

$$\tau = \frac{Julian\ Date - 2451545}{365250} + \frac{\Delta t}{31557600000} \qquad (2.6.1)$$

Using this time argument the construction of the time series describing the coordinates of the Earth as given by Bretagnon & Francou (1988) the heliocentric coordinates of the Earth are obtained that are later transformed into the geocentric coordinates of the Sun as follows:

Step-1: For heliocentric ecliptic longitude of the Earth, in line with (2.2.19) and 2.2.20 we have:

$$L_0 = \sum_{n=1}^{559} A_n \cos\left(B_n + C_n\tau\right) \qquad (2.6.2)$$

$$L_1 = \sum_{n=1}^{341} A_n \cos\left(B_n + C_n\tau\right) \qquad (2.6.3)$$

$$L_2 = \sum_{n=1}^{142} A_n \cos\left(B_n + C_n \tau\right) \tag{2.6.4}$$

$$L_3 = \sum_{n=1}^{22} A_n \cos\left(B_n + C_n \tau\right) \tag{2.6.5}$$

$$L_4 = \sum_{n=1}^{11} A_n \cos\left(B_n + C_n \tau\right) \tag{2.6.6}$$

$$L_5 = \sum_{n=1}^{5} A_n \cos\left(B_n + C_n \tau\right) \tag{2.6.7}$$

The values of A's, B's and C's for the shorter version are given by Meeus (Meeus, 1998, Appendix-III, pp. 418-421). Finally the heliocentric longitude of the Earth is given by

$$L = \frac{\sum_{i=0}^{5} L_i \tau^i}{10^8} \tag{2.6.8}$$

Step-2: For ecliptic latitude of the Earth:

$$B_0 = \sum_{n=1}^{184} A_n \cos\left(B_n + C_n \tau\right) \tag{2.6.9}$$

$$B_1 = \sum_{n=1}^{99} A_n \cos\left(B_n + C_n \tau\right) \tag{2.6.10}$$

$$B_2 = \sum_{n=1}^{49} A_n \cos\left(B_n + C_n \tau\right) \tag{2.6.11}$$

$$B_3 = \sum_{n=1}^{11} A_n \cos\left(B_n + C_n \tau\right) \tag{2.6.12}$$

$$B_4 = \sum_{n=1}^{11} A_n \cos\left(B_n + C_n \tau\right) \tag{2.6.13}$$

The values of A's, B's and C's for the shorter version are given by Meeus (Meeus, 1998, Appendix-III, pp. 418-421). Finally the heliocentric latitude of the Earth is given by:

$$B = \frac{\sum_{i=0}^{5} L_i \tau^i}{10^8} \tag{2.6.14}$$

Step-3: For heliocentric distance of the Earth:

$$R_0 = \sum_{n=1}^{526} A_n \cos\left(B_n + C_n \tau\right) \tag{2.6.15}$$

$$R_1 = \sum_{n=1}^{292} A_n \cos\left(B_n + C_n \tau\right) \tag{2.6.16}$$

$$R_2 = \sum_{n=1}^{139} A_n \cos\left(B_n + C_n \tau\right) \tag{2.6.17}$$

$$R_3 = \sum_{n=1}^{27} A_n \cos\left(B_n + C_n \tau\right) \tag{2.6.18}$$

$$R_4 = \sum_{n=1}^{10} A_n \cos\left(B_n + C_n \tau\right) \tag{2.6.19}$$

$$R_5 = \sum_{n=1}^{10} A_n \cos\left(B_n + C_n \tau\right) \tag{2.6.20}$$

The values of A's, B's and C's for the shorter version are given by Meeus (Meeus, 1998, Appendix-III, pp. 418-421). Finally the heliocentric distance of the Earth is given by:

$$R = \frac{\sum_{i=0}^{5} L_i \tau^i}{10^8} \tag{2.6.21}$$

As, Bs and Cs are all in radians for longitude L and latitude B. Bs and Cs are in radians and As in astronomical units for heliocentric distance R.

The corrections for Aberration and Nutation are done in the same way as Step-4 and Step-5 before. Finally the conversion to the equatorial coordinates and then to horizontal coordinates is also done the same way as was done for the Moon.

2.7 RISING AND SETTING

For the determination of precise timings of the setting or rising of an object one requires precise celestial coordinates of these objects at the instant of the occurring of the phenomena. However, these instants are the points of time that we require to find out so that a process of successive approximation is needed to arrive at these times. Such an iterative process is necessary because the objects under consideration (the Sun and the Moon) significantly change their position relative to the fixed celestial sphere during an interval around half a day. The whole process starts with an estimate for the time of transit of the object (over the local meridian) which then leads to initial estimates for the hour angles at the approximate time of the rising or setting of the object. These are in fact the estimates for the local sidereal times of the phenomena. From these estimates of the sidereal times of the events the universal mean solar time and then the local times can be calculated. These first approximations for the transit, the rising and the setting are obtained using the celestial coordinates of the object evaluated at 0^{hr} UT. At any point of the globe and for any of the events under consideration this moment (0^{hr} UT) may be an earlier or a later moment. This is the reason that the initial calculations are only approximate calculation. For these approximate times of the events the celestial coordinates of the object have to be calculated again and whole calculations mentioned above are repeated for better estimates. The details of these calculations are described in the following paragraphs.

For the Sun, the computations are simple as compared to those for the Moon. The local time of transit of the Sun can be initially considered at 12^{hr} local (zone) time whereas for the Moon it daily varies. Thus Universal Time of local transit is simply 12 – ZT, where ZT (zone time) is positive for the east longitudes and negative for west

longitudes. At universal time 12 - ZT will be a time of the same date in Greenwich as the local date. The time argument for this time and date is then formulated and the coordinates of the Sun are obtained. When an object is in transit its hour angle (HA) is zero and its right ascension (RA) is same as the Local Sidereal Time (LST) since:

$$\mathrm{HA} + \mathrm{RA} = \mathrm{LST} \tag{2.7.1}$$

Assuming that the object has RA = α at the time of local transit (2.7.1) shows that:

$$\alpha = \mathrm{LSTtr} \tag{2.7.2}$$

If JD is the Julian date for the day at 0^{hr} UT then with t = (JD – 2451545)/36525 measured in Julian centuries the Greenwich Mean Sidereal Time T_0 (GMST) is given by:

$$T_0 = 6^h 41^m 50^s.54841 + 8640184^s.812866 * t + 0^s.093104 * t^2 - 0^s.0000062 * t^3 \tag{2.7.3}$$

Then for the observer at the place with geographic longitude L (negative for west longitudes and positive for the east) the Greenwich sidereal time of transit is:

$$\mathrm{GSTtr} = \mathrm{LSTtr} - \mathrm{L} \tag{2.7.4}$$

And the Sidereal time elapsed since the 0^{hr} UT of the date is:

$$\mathrm{T_t} = \mathrm{GSTtr} - \mathrm{T_0} \tag{2.7.5}$$

This time is then converted to the UT by:

$$\mathrm{UTtr} = \mathrm{T_t} * 1.002737909375 \tag{2.7.6}$$

Finally the coordinates of the Sun are recalculated for UTtr and the following calculation is performed repeatedly:

$$LST = \alpha, \tag{2.7.7}$$

$$GSTtr = LST - L, \tag{2.7.8}$$

$$T_t = GSTtr - T_0 \tag{2.7.9}$$

$$UTtr = T_t * 1.002737909375 \tag{2.7.10}$$

So that there is a difference of less than a second between one value of UTtr and its next value.

For the local sunrise and sunset the base value is the UTtr and initial approximations for the sunrise is UTtr – 6^{hr} = UTrs and that of the sunset is UTtr + 6^{hr} = UTst. Depending on the local longitude L, UTrs and Utst may fall on previous or next day respectively so that a necessary date adjustment must be done. Further depending on the local latitude it is further possible that these phenomena simply don't occur.

The process begins with calculation of the coordinates of the Sun for UTrs (or Utst). The hour angle HA of any point of sky setting or rising over a local horizon is given by:

$$CosH = -Tan\varphi . Tan\delta \tag{2.7.11}$$

where φ is the geographic latitude of the observer and the altitude of the point of sky is assumed to be zero. However owing to the phenomenon of refraction a star, the Sun and a planet are well below the horizon when they are actually seen setting or rising. The average affect of refraction is that a star remains visible even if it has gone 34 arc minutes below horizon. This altitude is known as standard altitude denoted as a_0 and is considered

to be -50 arc seconds on average for the Sun, that includes the affects of the refraction and the semi diameter both. For a more accurate value the actual semi diameter SD_s should be calculated from the distance of the Sun and subtracted from the average affect of refraction. The change in temperatures in the middle latitudes may vary this by around 20 seconds of time and the barometric pressures may cause a variation of another 12 seconds of time. However as these variations can not be determined a priori the average affects are considered in calculations. Using the standard altitude a_0 the hour angle of the object is then evaluated as:

$$CosH_0 = \frac{Sina_0 - Sin\varphi.Sin\delta}{Cos\varphi.Cos\delta} \tag{2.7.12}$$

Thus the first approximation for the time of rising of the object is:

$$T_r = UTtr - H_0 \tag{2.7.13}$$

And that of the setting is:

$$T_s = UTtr + H_0 \tag{2.7.14}$$

These are only the first approximations for the times of sunrise and the sunset respectively. Formulating the time arguments for each of them separately, the coordinates of the Sun have to be calculated again. The Greenwich sidereal time corresponding to both T_r and T_s has to be obtained as:

$$\tau_i = T_0 + 360.985647 * T_i/15 \tag{2.7.15}$$

where T_i is either T_r or T_s so that local hour angle and the azimuth of the object is obtained using:

$$H_i = \tau_i - L - \alpha_i \tag{2.7.16}$$

And $$SinA_i = Sin\varphi.Sin\delta_i + Cos\varphi.Cos\delta_i.CosH_i \tag{2.7.17}$$

The corrections for the rising and setting are:

$$\Delta T_i = \frac{A_i - a_0}{360.Cos\delta.Cos\varphi.SinH_i} \tag{2.7.18}$$

Adding these ΔT_i into the appropriate T_i gives improved values for the two events. For further improved values calculate the coordinates of the Sun for respective values of T_i and repeat (2.6.12) and (2.6.18) to obtain the UT for the events.

For the Moon the issue is more complicated as affect of parallax is significant. The affect of refraction is to decrease the zenith distance so that the object is visible even if it is theoretical gone down the horizon but the affect of parallax is to increase the zenith distance so the object is well above the horizon and it appears to have set (or not risen still). Thus for the Moon the standard altitude is given by:

$$a_0 = \pi - 0.2725 * \pi - (0^0 34') \tag{2.7.19}$$

where π is the parallax of the Moon given by:

$$Sin\pi = \frac{\rho}{R} \tag{2.7.20}$$

ρ is the geocentric distance of the observer and R, the geocentric distance of the Moon. ρ is given by:

$$\rho = \frac{r\sqrt{1-(2e^2 - e^4)Sin^2\varphi}}{\sqrt{1-e^2 Sin^2\varphi}} \tag{2.7.21}$$

r is the equatorial radius of the Earth, $e = \frac{\sqrt{r^2 - r_p^2}}{r}$ and r_p is the polar radius of the Earth. The rest of the calculations for the transit, rising and the setting of the Moon are the same as that for the Sun.

2.8 NEW LUNAR CRESCENT VISIBILITY PARAMETERS

A number of parameters have been considered important for determining whether the new crescent would be visible at a location on the Earth or not. These were briefly discussed and listed in the beginning of chapter 1. Once the time of the Birth of a particular New Moon or the conjunction has been determined a number of parameters are required to be determined. These include (i) Time of Sunset T_s, (ii) Time of Moonset T_m, (iii) LAG $T_m - T_s$, (iv) Best Time of Visibility T_b, (v) Age of the Moon at T_b, AGE, (vi) Arc of Vision ARCV, (vii) Relative Azimuth DAZ, (viii) Arc of Light (Elongation) ARCL, (ix) Phase of Crescent P, and (x) Width of Crescent W. In view of the discussion of the astronomical algorithms and techniques in this chapter these circumstances are re-considered to explore computations of various parameters that are important for the analysis of local visibility of the new lunar crescent on the day of conjunction or the day after.

The first of these parameters is the conjunction of the Moon with the Sun or the time of Birth of New moon. The algorithm for the computation of this time was presented in article 2.3. Using this algorithm the time of "geocentric birth" of the New moon is obtained. The algorithm takes year as input and gives the Julian date of the time of the birth of new moon. It is important to note that the input "year" is not a whole number, it is a real number calculated on the basis of the expected date of the New Moon. For instance the New Moon in the month of April, 2007 is expected around 17th day of the month, thus:

$$year = 2007 + (3*30+17)/365$$

An approximate value of "year" with an error of few days works well. If the "year" is a whole number the algorithm gives the Julian Date for the New Moon that occurs closest to the beginning of the "year".

Once the Julian Date of the birth of the New Moon closest to the expected day has been found the same is initially converted to the Universal time and date and consequently the local time and date. For these conversations from Julian Date to the local time and date the techniques of the article 2.4 are used. Before this time the new "lunation" has not begun so no question of the visibility of the new lunar crescent on the evening before this time. Before this time only the "old crescent" can be last seen before the sunrise on the day of conjunction or a day or two before.

At the time of conjunction the Moon may be anywhere within a strip of width 10^0 18' around the ecliptic, i.e. within 5^0 9' of the Sun. Apart from the occurrence of a solar eclipse the "crescent" exists but due to its extreme closeness to the glaring sun it can not be seen and has never been seen.

The next important parameter is the local time of sunset and the Moon set. These can be computed for any day of the year using the techniques of the article 2.7. However these techniques require the determination of the coordinates of both the Sun and the Moon for an expected time of transit, rising and setting of each of them. The coordinates of the Sun are obtained using the VSOP87 theory of Bretagnon and Francou (or a simplified version given by Meeus) and briefly presented in article 2.2 and 2.6. Similarly the precise coordinates of the Moon are obtained using the ELP2000 of Chapront-Tozė and Chapront presented in article 2.2 and 2.5. The coordinates of the Sun and the Moon this way are the geocentric spherical polar coordinates (distance R, celestial longitude λ and the celestial latitude β). After correction for aberration the same are transformed to the geometric equatorial coordinates, right ascension α and declination δ. Finally the topocentric right ascension and declination are obtained taking into consideration the

affects of parallax. Using the local sidereal time the coordinates are then transformed into local horizontal coordinates the altitude and azimuth.

Once the time of local sunset (T_s) and that of Moon set (T_m) are obtained the parameter LAG = $T_m - T_s$ is calculated. Unless the LAG is positive for the New Moon and negative for old Moon the crescent can never be seen.

Suppose $(r_m, \lambda_m, \beta_m)$ and $(r_s, \lambda_s, \beta_s)$ are the precise distances, ecliptic longitude and the latitude of the Moon and the Sun, respectively, referred to the mean equinox of the time of sunset at any location on the Earth with terrestrial coordinates (l, ϕ) on the day, or day after, the birth of New Moon (also called the Geocentric Conjunction of the Moon). The first step in the determination of the visibility of the new lunar crescent, on the day of conjunction or the day after, is to determine the actual dynamical time (TD or TT) T_c, of the conjunction. Next one requires considering the local times of setting of the Sun and the Moon. Let, T_s and T_m (Coordinated Universal Time TUC) be the times of the local sunset and the moonset, with $T_s < T_m$.

Using the ecliptic coordinates of the sun and the Moon recalculated for the T_b, the equatorial coordinates of the two bodies (α_m, δ_m) and (α_s, δ_s) are calculated using (Meeus, 1998):

$$Tan(\alpha) = \frac{Sin(\lambda)Cos(\varepsilon) - Tan(\beta)Sin(\varepsilon)}{Cos(\lambda)} \tag{2.8.2}$$

and
$$Sin(\delta) = Sin(\beta)Cos(\varepsilon) + Cos(\beta).Sin(\varepsilon)Sin(\lambda) \tag{2.8.3}$$

Where α, the right ascension is in the same quadrant as λ, and ε, the obliquity of the ecliptic is also adjusted for the date and the best time T_b. Local Hour Angle H is then obtained from the difference of the local Sidereal Time (*LST*) and the right ascension. This finally gives the local horizontal coordinates azimuth (A) and the altitude (h) by:

$$Tan(A) = \frac{Sin(H)}{Cos(H)Sin(\phi) - Tan(\delta)Cos(\phi)} \quad (2.8.4)$$

and $$Sin(h) = Sin(\phi)Sin(\delta) + Cos(\phi)Cos(\delta)Cos(H) \quad (2.8.5)$$

After adjusting for the refraction and the height of the observer's location above see level the topocentric coordinates (A_m, h_m) and (A_s, h_s) of the Moon and the Sun, respectively are obtained.

In almost all the models for earliest moon-sighting, the ancient as well as the modern, the difference of azimuths (DAZ = $|A_s - A_m|$, called relative azimuth) and that of altitudes (ARCV = $h_m - h_s$, called arc of vision) as shown in the Fig: 1, at the time of local sunset T_s and/or at the best T_b play a vital role:

As the angular separations involved between the Sun and the lunar crescent at these times are small, without much error the arc of light (ARCL) is given by:

$$\text{ARCL} = \sqrt{(DAZ)^2 + (ARCV)^2} \quad (2.8.6)$$

Whereas for larger angles or more accurate results the arc of light should be calculated using:

$$\text{ARCL} = Cos^{-1}(Cos(h_m)Cos(h_s)Cos(DAZ) - Sin(h_m)Sin(h_s)) \quad (2.8.7)$$

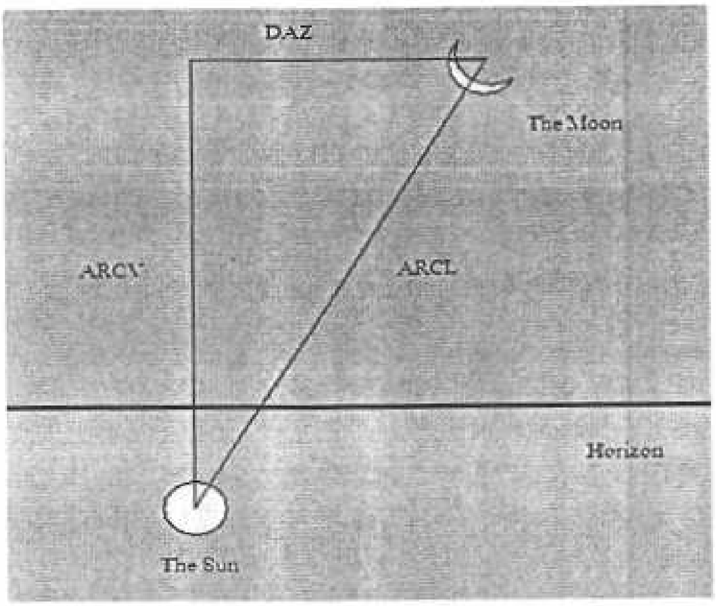


Fig. 2.8.1

Apart from the relative azimuth, the arc of vision and the arc of light the criteria for earliest visibility of lunar crescent requires to take into consideration a number of other parameter. One such parameter is the Age of the Moon (AGE) defined as the time elapsed since the last conjunction till the time of observation. Another important factor is the Width of Crescent (W) that depends on the distance of the Moon. The great astronomer Al-Battani of Baghdad had realized the importance of crescent width a thousand years ago (Bruin 1977). As the distance of the Moon from the earth varies from around 0.34 million km to around 0.4 million km the semi-diameter of the lunar disc varies from 15 arc minutes to 16.5 arc minutes. Thus if the Moon is closest to the earth at the time of observation the crescent would be widest and thus brightest. Width of the crescent is directly proportional to the Phase (P) of the Moon that is a function of the ARCL:

$$P = \frac{(1 - Cos(ARCL))}{2} \qquad (2.8.8)$$

And then the crescent width is given by:

$$W = P \times \left(\frac{Radius\ of\ Moon}{Dist.of\ Moon} \right) \qquad (2.8.9)$$

Since the ancient and the medieval times the AGE, the time elapsed since the birth of New Moon till the sunset of the day in question, was generally considered a significant parameter. However, in the time of Muslim/Arab astronomers it had already been realized that the AGE is not a fundamental parameter, still the parameter is important to calculate. This is because of the fact that depending on the coordinates of the Moon and the seasons at time a very "young" crescent can be seen. Amongst both the amateur as well as the professional astronomers there is always a competition for having the record to see the "youngest" crescent either with optical aid or without it.

Amongst the early models of new crescent visibility astronomers used the relation between the relative altitude (ARCV = altitude of the Moon – altitude of the Sun) and the relative azimuth (DAZ = |azimuth of the Sun – azimuth of the Moon|). The two parameters are still considered important as if DAZ = 0, the crescent is vertically above the point of sunset and under such a circumstance the youngest as well as the thinnest crescent can be seen. With larger DAZ values only older and the thicker crescent can be seen.

How thick or thin is the crescent at any time can be determined once the separation (elongation) between the Sun and the Moon has been determined using (2.8.7) with either horizontal coordinates or the equatorial coordinates. This elongation or Arc of Light (ARCL) leads to the Phase (P fraction of the illuminated lunar disc facing the observer) of the Moon using (2.8.8). However, the thickness or the Width (W) of the central part of the crescent does not only depend on the phase of the Moon, it also depends on the Earth-Moon distance, the same can be obtained using (2.8.9). The best time of the crescent visibility suggested by Yallop is critical for sighting the crescent under marginal conditions and can be computed using (2.8.1). Sighting is critical when the LAG is very small.

2.9 THE SOFTWARE HILAL01.CPP

In this work a software is developed for the analysis of the first visibility of new lunar crescent that is similar in nature as the MoonCalc by Manzur (Manzur, 2001) and Accurate Times by Odeh (Odeh, 2006) but that can be used to compare all the computational and the ancient and modern visibility models. The listing of the program Hilal01.cpp is given in Appendix 4. The program features are briefly described below:

The program LinAnal uses three data files, two for input and one for output. The two input data files are the files ***vsopeart.txt*** that contains the parameters a_i, b_i and c_i (as described in article 2.6 above). The parameters of this data file are arranged in tables 2.1 (A) to 2.3 (F) in appendix 2. The other input file is the ***elp2000.txt*** that contains the parameters as described in the article 2.5 above. The parameters of this data file are arranged in tables 1.1 (A) to 1.3 (D) in appendix 1.

The third file used by the program is ****.txt*** that is optional and is used only when the results of the computations in the program are required to be stored. The version of the program Hilal01 given in appendix 4 has the file name ***schrnge.txt*** that stores the computational results of a single execution of the program. The informations stored in this output data file are:

1. O. No., the observation number that is generally assigned by Odeh (Odeh, 2004) ,
2. Date, date of observation,
3. Long., the longitude of the place from where the crescent is observed
4. Latit., the latitude of the place
5. Elev., the elevation of the place above sea level,
6. Temp., estimated temperature of the time of observation,
7. Humid., estimated relative humidity of the place at the time of observation
8. Sunset, the local time of sunset

9. JD of Conjunct., the Julian Date of the birth of new Moon or the last conjunction

10. Age, the age of the Moon at the best time according to Yallop (Yallop, 1998), in hours.

11. LAG, the difference between the Moon set and the sunset, in minutes.

12. ARCL, arc of light or elongation of the Moon from the sun at the best time, in degrees.

13. ARCV, the relative altitude of the crescent at the best time, in degrees.

14. DAZ, the relative azimuth at the best time, in degrees.

15. Width, the central width of the crescent at best time, in arc minutes.

16. q-val, the visibility parameter defined by Yallop (Yallop, 1998) to be discussed in detail in chapter 4.

17. Phi+delta, the angle that the ecliptic makes with the vertical on the western horizon, in degrees.

18. Mlatit, Moon's ecliptic latitude in degrees.

19. Mlongit, Moon's ecliptic longitude, in degrees.

20. Slongit., Sun's ecliptic longitude, in degrees.

21. M-SD, angular semi-diameter of the Moon, in arc minutes.

22. As-Fact, arc of separation factor defined in later chapters, in degrees.

23. R-Est, estimated Ripeness Function value defined in chapter 3.

24. R-Aver, average Ripeness function value defined in chapter 3.

25. R-actual, actual Ripeness Function value defined in chapter 3.

26. DR-est, the difference of R-actual and R-Estimated.

27. DR-act, the difference of R-actual and R-average.

28. Moon's Mag, Moon's magnitude at the best time of Yallop,

29. Lim-Mag(time), the Limiting Magnitude of the sky near the crescent as defined by Schaeffer (Schaefer, 1988b) along with the Yallop's best time in universal time discussed in chapter 4.

30. Start(DMag), the universal time when the contrast of the sky brightness and the Moon's brightest just turn in favour of the Moon and the difference of the

magnitude of the Moon and the limiting magnitude of the sky at that moment, discussed in chapter 4.

31. Best(Dmag), the universal time when the contrast of the sky brightness and the Moon's brightest is best in favour of the Moon and the difference of the magnitude of the Moon and the limiting magnitude of the sky at that moment, discussed in chapter 4.

32. Last(Dmag), the universal time when the contrast of the sky brightness and the Moon's brightest is last in favour of the Moon and the difference of the magnitude of the Moon and the limiting magnitude of the sky at that moment, discussed in chapter 4.

As the execution of the program begins it prompts for observation number, date, month, year, longitude, latitude and elevation above sea level of the place and the estimated temperature and estimated relative humidity of the place. This prompt is initiated by the function ***inputdatetime*** called by the function ***mainroutine***.

Then the program calls for the function ***month_change***. This function first determines the Julian Date of the time of nearest conjunction or the birth of new Moon calling the function ***dt_new_moon***. The function ***dt_new_moon*** is based on algorithm due to Meeus (1998) discussed in article 2.3 above (formulas 2.3.1. to 2.3.25). The function ***month_change*** then calls the function ***setting_rt*** that determines the times of the sunset and the Moon set through functions ***sun_set*** and ***moon_set***. The functions ***sun_set*** and ***moon_set*** are based on the algorithm discussed in article 2.7 above. The function ***setting_rt*** also calculates the best time for crescent visibility according to condition due to Yallop (1998) discussed in a later chapter. This is followed by computation of the Julian Date and the Extended Julian date corresponding to the best time using the function ***juliandate*** that is based on the algorithm presented above in article 2.4. This leads to formulation of the time argument (also discussed in article 2.1) for the ELP2000 and VSOP87 theories for the calculation of coordinates of the Moon and the Sun respectively. These coordinates are calculated according to the algorithms in articles 2.5 and 2.6 implemented in functions ***moon_coord*** and the ***sun_coord*** respectively. Before

these coordinates are calculated the effect of nutation and the sidereal time corresponding to zero hour universal time, for the date considered, are calculated using the functions ***nutation*** and ***sid_time***, respectively. The information/data thus far generated is displayed on screen using the functions ***outinfo***, ***display_scoord*** (coordinates of the sun) and ***display_mcoord*** (coordinates of the Moon).

Finally, the function ***mainroutine*** calculates and displays on screen all the visibility parameters discussed in previous article (2.8) and listed as 10 to 16 and 21 above in the current article. Other parameters (17 to 20 and 22 to 32) are computed in other functions (17 to 20 in function ***moon_coord*** and 22 to 32 in ***limmagit***) but displayed along with these parameters. All the parameters listed as 1 to 32 are written to the output data file ***schrnge.txt*** in functions ***fileout1*** (parameters 1 to 29) and ***time_range*** (parameters 30 to 32). The function ***limmagnit*** is the reproduction of Schaefer's program (Schaefer, 1998, Bogan, 2004) for determining the limiting magnitude of any point of sky.

The rest of the functions used in the program are listed and briefly described below:

- ***leapcheck*** checks whether the year used is leap year (according to Gregorian rule) or not? If the year is leap the function returns 1 otherwise 0.
- ***convert_dms*** converts an angle into degrees, arc minutes and arc seconds.
- ***convert_hms*** converts time in hours into hours, minutes and seconds.
- ***modfunc*** returns the remainder after dividing the input by 360. this function is used obtain the angle in the range 0 degrees to 360 degrees.
- ***paralx*** calculates the affects of parallax in right ascension and declination to get the topocentric right ascension and declination and is based on step 8 of algorithm in article 2.5.
- ***inc_sec*** through ***inc_mon*** and ***dec_sec*** through ***dec_mon***: These functions allows changing time (at levels of hours, minutes and seconds) and date (at levels of date and month).

In fact within the function ***mainroutine*** the program Hilal01 has a *do-while* loop that terminates when the identifier *nexts* receives '\r' (pressing enter key) from the user. If any other key is pressed the program remains in wait state. Pressing particular keys as is obvious in the *switch-case* combination various actions are initiated like increasing or decreasing time and date, changing temperature or humidity or writing to the output file ***schrnge.txt***. Initiating any of these actions repeats the execution of all the computations with new time, date, temperature or humidity. In case of pressing *p* all the data is written to the out file ***schrnge.txt*** in one line and pressing *q* writes only the selected values of time and corresponding value of the difference of moon's magnitude and the limiting sky magnitude. These features of the program Hilal01 (varying time, date and weather conditions) make the program more dynamic as compared to MoonCalc of Manzur (Manzur, 2001) and Accurate Time of Odeh (Odeh, 2006).

In view of the problem of determining the day of the first sighting of new lunar crescent in this chapter we explored all the major aspect of computational efforts. These include:

- The semi-analytical dynamical theories VSOP87 and ELP2000 that describe the motion of planets round the Sun and of the Moon round the Earth. These are the most recent and most accurate available tools for the determination of ephemeris of the Sun and the Moon.

- The algorithm that leads to the determination of the dynamical time of the lunar conjunction or the birth of new Moon.

- The problems associated with the dynamical and universal time and related issues. Without having a complete know how of these issues appropriate time argument for the determination of lunar and the solar coordinates can not be formulated.

- The all important algorithms for the determining the universal and local zone times of the sunset and the moonset. Without accurate determination of these times the parameters associated with the problem of earliest sighting of new lunar crescent can not be obtained.

- With a thorough understanding of all these details of astronomical techniques and algorithms a computer program is generated to test all the new lunar visibility criteria.

Thus the chapter encompasses all the computational details associated with the problem of the determining the day of first sighting of new lunar crescent.

Chapter No. 3

ANCIENT, MEDIEVAL & EARLY 20TH CENTURY MODELS

For calendarical purposes as well as an interesting and challenging astronomical problem, in the history of mankind there have been consistent efforts to determine when and where the new lunar crescent will be first seen. The results of these efforts have been numerous and have been based on different techniques and tools. These results can be termed as "Models/Criteria for the earliest visibility of new lunar crescent". Most of the early models are derived directly from the observations of the new lunar crescent and are empirical in nature. Some are based on theoretical considerations. Most important of these efforts include:

i) The Babylonian rule of thumb "when the age of the new Moon is 24 hours only then the crescent can be seen". The rediscovery of the more mathematically involved criterion of Elongation + LAG > 21° that was enunciated well before Christian era (Fatoohi et al, 1999).

ii) The criteria developed in the medieval periods by Muslims/Arabs based on observation, spherical trigonometry and the ephemerid of the Moon and the Sun (Bruin, 1977). This happened as early as the 10th century AD.

iii) The exploration of Fotheringham (1910), Maunder (1911) and others in the first quarter of the twentieth century. These efforts were more of statistical in nature based on the observations mostly in Room by Schmidt and others.

In this chapter we start with a brief discussion on relatively recent re-discovery of the Babylonian criteria for the earliest visibility of new lunar crescent. A set of observational data selected from the recent literature is used to examine the merits of the Babylonian criterion. This is followed by some spherical trigonometric considerations that are significant for the conditions on which the visibility or the invisibility of the new lunar crescent may depend.

On the basis of the trigonometric consideration the Lunar Ripeness function attributed to the Arabs is explored and a Modified Lunar Ripeness Law is suggested and examined in view of the recorded observations in the data set mentioned above. This is done using the current knowledge, tools and techniques and the results of this exploration are presented. All computational work is done using the software Hilal01.

In the end of the chapter the empirical models due to modern astronomers of the early 20^{th} century are re-visited and comparison of the same is done with the Babylonian and the medieval models.

3.1 BABYLONIAN CRITERIA

No trigonometry or spherical trigonometry existed till the mathematical developments in Arabia in the 9^{th} and the 10^{th} century AD; the problem was only described in terms of certain observable parameters shown in the Figure No.3.1.1 in the ancient and the early Christian era. If any visibility criterion was deduced must have been based on the observations and measurements of these observable quantities. The figure shows the western horizon at a time when the sun S has gone s degrees below horizon (the solar depression) and the crescent at M is just visible. AB is the equator that makes an angle φ, the geographical latitude of the place, with the normal to the horizon. SM is the separation between the Sun and the Moon commonly known as "arc of light" and

abbreviated as ARCL but shown as a_L in the figure. SD is the difference between the altitudes of the sun and the Moon know as "arc of descent" denoted as a_D (also called "arc of vision" and abbreviated as ARCV). Note that the altitude of the crescent above horizon is h so that $a_D = s + h$. As the point A is at the same altitude as the crescent and B at the same depression as the sun, AB is equatorial separation between the Sun and the Moon called "arc of separation" and is equivalent to the Moonset – Sunset Lag shown in the figure as a_S. The figure can be used to measure the arcs of descent and light for a given latitude φ once an accurate enough ephemeredes shows the arc of separation and where to look for the dimly illuminated crescent against the bright evening twilight.

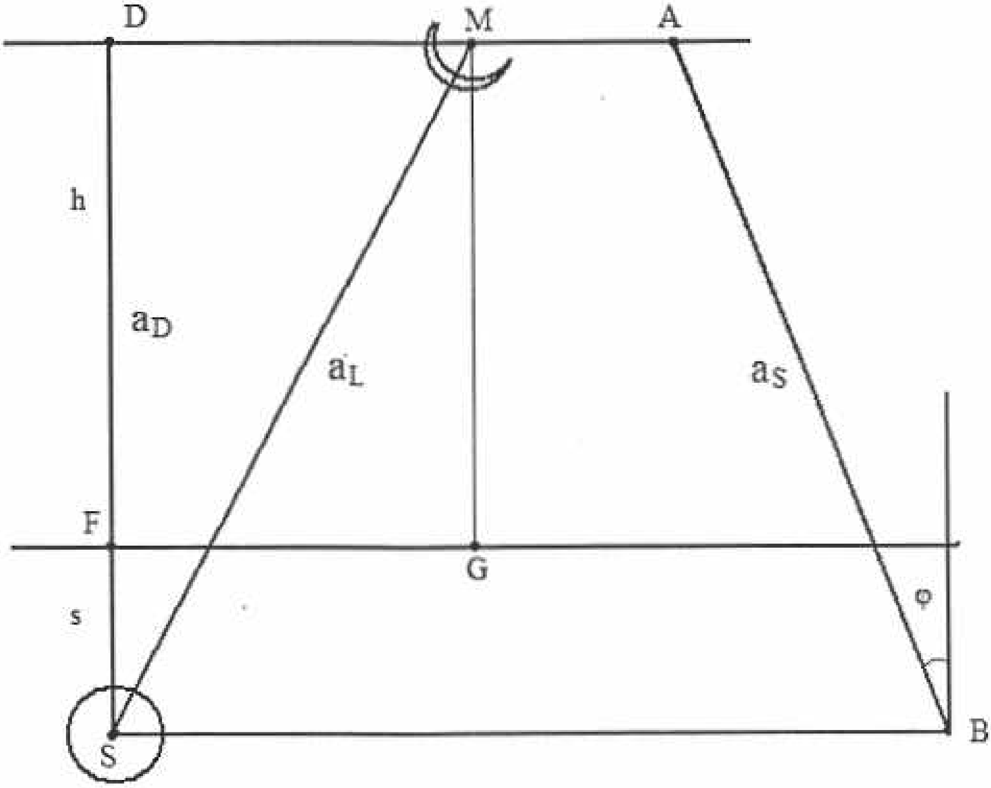


Fig No. 3.1.1 Angular Parameters associated with sighting of new lunar crescent

In modern times the earliest reference of any systematic study of the earliest sighting of new lunar crescent is that due Fotheringham (Fotheringham, 1910). He refers to 12[th] century Jewish philosopher Maimonides. According to Fotheringham, the work appeared in the Treatise by Maimonides (Moshe ben Mamoon) on the *Sanctification of the New Moon* (Mishneh Torah: Sefer Zemnim – Hilhot Kiddush HaHodesh, 1178 that

was translated by S. Gandz as *Code of Maimonides, Book Three, Treatise Eight, Sanctification of the New Moon,* Yale Judaica Series, volume XI, Yale University Press, New Heaven CT, 1956). Maimonides work was studied by von Littrow in *Sitzunghberichte der Wiener Akademie, Math-Nature, Classe,* lxvi, (1872), pp. 459-480. In his work Maimonides makes the smallest visible phase of the Moon dependent on two variables, as claimed by Fotheringham. These variables have been termed as *the true elongation of the Moon* and the *apparent "angle of vision"*. Fotheringham states that if by *angle of vision* Maimonides means the difference of the Sun and the Moon in zenith distances then the rule of Maimonides and his own rule for minimum visible phase of Moon are nearly the same. However he further says that Maimonides rule gives slightly lower minimum altitudes that could be due to better observing conditions in Jerusalem than in Athens. Unfortunately Fotheringham admits that he could not sufficiently understand Maimodies' arithmetical method and has not given a detailed account of the same (Fotheringham, 1910).

Later, Bruin (Bruin, 1977) gives a more detailed account of any medieval attempt of solving the problem of finding astronomical conditions for the first visibility of crescent. Describing the Islamic astronomical procedures Bruin says that Al-Khwarizimi gives mathematical rules and tables for predicting the new crescent whereas Al-Battani presents a complete solution. He also mentions the later account of Moses ben Mamoon and claims that ibn Mamoon largely follows Al Battani. One of the most important aspect of Bruin's account of the efforts of the medieval era is that Arabs/Muslims of this era had already realized the significance of crescent width. This important aspect remained missing from all the major contributions of twentieth century before Bruin. Based on this description and the modern knowledge in this work the same has been explored more extensively and is presented later in the chapter.

In most of the literature that appeared during 20th century the general rule of thumb attributed to the Babylonians for first visibility of new lunar crescent is:

The Age of the Moon at the time of local sunset should be greater than 24 hours and the Lag between the sunset and the moonset should be greater than 12 time degree. (3.1.1)

12 time degrees is equivalent to $\frac{4}{5}$th of an hour or 48 minutes (Bruin, 1977, Ilyas, 1994a, Yallop, 1998).

However, it was Fatoohi, Stephenson and Al-darghazalli who have explored the efforts of the Babylonians extensively and have reported that according to the historical records of new crescent sightings of pre Christian era the criterion attributed to Babylonians is an over simplification (Fatoohi et al, 1999). According to their study of 209 records of positive new crescent sighting extracted from Babylonians Astronomical diaries, the Babylonians had succeeded in formulating a truly mathematical lunar theory which they used to predict various parameters of lunar motion. Unfortunately, without presenting any theoretical details of Babylonian era they appear to be critical about Bruin's suggestion that Babylonian criterion was what is mentioned above (3.1.1). Instead, they claim that Babylonian criterion was as Follows:

The new crescent is seen if:
"*Elongation (ARCL) + moonset-sunset lag time (LAG) in degrees > constant*". (3.1.2)

Even, this criterion is reported on basis of the suggestions of Neugebauer (Neugebauer, 1955) and not as a result of any description of a mathematical theory. They have mentioned two systems of solar motions on which the lunar theory of Babylonians was based, but how the new rule they have attributed to the Babylonians was arrived at is not given. They have gone on to present different values for the constant on the right hand side of (3.1.2). These values range from a minimum of 21° to a maximum of 23°. In the present work we adopt the following as a better criterion attributed to the Babylonians as suggested by Fatoohi et al:

$$ARCL + LAG > 22^0 \qquad (3.1.3)$$

Table No. 3.1.1

Ser.	Date	Latit	Long	Visibility			Age	LAG	ARCL	ARCV	DAZ	ARCL+LAG
No.		Deg	Deg	N	B	T	Hrs	Min	Deg	Deg	Deg	Deg
549	7/9/2002	31.1	56.5		V		11.6	34.8	8.44	8	2.7	17.15
477	19/8/2001	29.5	56.8		V		12.2	35	8.47	7.95	2.92	17.23
682	16/8/2004	32	35.9			V	15.2	36.1	8.58	7.96	3.21	17.6
455	25/3/2001	-34	18.4	V			15.8	36.9	9.06	8.3	-3.6	18.29
274	25/2/1990	35.6	-83.5	V		V	14.8	39.3	8.53	8.51	-0.6	18.35
275	25/2/1990	35.6	-83.5			V	14.8	39.3	8.53	8.51	-0.6	18.35
478	19/8/2001	30.2	35.5		V	V	13.6	37.8	9.24	8.47	3.69	18.7
375	10/9/1999	30.4	35.5			V	18.1	35.9	9.84	8.31	5.26	18.81
557	5/11/2002	29.9	56.2		V		17.1	35.2	10.1	7.82	6.4	18.91
416	31/7/2000	6.5	3.4	V			15.9	37.7	9.58	9.41	1.8	19
386	8/12/1999	36.8	10.4		V		17.8	41.6	8.68	8.07	3.21	19.08
312	20/1/1996	34.1	-118			V	12.7	41	8.92	8.78	-1.6	19.17
558	5/11/2002	30.1	52.1		V		17.4	35.7	10.3	7.9	6.56	19.18
559	5/11/2002	29.6	52.5		V		17.4	35.8	10.3	7.95	6.48	19.19
391	7/1/2000	32.7	52.3		V		19.7	40.9	9.01	8.36	3.36	19.24
412	2/7/2000	2.3	102.4		V	V	16.3	39	9.69	9.5	1.92	19.43
484	17/10/2001	2.3	102.4		V	V	15.9	38.1	9.93	9.92	-0.4	19.47
389	7/1/2000	-34	18.4	V			24.1	35.1	11	7.19	-8.3	19.74
566	2/4/2003	30.2	35.5		V	V	20.9	39.4	10.1	9	4.48	19.9
638	22/1/2004	30	51.7		V		17.2	36.3	10.9	7.7	7.69	19.96
560	5/11/2002	31.9	35.8		V		18.4	37.2	10.9	8.04	7.31	20.15
593	2/5/2003	5	114.9			V	22.5	40	10.2	10.1	-1.3	20.17
135	15/3/1972	35.5	-118		V		14.8	42	9.69	9.35	-2.5	20.19
432	26/12/2000	30.2	35.5		V	V	21.7	41.9	9.87	8.67	4.72	20.34
281	24/5/1990	35.6	-83.5			V	12.7	47	8.67	8.66	-0.5	20.42
321	7/5/1997	32.7	52.3		V		18.8	37.8	11.1	8.06	7.57	20.5
404	5/4/2000	5.3	102.9		V		17.4	40	10.6	10.4	2.12	20.62
301	1/1/1995	33	-106			V	13.5	46.5	9.05	9.05	-0.3	20.68
594	2/5/2003	3.2	101.7			V	23.4	41.3	10.6	10.4	-1.7	20.88
543	9/8/2002	2.3	102.4			V	16.4	41.6	10.5	10.4	-1	20.89
607	28/8/2003	5.3	102.9		V	V	18.2	42.4	10.9	10.9	0.4	21.48
304	31/1/1995	35.6	51.3		V		15.6	47	9.82	9.66	-1.8	21.57
573	3/1/2003	32.5	3.7		V		20.8	40.1	11.6	7.96	8.46	21.62
715	13/11/2004	4.9	114.8		V		19.9	40.3	11.6	9.86	6.16	21.7
688	15/9/2004	36.6	59		V		24	37.3	12.4	8.14	9.41	21.77
319	7/5/1997	31.8	34.9	V			19.9	40.9	11.6	8.74	7.68	21.86
290	15/2/1991	33.4	73.1	V			19.7	46.9	10.2	10.1	-0.9	21.89
434	26/12/2000	-32	20.8	V			24.7	42.9	11.2	8.62	-7.2	21.95
435	26/12/2000	-32	20.8		V		24.7	42.9	11.2	8.64	-7.2	21.95

The reason at arriving this criteria as described by these authors is that (i) the arc of light describes how bright is the crescent and (ii) the moonset-sunset lag describe how long the crescent would remain above horizon after the sun set. More is the value of each of these parameters more is the possibility of sighting of the crescent. However, there has to be a combined minimum of the two and the sighting records were the only means to verify the criterion or arriving at the criterion. Using this condition Fatoohi et al have reported the results of their computations for 399 new crescent sightings that include both the Babylonian crescent sighting records and the sighting records reported in the 20^{th} century literature. The authors have reported that out 399 cases the Babylonian criterion was successful in 98.7% of the positive sighting (crescent claimed to have been seen) cases but failed in 45.7% cases of the negative sighting (crescent not seen) cases.

In recent times a number of organizations have arranged for collection of crescent sighting or non-sighting records. These include Islamic Crescent Observation Project, the South African astronomical Observatory and their websites. Similar records are reported to and collected at the website www.moonsighting.com. Moreover, the largest data set yet available in the published papers is that by Odeh (Odeh, 2004). We have selected 463 of these records of evening crescents selected for the comparison of the models studied in this work.

Using (3.1.3) applied to the data set selected for this work the results are presented in the table no. 3.1.1. In this table only those records are presented when the crescent was reported to have been seen with or without any optical aid and ARCV + LAG is less than 22 degrees. The table is sorted on ARCV + LAG. It shows that there are 7 claims of optically unaided sightings of new crescent not satisfying the Babylonian criterion given by (3.1.3). The complete data set is presented in the Appendix-II. The table in Appendix-II is sorted on visibility column N in descending order, so that all optically unaided visibility cases of crescent sighting in order of ARCV + LAG (in degrees) appear at the top of the table. The table in Appendix-II shows S. No., the observation No. (Odeh, 2004), date of observation, latitude and longitude of the location of observer, visibility 'N' for unaided visibility, 'B' for visibility through binocular, 'T'

for visibility through telescope. The visibility columns are empty if the crescent was not seen and contains 'V' in an appropriate column if the crescent is seen. The rest of the columns of the table contain age of Moon , LAG, ARCL, ARCV and DAZ. The last five columns are for the five models considered in this chapter. The column headed A is for the Babylonian model.

This table shows that out of 196 claims of optically unaided sightings of the new crescent there are only 7 cases not satisfying the Babylonian criterion. However, out of 267 cases when the crescent was not seen without any optical aid ARCV + LAG is greater than or equal to 22 degrees in 107 cases. Thus for positive sightings the criterion is found to be successful in 96.4% cases and amongst the negative sightings it successful in 59.92% cases. The success of a model in describing positive sightings alone is not a real success of a model. The model is not suited well if it is not able to describe the negative sighting as well as it does the positive sightings.

3.2 SOME SPHERICAL TRIGONOMETRIC CONSIDERATIONS

This has been mentioned earlier that for the problem of sighting of new lunar crescent the orientation of the ecliptic plays and important role (article 1.2). Therefore, it is important to revisit the methods that lead to the determination of the angle that the ecliptic makes with the horizon at the time of sunset on the western horizon (it is particularly significant in view of the fact that the new crescent appears close to it). This angle varies as against the fixed celestial equator the orientation of the ecliptic slowly varies through the year. The figure no. 3.2.1 on the next page shows the western part of the celestial sphere with important points and angles are as described below:

- W, the west cardinal point,
- γ, the vernal equinox, intersection of ecliptic and equator,
- S, the setting sun,
- P, the North celestial pole,
- Z, the zenith,

- $\angle S\gamma W = \varepsilon$, the obliquity of the ecliptic,
- $\angle SW\gamma = 90^0 + \varphi$, φ being the latitude of the place,
- $\angle ZS\gamma = \varphi + \Delta$, the angle of the ecliptic with the vertical,
- $PS = 90^0 - \delta$, δ being the declination of the Sun,
- $PZ = 90^0 - \varphi$,
- $S\gamma = \lambda$, the longitude of the Sun

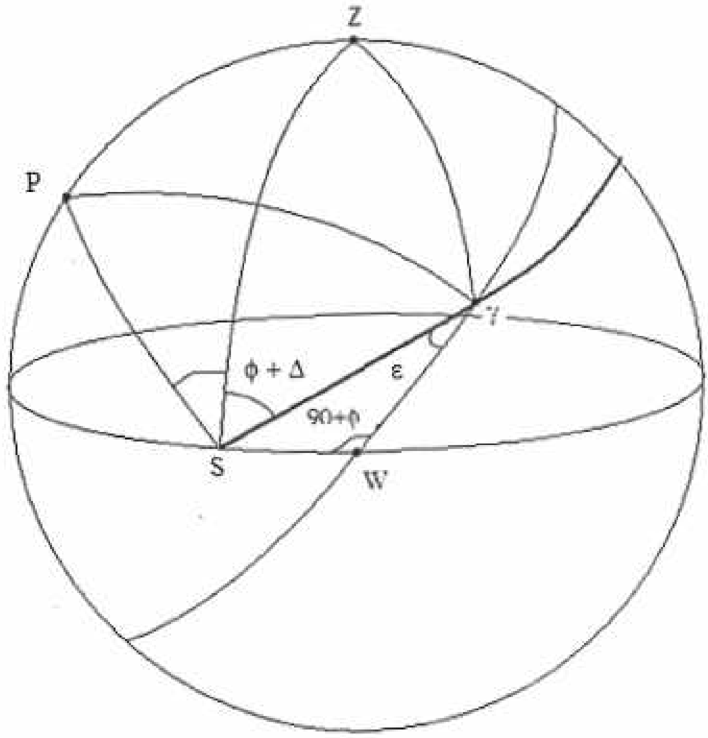


Fig No. 3.2.1

From the figure 3.2.1 in the spherical triangle SPZ applying law of cosine of spherical trigonometry starting from the side PZ one gets:

$$\sin\varphi = \cos\delta\cos P\hat{S}Z \tag{3.2.1}$$

The general relation between declination δ and longitude λ is:

$$\sin\delta = \sin\beta\cos\varepsilon + \cos\beta\sin\varepsilon\sin\lambda$$

as $\beta \approx 0$ for the sun,

$$\sin\delta = \sin\varepsilon.\sin\lambda \qquad (3.2.2)$$

$$\Rightarrow \qquad \cos\delta = \sqrt{1-\sin^2\varepsilon.\sin^2\lambda} \qquad (3.2.3)$$

From triangle SPγ, starting at Pγ:

$$\cos(P\hat{S}Z + \varphi + \Delta) = \tan\delta.\cot\lambda \qquad (3.2.4)$$

Using (3.2.2) and (3.2.3):

$$P\hat{S}Z + \varphi + \Delta = \cos^{-1}\left(\frac{\sin\varepsilon.\cos\lambda}{\sqrt{1-\sin^2\varepsilon\sin^2\lambda}}\right) \qquad (3.2.5)$$

and (3.2.1) together with (3.2.3) leads to:

$$P\hat{S}Z = \cos^{-1}\left(\frac{\sin\varphi}{\sqrt{1-\sin^2\varepsilon\sin^2\lambda}}\right) \qquad (3.2.6)$$

(3.2.5) and (3.2.6) then give:

$$\varphi + \Delta = \cos^{-1}\left(\frac{\sin\varepsilon.\cos\lambda}{\sqrt{1-\sin^2\varepsilon\sin^2\lambda}}\right) - \cos^{-1}\left(\frac{\sin\varphi}{\sqrt{1-\sin^2\varepsilon\sin^2\lambda}}\right) \qquad (3.2.7)$$

This shows that $\varphi + \Delta$ or the angle that the ecliptic makes with the vertical is season dependent as it depends on the longitude of the sun only for a fixed place (or latitude φ). In the rest of the discussion in this article the angle $90^0 - \varphi + \Delta$ is denoted as ψ. In the next article where ever $\varphi + \Delta$ is used it is calculated on the basis of (3.2.7).

In case the declination of the Moon is south of that of the Sun in Northern Hemisphere (and north of the Sun in the southern hemisphere) it is possible that even after conjunction the new lunar crescent sets before the sunset in which case it is simply

impossible to see the crescent. These circumstances are shown in the figure 3.2.2 on the next page that describes:

i) $C\gamma$, the celestial equator, γ is the vernal equinox.

ii) TS is the diurnal path of the Sun that is just setting at S.

iii) DE is the diurnal path of the Moon that set before the sunset at E.

iv) DS = $\delta_M - \delta_S$, perpendicular to the celestial equator is the difference of the declination δ_M of the Moon and the declination δ_S of the sun. Declination of the Moon is south of the sun and the upper limit of DS is $5^0\ 9'$, the inclination of lunar orbit to the ecliptic.

v) $\angle CWS = 90^0 - \phi$ is the angle between the celestial equator and the horizon NS where ϕ represents the latitude of the place. The $\angle ESD = \phi$.

vi) DM = $\alpha_M - \alpha_S$, the difference of right ascension α_M of Moon and the right ascension α_S of the Sun. For higher latitudes DE may be large allowing large values of negative LAG after conjunction.

vii) SF = $\beta_M - \beta_S \cong \beta_M$ is perpendicular to the Ecliptic is the difference between the celestial latitudes of the Moon and the Sun. As for DS, SF never exceeds the limit of $5^0\ 9'$.

viii) FE' (E' not shown in the figure as it almost coincides with E) is parallel to the ecliptic.

ix) γ is the Vernal Equinox and $\angle \gamma SE = \psi$ is the angle of the Ecliptic with the horizon. Depending on latitude ϕ and the season, ψ may vary from zero (when ecliptic is along the horizon) to 90 degrees (when ecliptic is perpendicular to the horizon). In case when ψ is small DE' that has to be parallel to the ecliptic is much larger than DE (that is parallel to the equator) and hence E and E' are much separated. The determination of this angle and its significance shall be discussed in the next article.

x) The figure shows the Sun is just about to set in a place of small to medium latitude ϕ, the Moon having declination south of the Sun has already set (and was set at point E) so that the LAG = Time of Moonset – Time of sunset is negative.

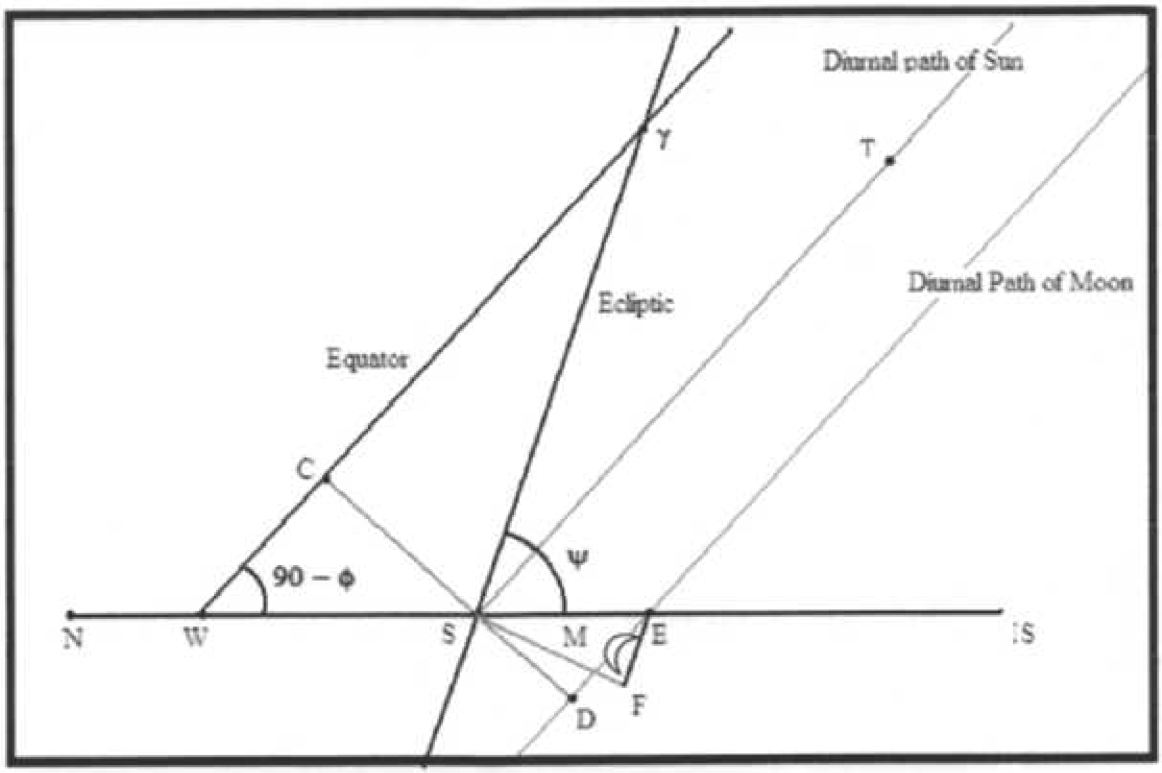


Fig. 3.2.2: Geometry of Positive Age and Negative Lag

According to the lunar calendar based on the birth of New Moon before the sunset the new lunar month begins at the time of sunset on this very evening. However, for the lunar calendars based on the visibility of the new crescent, new lunar month does not begin on this evening as it is simply impossible to see the new lunar crescent in this circumstance. For places with large latitudes that allow for large values of DE and hence DM the triangles under considerations can not be small enough and must be treated as spherical triangles. Here only small to medium latitudes are considered so that the triangle EDS is a small right angled triangle on sky with the angle at D between E and S is 90^0 and the angle at S between D and E is φ, the latitude of the place. Therefore:

$$\cos\varphi = \frac{SD}{ES} \qquad (3.2.8)$$

Similarly the triangle ΔEFS is also small and can be considered a plane right triangle with right angle at F so that:

$$\sin\psi = \frac{SF}{ES} \tag{3.2.9}$$

Replacing SD by $\delta_M - \delta_S$, SF by $\beta_M - \beta_S$ and eliminating ES from (3.2.8) and (3.2.9) we get:

$$\beta_M - \beta_S = (\delta_M - \delta_S)\frac{\sin\psi}{\cos\phi} \tag{3.2.10}$$

Again in ΔSDE we have:

$$\tan\phi = \frac{DE}{SD} \qquad \Rightarrow \qquad DE = (\delta_M - \delta_S)\tan\phi \tag{3.2.11}$$

In the figure 3.2.3 which is only the plane triangle part SDE of the previous figure a requirement for negative LAG is to have DM < SE, but:

$$DM = (\alpha_M - \alpha_S)$$

$$(3.2.11) \Rightarrow \qquad DM = (\alpha_M - \alpha_S) < (\delta_M - \delta_S)\tan\phi \tag{3.2.12}$$

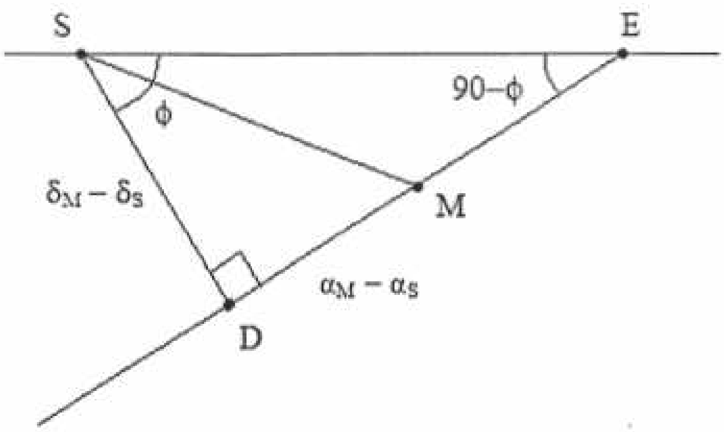


Fig. 3.2.3

using (3.2.10) it translates into:

$$(\alpha_M - \alpha_S) < (\beta_M - \beta_S)\sin\phi / \sin\psi \tag{3.2.13}$$

Thus whenever the New Moon is born just before local sunset the LAG should be negative if condition (3.2.12) or (3.2.13) is satisfied if the crescent is south of the Sun.

In the modern setup one can just calculate the relative altitude ARCV of the Moon at the time of Sunset. If ARCV is negative the LAG has to be negative. Still the conditions (3.2.12) and (3.2.13) show the dependence of the phenomenon on the (i) equatorial coordinates of the Sun and the Moon, (ii) the latitude φ of the place, (iii) the ecliptic latitudes of the Sun and the Moon and (iv) the angle ψ of ecliptic with the horizon. The Table 3.2.1 shows some of the negative LAG cases during the years 2000 to 2010 AD for Karachi, Pakistan (latitude 24.85 degrees, longitude 67.05 degrees).

Negative LAG cases on the day of conjunction rarely occur from small to medium latitudes but are significant as a clear dividing line between conjunctional lunar calendars and the observational lunar calendars. For higher latitude places negative LAG ecen after conjunction may occur more frequently. The columns of the table in sequence from left to right are described below:

- Local date of conjunction.
- Local Time of Conjunction,
- Zone (PST) time of local sunset,
- Age of Moon at local sunset in hours,
- Elongation of Moon from the Sun in degrees,
- Declination of Moon in degrees,
- Declination of Sun in degrees,
- Right Ascension of Moon in degrees,
- Right Ascension of Sun in degrees,

- Latitude of Moon in degrees,
- Angle of Ecliptic with Horizon in degrees,
- LAG, Moonset –sunset in minutes.

Table No. 3.2.1											
Date	Conj	Sunset	Age	ARCL	δ(M)	δ(S)	α(M)	α(S)	β(M)	ψ	LAG
5/2/2000	18:03:18	18:18	0.23	1.14	-17.08	-16.03	21.26	21.23	-1.14	81.4	-3.69
2/6/2000	17:13	19:17	2.03	3.64	19	22.27	4.839	4.724	-3.44	70.5	-3.75
24/1/2001	18:06	18:10	0.03	1.73	-20.78	-19.11	20.49	20.46	-1.73	77.4	-5.27
1/5/2003	17:14:43	19:01	1.77	1.8	13.77	15.06	2.644	2.558	-1.62	82.4	-0.67
23/12/2003	14:43:00	17:48	3.06	3.97	-26.95	-23.44	18.23	18.09	-3.54	63.3	-3.63
22/9/2006	16:45:01	18:28	1.73	0.9	-0.481	0.235	12	11.96	-0.44	41.7	-2.31
11/9/2007	17:44:14	18:40	0.93	1.17	3.403	4.568	11.29	11.29	-1.08	42	-5.42
8/1/2008	16:37:00	17:58	1.36	3.38	-25.48	-22.28	19.35	19.27	-3.31	70.4	-6.19
27/12/2008	17:22:27	17:51	0.48	2.86	-26.13	-23.3	18.47	18.45	-2.85	65.4	-8.81
20/8/2009	15:01:25	19:02	4.01	3.5	9.013	12.28	10.08	9.991	-2.62	44.4	-3.34
16/12/2009	17:02:09	17:45	0.73	2.34	-25.66	-23.33	17.63	17.61	-2.31	60.4	-8.03
8/9/2010	15:29:41	18:43	3.24	5.07	0.529	5.595	11.13	11.13	-4.68	42.2	-13

Table No. 3.2.1: Positive Age Negative Lag cases 2000-2010 for Karachi, Pakistan

3.3 LUNAR RIPENESS LAW & ITS MODIFICATION

Before the times of Ptolemy there was no known systematic description of the dynamics of the planets and the Moon. Therefore Ptolemy's exploration (the epicycle based description of the planetary motion) seems to be the first serious, scientific and systematic attempt to describe the dynamics of solar system objects. Muslims used this system to explore the conditions for the first visibility of new lunar crescent. Ptolemaic theory had described the planetary motion using the epicycles theory that could predict the planetary, lunar and solar ephemeris to a good enough degree of accuracy. The Muslims had put to use this theory well and had developed solar and lunar tables. The position of the Sun was predictable to a good degree of precision and consequently the angle ψ that the ecliptic makes with the horizon at the time of sunset could be calculated using the spherical trigonometry that Muslim mathematician have developed themselves. As described by Bruin (1977) using the angle of ecliptic ψ with the horizon at the time of sunset, the latitude of the place of observation, angle of separation between Moon and

Sun parallel to equator (difference of their right ascensions), the angle of descent (difference of altitudes or ARCV) and the width of crescent the Muslim astronomers were able to develop a highly reliable criterion for the visibility or invisibility of the new lunar crescent.

Figure (3.3.1) below which is a reconstruction of figure 4a of Bruin (1977) shows that the sun has just set south of west. The line WX passing through west cardinal point W, is the equator inclined at an angle φ (the latitude of the place) from the normal to the horizon. AS is the diurnal path of the Sun that is parallel to the equator is shown as broken line. The diurnal path of the Moon MZ is also shown with a similar broken line parallel to the equator. The line HS is the Ecliptic that makes an angle Δ with the diurnal path of the Sun or the equator. Thus the Ecliptic makes and angle $\varphi + \Delta$ with the normal. MS is the separation between the Moon and the Sun (arc of light).

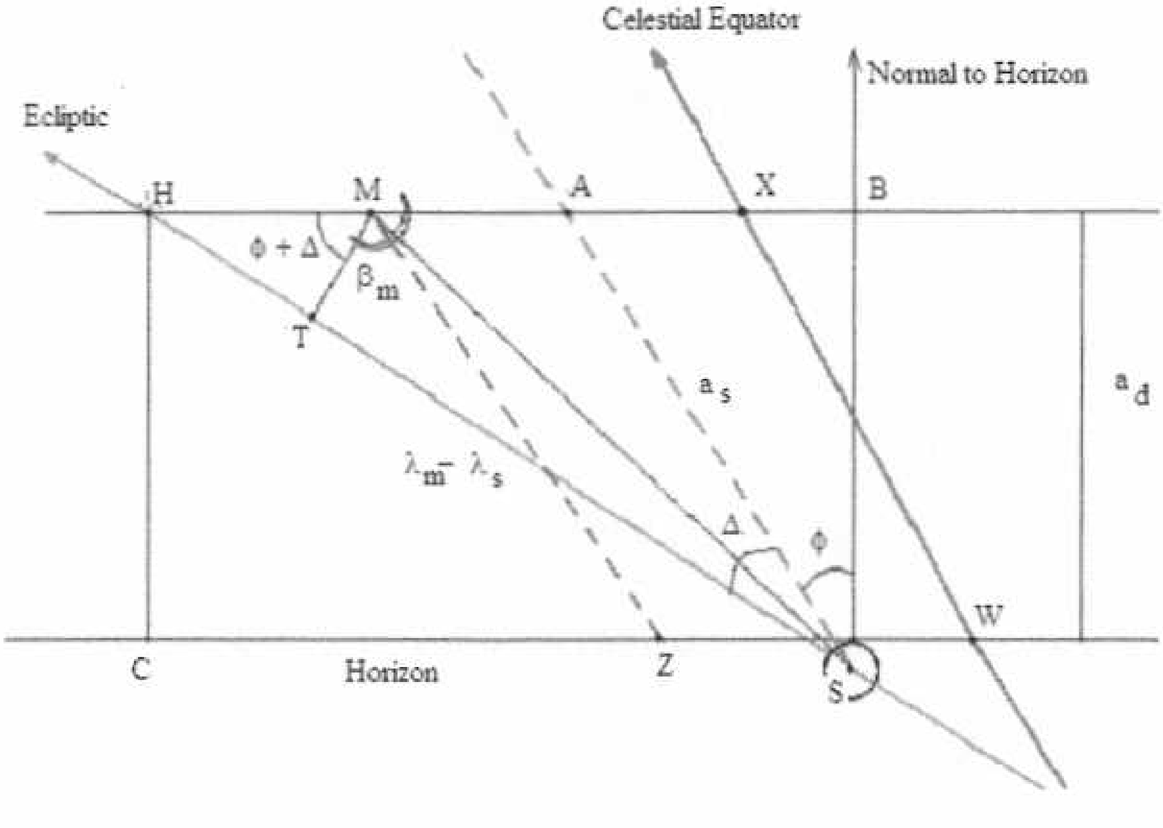


Fig No 3.3.1: Trigonometric description of conditions of new lunar crescent

MT is perpendicular to the ecliptic HTS such that TS = $\lambda_M - \lambda_S$. In the small triangle MTH, MT = β_M and the angle at H between Ecliptic (HTS) and the Horizontal (HM) = $90^0 - (\varphi + \Delta)$. Thus as triangle HTM is right angled at T we have angle at M between H and T = $\varphi + \Delta$. It then simply leads to the relation:

$$HS = HT + TS = \beta_M \tan(\varphi + \Delta) + \lambda_M - \lambda_S \qquad (3.3.1)$$

this relation is same as equation (1) of Bruin (1977). If triangle HSB is treated as a plane triangle then:

$$HS = \frac{a_D}{Cos(\varphi + \Delta)} \qquad (3.3.2)$$

and triangle ABS leads to:

$$a_D = a_S \cos\varphi \qquad (3.3.3)$$

$$\Rightarrow \qquad HS = \frac{a_S \cos\varphi}{\cos(\varphi + \Delta)} \qquad (3.3.4)$$

that leads to:

$$HS = \frac{a_S}{\cos\Delta - \tan\varphi Sin\Delta} \qquad (3.3.5)$$

This result (3.3.5) is different from what Bruin (1977) has given in his equation (4) that he claims has been obtained by using plane trigonometry. If the same figure is treated using spherical trigonometry, the triangle ABS gives:

From spherical triangle HTM:

$$\frac{\sin HT}{\sin(\varphi+\Delta)}=\frac{\sin\beta_M}{\cos(\varphi+\Delta)}$$

so that (3.3.1) $$HS=\sin^{-1}\left(\sin\beta_M.\tan(\varphi+\Delta)\right)+\lambda_M-\lambda_S \qquad (3.3.6)$$

In spherical triangle ABS:

$$\sin a_D=\sin a_S\cos\varphi \qquad (3.3.7)$$

and in spherical triangle HCS:

$$\sin HS=\frac{\sin a_D}{\cos(\varphi+\Delta)} \qquad (3.3.8)$$

(3.3.6) $\Rightarrow$ $$\sin HS=\frac{\sin a_S.\cos\varphi}{\cos(\varphi+\Delta)}$$

$$HS=\sin^{-1}\left(\frac{\sin a_S.\cos\varphi}{\cos(\varphi+\Delta)}\right)$$

or $$HS=\sin^{-1}\left[\frac{\sin a_S.}{\sin\Delta-\tan\varphi.\cos\Delta}\right] \qquad (3.3.9)$$

If angles HS and a_S are small equations (3.3.6) and (3.3.9) yield almost same results as those by (3.3.1) and (3.3.4). If the value of φ is large, then HS and a_S are not small and the spherical trigonometric results (3.3.5) and (3.3.9) should be used for more accurate results.

The significance of (3.3.6) is that if the ephemeredes of both the Sun and the Moon are known accurately (as they are now) and the angle $90^0-(\varphi+\Delta)=\psi$ of ecliptic with the horizon calculated from (3.2.7) the value of HS can be evaluated for the time of sunset for any day of the year and particularly for the day or day after the birth of new

Moon. Whereas, once the minimum angle of separation a_S (equivalent to LAG) for the visibility of new lunar crescent is known for the day the corresponding angle HS using (3.3.9) can also be evaluated. However, the use of (3.3.6) is independent of any visibility condition and is fixed for the day it depends only on the positions of the Sun and the Moon for the time of observation and the location of the observer. On the other hand the use of (3.3.9) depends on the visibility conditions, namely the minimum angle of separation a_S, that can be known only on the basis of a large number of observations for the place in question.

At the time of Muslim astronomers the ephemeredes of the Moon may not have been so accurately known as it is today, but the religious keenness of the seeing the new lunar crescent must have lead to more accurate values of a_S. Bruin (1977) has indicated that Muslim astronomers were well aware of the fact that distance of the Earth and the Moon and hence the width of crescent for same arc of light varies. Though explorations of the ancients considered this variation to behave linearly we now know that it involves the trigonometric function.

In this work this problem is handled by considering the actual semi-deimeter of the Moon and the fact that Muslims/Arabs observed that at shorter distances the crescent was seen when the LAG or arc of separation a_S was 10 degrees or 40 minutes of time and at larger distances the crescent was seen when the arc of separation a_S was 12 degrees or 48 minutes of time. This leads to a simple relation between a_S and the actual semi-diameter of the Moon D:

$$a_S = 25.5 - \frac{D}{2.2} \qquad (3.3.10)$$

Using this relation the arc of separation needed for crescent visibility called arc of separation factor is calculated in the software Hilal01.

As mentioned above the Muslim astronomers have already deduced from observations that for very thin but visible crescent minimum a_S was 12 time degrees (48 minutes) and for a wider but barely visible crescent minimum a_S was only 10 time degrees (40 minutes). This minimum a_S is actually dependent on the width of the crescent that can be derived using (2.8.8) and (2.8.9) and the relative altitude (ARCV) that can be obtained using (2.8.5) for the sun and the Moon. The issue shall be discussed in more detail later. Using (3.3.10) that shows a_S being dependent on the visual diameter (in arc minutes) of the Moon in our sky one can compute the "Lunar Ripeness Function" $R(\lambda,\varphi)$ as mentioned by Bruin (Bruin, 1977) or that is given by (3.3.6). According to Bruin the rule deduced by Muslims was that if the value of HS calculated using (3.3.9) equals or exceeds that calculated by (3.3.6) the crescent would be visible otherwise not. In this work this rule has been named Muslim Lunar Ripeness Law or simply "Lunar Ripeness Law".

According to Bruin (1977) the values of HS denoted as $R(\lambda, \varphi)$ obtained from (3.3.6) were presented in the so-called *Lunar Ripeness Tables* and first visibility of new Lunar crescent was evaluated by Muslim astronomers using the above rule. In this work HS as derived from (3.3.6) is denoted as R_{day} and:

$$R_{\text{day}} = \sin^{-1}\left[\sin\beta_M \cdot \tan(\varphi+\Delta)\right] + \lambda_M - \lambda_S \qquad (3.3.11)$$

For a fixed place (φ constant) it is noted that for small values of β_M, $\sin\beta_M$ is also small and R_{day} is almost same as $\lambda_M - \lambda_S$. But when the ecliptic is well inclined towards the horizon ($\varphi + \Delta$ is large but less than 90^0) R_{day} may become more than $\lambda_M - \lambda_S$ (if $\beta_M > 0$ and ecliptic is towards north) or remain less than $\lambda_M - \lambda_S$ (if $\beta_M < 0$ and ecliptic is towards north). However R_{day} is entirely independent of the season and depends only on the relative coordinates of the sun and the Moon. As the major component in (3.3.11) is the difference of longitudes of the sun and the Moon R_{day} is closely linked to the AGE of the Moon.

Moreover, if $\beta_M \approx 0$, $R_{day} \approx \lambda_M - \lambda_S$. But when both β_M and $\varphi + \Delta$ have the same sign (3.3.11) shows that R_{day} would be more than $\lambda_M - \lambda_S$ and if β_M and $\varphi + \Delta$ have different signs R_{day} would be less than $\lambda_M - \lambda_S$. But these variations are not seasonal.

The values for HS obtained from (3.3.9) are denoted as R_{vis} and:

$$R_{vis} = \sin^{-1}\left(\frac{\sin a_S . \cos\varphi}{\cos(\varphi + \Delta)}\right) \qquad (3.3.12)$$

For a fixed place (φ constant) R_{vis} depends on arc separation a_s (the equatorial LAG between the sun and the Moon) and the season as $\varphi + \Delta$ is season dependent that shall be shown later. Using the tools and techniques discussed in chapter 2 R_{day} for the day or the day after conjunction for a place of observation is evaluated. The minimum value of a_S is calculated using the technique described at the end of previous article so that R_{vis} is deduced. If R_{vis} is calculated using an estimated value of a_s then we call it R_{est} (an estimated value of Lunar Ripeness function). If it is calculated using an average value 10.5 degrees of a_s then we call it R_{avr} (an average value of Lunar Ripeness function). Accordingly $\Delta R_{est} = R_{est} - R_{day}$ and $\Delta R_{avr} = R_{avr} - R_{day}$. The simplest form of the Lunar Ripeness Law is then:

The crescent is visible if $\Delta R_{avr} > 0$ or:

$$\sin^{-1}\left(\frac{\sin a_S . \cos\varphi}{\cos(\varphi + \Delta)}\right) - \left\{\sin^{-1}\left[\sin\beta_M . \tan(\varphi + \Delta)\right] + \lambda_M - \lambda_S\right\} > 0$$

(3.3.13)

Thus Lunar Ripeness Law that provide a solution of the problem of determining the first day of visibility of new lunar crescent is based on the Angle that the Ecliptic makes with the horizontal or the angle $\varphi + \Delta$ that it makes with the vertical. On the day or the day after conjunction once the coordinates of the Sun and the Moon at the time of sunset for any location are calculated, (3.3.11) allows one to calculate R_{day} and using an appropriate value of a_s as given by (3.3.10), in (3.3.12) the value of R_{vis} can be found. In

view of studying the behaviour of the Ripeness function over a year for any place calculating R_{day} for every day of the year is not useful. This is because of the fact that year to year and day to day variations in latitude of the Moon and hence R_{day} are not dependent on the time of year. However, with possible values of a_s one can calculate relevant values of R_{vis} for each day of a year and study its variations seasonally as well as with changing latitude of place. As possible values of a_s vary from 10 degrees to 12 degrees an appropriate value for the day or day after conjunction is obtained only on the basis of the true distance or semi-diameter of the Moon. Still for a comparison values of R_{vis}, for different latitudes, are calculated for both the extreme values $a_s = 10^0$ and $a_s = 12^0$ and the curves R_{vis} against the longitude of the Sun (from March 21) are plotted in Figures (3.3.2) to (3.3.5). These curves show interesting features that can be summarized as follows:

i) From fig. 3.3.2 it is clear that for $\varphi = 0^0$ i.e. for a place on the equator R_{vis} is sinusoidal with maxima at March 21, September 21 and minima at June 21, December 21. Smaller values of R_{vis} means smaller values R_{day} or relatively younger crescent may be seen. Larger values of R_{vis} means larger values R_{day} or relatively older crescent may be seen. Thus close to equator older crescent may be visible near equinoxes and relatively younger crescent near solstices. If the Moon is near its apogee then it is moving faster and appears thicker in sky relatively younger crescents become ripe for visibility. Further the presence of two maxima and two minima indicates four strong inflection points. There are two regions of upward concavity (around solstices) and two of downward concavity (around equinoxes).

ii) As the latitude of the place increases (one move to the north of equator) the maximum of R_{vis} at the vernal equinox lowers making it easier to see a a younger crescent but the maximum of the vernal equinox rises so that it becomes more difficult to see a younger crescent (fig. 3.3.3). On the other hand the minimum of the summer solstice moves towards spring (vernal equinox) and that of the winter solstice moves towards autumn (autumnal equinox) and in either case decrease further making it easier to see a younger

crescent. Downward concavity of the curve sharpens close to the autumnal equinox whereas the upward concavity around the solstices flattens.

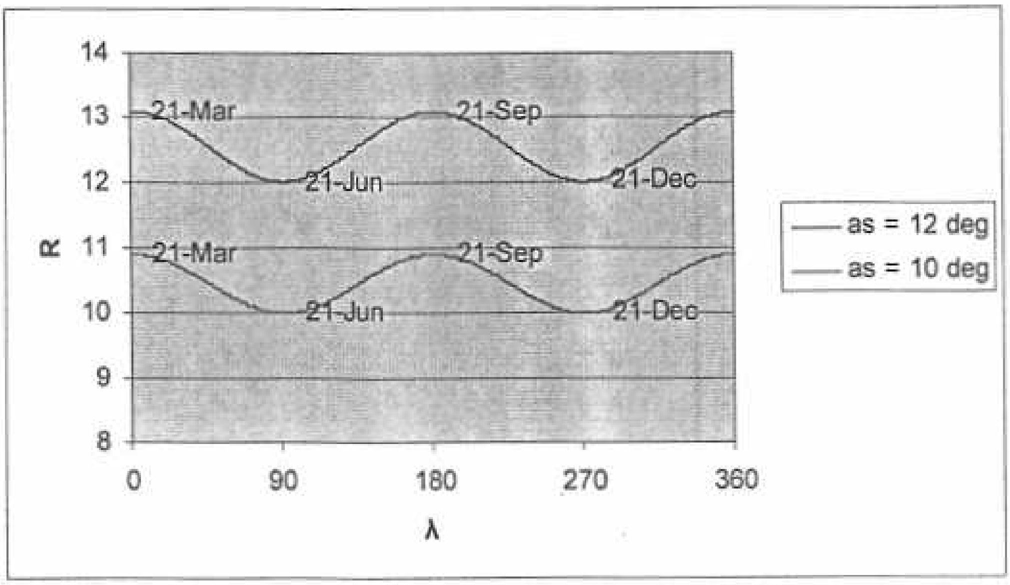


Fig. No. 3.3.2 R_{vis} for $\varphi = 0^0$

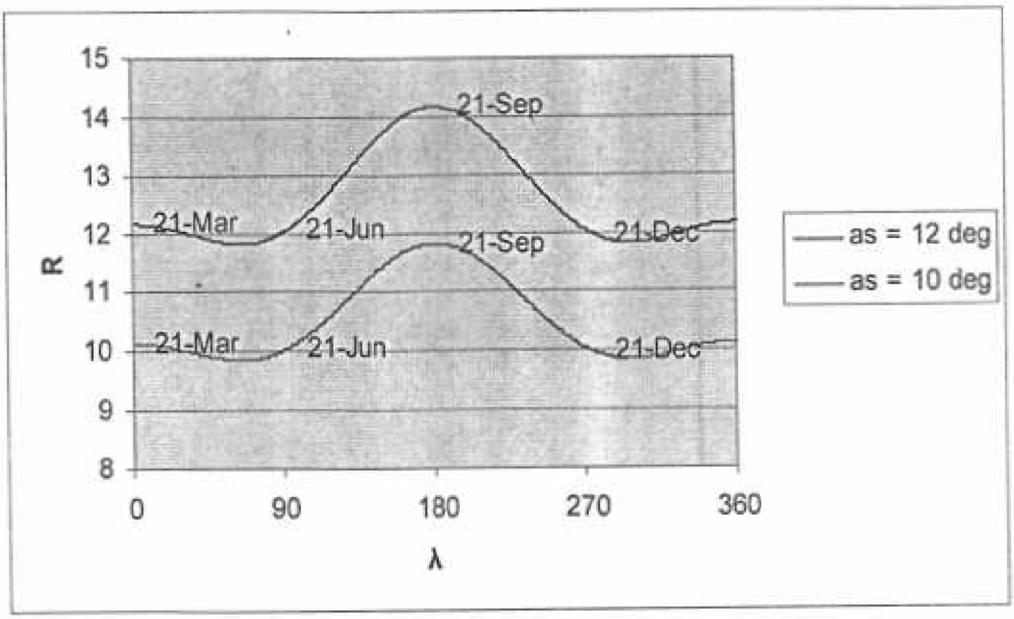


Fig. No. 3.3.3 R_{vis} for $\varphi = 10^0$

iii) Beyond tropic of cancer two of the inflection points are simply gone and the maximum at the autumnal equinox rises further making it more difficult to see-

a younger crescent (fig. 3.3.4). The minimum of the curves rests at the vernal equinox making it easier to see younger crescents near to vernal equinox.

iv) For further increase in the latitude the maximum becomes higher and higher and the minimum keeps decreasing showing that for higher latitudes it is generally easier to see younger crescents close to vernal equinox and difficult close to autumnal equinox.

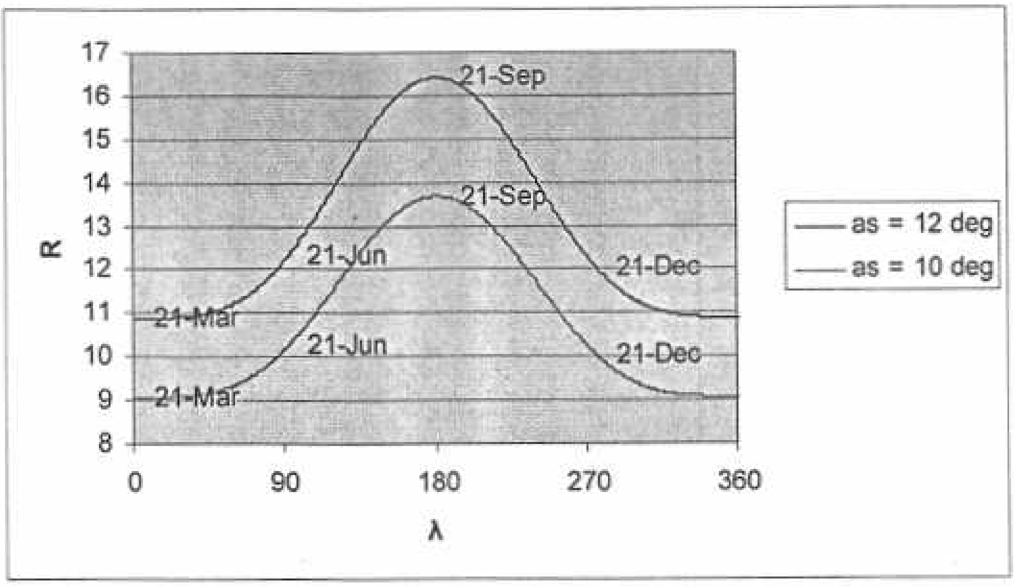


Fig. No. 3.3.4: R_{vis} for $\varphi = 25^0$

v) The situation reverses entirely in favour of autumnal equinox for the southern hemisphere.

vi) For the Arctic Circle and to its north (latitudes greater than 63.5 degrees) it is simply not possible to see the new lunar crescent close to autumnal equinox as the value of R_{vis} becomes greater than 90^0. The values as small as 5^0 of R_{vis} close to vernal equinox indicate simply that crescents younger than as compared to those close to autumnal equinox can be seen.

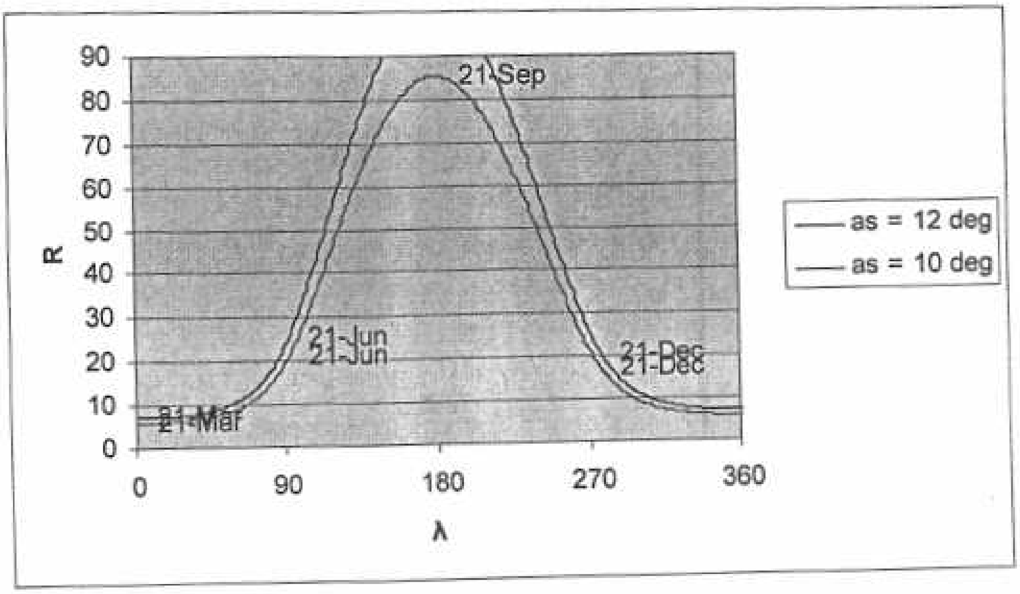


Fig. No. 3.3.5: R_{vis} for $\varphi = 63.5^0$

However the matter is not so simple because the role of latitude of the crescent, i.e. whether the crescent is south or north of the Sun plays an important role that determines the value of R_{day} that is strongly latitude dependent. Now the strong maximum at the vernal equinox for higher latitudes indicates that an older Moon may not be visible but as the AGE of the Moon increases so does its elongation and brightness. Therefore it is possible that invisibility suggested by the Ripeness function values may be misleading. This may lead to allowance for smaller LAG values and smaller values of R_{vis}. In this work the Ripeness function values for the observational data available in literature are calculated and presented in Appendix-II. Out of this data set the cases when the crescent was reportedly seen but are not in agreement with the Lunar Ripeness law are also shown in table no. 3.3.1. The observational data is selected from that reported by Odeh (Odeh, 2004) from which cases of evening crescent observations are considered only. Table 3.3.1 shows O. No. (the observation number (Odeh, 2004)), date of observation, latitude and longitude of the location of observer, Moon's age and LAG, angle ψ of ecliptic with horizon, latitude of Moon, longitudes of the Moon and the sun, $R_{est.}$ given by (3.3.12) with a_S given by (3.3.10), R_{avr} given by (3.3.12) with a_S = 10.5 degrees, R_{day} given by (3.3.11) and $\Delta R_{avr} = R_{day} - R_{avr}$.

The table in Appendix-II shows S. No., the observation No. (Odeh, 2004), date of observation, latitude and longitude of the location of observer, visibility 'N' for unaided visibility, 'B' for visibility through binocular, 'T' for visibility through telescope. The visibility columns are empty if the crescent was not seen and contains 'V' in an appropriate column if the crescent is seen. The rest of the columns of the table contain age of Moon , LAG, ARCL, ARCV and DAZ. The last five columns are for the five models considered in this chapter. The column headed A is for the Babylonian model, and B for the Lunar Ripeness model. The column headed B contains $\Delta R_{avr.}$.

The table in Appendix-II shows that out of 196 cases in which the crescent has been reported to have been seen without optical aid there are only 14 cases that do not obey the Lunar Ripeness Law stated above. The details of these observations are further explored below when a Modified Lunar Ripeness Law is suggested. The modification is needed in order to separate the cases of naked eye visibility and the cases when optical is used for crescent visibility.

A total number of 32 positive observations in this data have been with the help of binoculars and telescopes when the crescent was not visible without optical aid and the Lunar Ripeness Law is not satisfied. This is logically valid as the Lunar Ripeness Law was deduced only for naked eye observations. This has been the motivation during this work to modify the Lunar Ripeness Law to encompass the optically aided observations.

The columns of the table contain Observation Serial Number as used by Odeh (2004), Date of observation, Latitude of the place, Longitude of the place, Visibility columns N (for unaided visibility), B (with binocular) and T (with telescope). These visibility columns are empty for invisible crescent and contain V for visible. Rest of the columns contains Age of crescent in hours, for the best time (Yallop, 1998), Lag in minutes, Separation between the Sun and the Moon Arc of Light (ARCL), the angle $\psi = 90^0 - (\varphi + \Delta)$, that the ecliptic makes with the horizon on the western horizon for the day of calculation/observation, β_M the latitude of the Moon, λ_M the longitude of the Moon, λ_S

the longitude of the Sun, Arc-of-separation factor, given by (3.3.10), Estimated Ripeness Function, R_{est} calculated equation (3.4.2), Arc-of-separation factor, Average Ripeness Function R_{avr} calculated using average a_S = 10.5 degrees and the equation (3.4.2), Actual Ripeness Function R_{day} calculated using equation (3.4.1), ΔR_{avr} the difference of Average Ripeness Function & the Actual one. The table 3.3.1 does not show the visibility columns as this table comprises of cases when the crescent was reportedly seen without optical aid.

The table is sorted on the values of ΔR_{avr}. On the basis of a close analysis of the results of comparing ΔR_{avr}, the differences of Average Ripeness Function values and the Actual Ripeness Function values we observe that:

The Table shows that there are only 14 positive sightings out of a total number of 196 positive sightings that are not according to the Lunar Ripeness Law ($\Delta R_{avr} = R_{avr} - R_{day} < 0$).

There are no positive sighting with $\Delta R_{avr} < -3.58$ with or without optical aid the range in which 38 attempts have been mentioned in the literature. We consider it as Group-A for the Modified Muslim Lunar Ripeness Law that we state in this work, as:

"If the difference of Average Ripeness Function R_{avr} and the Actual Ripeness Function R_{day} (= ΔR_{avr}) is less than -3.6 it is impossible to see the new lunar crescent for all latitudes with or without optical aid".

The next 127 cases are grouped as Group-B. There are 14 (11%) naked eye visibility cases, 25 (19.7%) binocular visibility cases and 16 (12.6%) telescopic visibility cases for values of ΔR_{avr} lying between -3.58 and 0.0. Out of the 14 naked eye visibility cases in this range, three very low ΔR_{avr}-value cases have common characteristics. These are No. 286, 2 and 272 with ΔR_{avr} value -3.5, -3.47 and -2.88. All these cases are near autumnal equinox (Sept. 20, Oct. 21 and Oct. 1 respectively), crescent has older age (39.13, 39.24 and 41.91 hours respectively) and have consequently larger phase.

Table No. 3.3.1

O.No.	Date	Latit.	Long.	Age	LAG	ψ	β_M	λ_M	λ_S	a_S	R_{est}	R_{avr}	R_{day}	ΔRav
286	20/9/1990	31.8	34.7	39.13	29.5	55.2	-4.8	196.5	177.4	11.78	17.56	15.65	12.15	-3.5
2	27/10/1859	38	23.7	39.24	33.62	58.9	-4.98	234.6	213.7	11.25	17.2	16.06	12.59	-3.47
272	1/10/1989	31.3	34.6	41.91	32.07	54.5	-4.55	207.4	188.4	12.14	17.89	15.48	12.6	-2.88
727	12/12/2004	32.4	-110.7	23.15	43.24	39.3	-4.59	275	261.3	10.33	11.28	11.47	9.923	-1.55
316	8/2/1997	-33.9	18.4	26.91	33.27	-53.6	2.049	336	320	10.31	14.43	14.69	13.19	-1.5
274	25/2/1990	35.6	-83.5	14.81	39.26	13.3	2.532	345.3	337.1	10.78	9.05	8.812	8.797	-1.19
416	31/7/2000	6.5	3.4	15.94	37.68	22.0	2.114	138.2	128.8	10.42	11.18	11.26	10.2	-1.06
314	21/1/1996	-33.9	18.4	29.39	35.22	-48.9	4.441	318.3	301	10.43	13.19	13.28	12.25	-1.03
389	7/1/2000	-33.9	18.4	24.05	35.12	-43.2	0.548	297.7	286.8	12.02	13.92	12.16	10.41	-0.95
341	18/1/1999	-33.9	18.4	26.5	37.42	-48.1	0.974	311.5	298.2	11.37	13.15	13.06	12.19	-0.87
633	24/12/2003	49.6	8.7	30.16	53.8	55.0	-4.48	290	272.4	10.52	11.91	11.88	11.16	-0.72
455	25/3/2001	-33.9	18.4	15.77	36.93	-57.2	-5	12.68	5.11	11.75	18.04	16.12	15.35	-0.62
716	13/11/2004	36.8	-81.8	32.21	46.05	54.1	-3.25	250.3	231.9	10.52	13.36	13.33	13.9	-0.43
639	22/1/2004	32.5	3.7	20.38	44.59	21.8	-5.01	313.5	302	10.78	9.8	9.541	9.442	-0.1

All these conditions favour the visibility and it was mentioned above in view of (3.4.1) that for older age crescents ($\lambda_M - \lambda_S$ large) smaller arc of separation may be allowed. This explains the very small LAG (29.5, 33.62 and 32.07 respectively) in these cases. Moreover, in all these cases the arc of vision is small (ARCV 6.85, 6.8 and 7.34 degrees respectively) but relative azimuths are large (DAZ = 18.4, 20.3 and 18.1 degrees respectively) from the sun and larger widths (53, 65 and 51 arc seconds respectively). All these factors support the claims of visibility and were anticipated above when it was suggested that smaller LAG values may be allowed in such cases. All these claims are from latitude just more than 30 degrees. The smaller values of R_{day} in comparison to R_{avr} in these cases is due to large values of $\varphi + \Delta$ (more than 54 degrees), the angle of the ecliptic with vertical and large negative values of latitude of Moon λ_M (less than -4.5degrees) that reduces R_{day} to make it much smaller than $\lambda_M - \lambda_S$. All these three observation are in agreement with the Babylonian criterion.

There is no further case of crescent visibility till a value of $\Delta R_{avr} < -1.6$. From amongst other positive cases with $-1.6 < \Delta R_{avr} < 0$, two are very young crescents. These

are observation no. 274 (ΔR_{avr} = -1.19) and 416 (ΔR_{avr} = -1.06) with age 14.8 and 15.9 hours respectively. Despite being very young having relatively larger LAGs (39.3 and 37.7 minutes respectively) in both cases the Moon was very close to the Earth resulting in large visual diameter but the crescent widths were small (10.7 and 13.9 arc seconds only). With small the relative azimuths (-0.6 and 1.8 degrees) the crescent was almost vertically above the sun that bring it in the ideal condition for visibility but the small relative altitudes (8.5 and 9.1 degrees) make these claims highly optimistic. Both these observations are in disagreement with the Babylonian criterion.

Out of the 14 positive observation with $\Delta R_{avr} < 0$, three were very faint crescents i.e. observation numbers 389 (ΔR_{avr} = -0.94), 341 (ΔR_{avr} = -0.87) and 455 (-0.62). These crescent were low in altitude (7.2, 7.8 and 8.3 degrees respectively) and had small elongation (10.9, 13.3 and 9 degrees respectively). This makes these claims to be highly optimistic as well. Two of these (389 and 455) are also in disagreement with Babylonian criterion whereas 341 is a marginal case in Babylonian criterion (with ARCV + LAG = 22.7 degrees).

However, for all these 119 cases when the crescent was claimed to have been seen with naked eye, the common feature was relatively high latitudes (generally greater than 30 degrees on either side of the equator) except for observation no. 416. The claim 416 is from latitude 6.5 degrees north. Apart from this lonely case it appears that for $\Delta R_{avr} < 0$ it is impossible to see the crescent at places with latitudes less than 30 degrees (both North and South).

In the 119 cases of group B, the frequency for optically aided visibility (both with binoculars and telescopes) increases and one can easily generalize that when ΔR_{avr} lies between -3.58 and 0.0 there is a high possibility of crescent visibility with some optical aid for both the high latitude as well as low latitude observers. Thereby in this work the second part of the Modified Muslim Lunar Ripeness Law is sated as:

"if the ΔR_{avr} lies between -3.5 and 0.0 the possibility of visibility of first crescent with and without optical aid increases with increasing values of ΔR_{avr}. For higher latitudes, generally greater than 30 degrees north and south and ΔR_{avr} being in the range -3.5 to 0.0 the possibility of naked eye visibility also increases. However, in the range of these values of ΔR_{avr} for smaller latitudes the naked eye visibility is almost impossible".

Next is the Group-C that contains 76 cases with ΔR_{avr} in the range 0.0 to 1.6. In this group there are 12 naked eye visibility cases (15.8%), 26 binocular (34.7%) and 15 telescopic visibility cases (19.7%) so that visibility with both naked eye and with optical aid becomes more probable. Unfortunately the data is heavily inclined towards the high latitude cases and a clear demarcation for unaided visibility for smaller latitude observers can not be made. Still, the third part of the Modified Muslim Lunar Ripeness Law is stated as:

"if the values of ΔR_{avr} lie between 0.0 and 1.6 the possibility of visibility of first crescent with and without optical is strong for higher latitudes".

The Group-D containing a total number of 221 cases has 170 cases of optically unaided visibility of lunar cases for $\Delta R_{avr} > 1.6$. It is therefore inferred that:

"if ΔR_{avr} >1.6 the possibility of visibility of first crescent without optical is strong for both lower and higher latitudes".

Finally, the summarized Modified Muslim Lunar Ripeness Law is:

1. ***$\Delta R_{avr} < -3.5$ impossible to see the new lunar crescent for all latitudes with or without optical aid.***
2. ***$-3.5 < \Delta R_{avr} < 0.0$ impossible to see the new crescent with or without optical aid for smaller latitudes. For higher latitudes there is a high possibility of visibility of first crescent with optical.***

3. ***0.0 < ΔR_{avr} < 1.6 possibility of visibility of first crescent with and without optical is strong for higher latitudes. Locating crescent with optical aid and then try seeing it with naked eye has a good chance of optically unaided visibility.***

4. ***ΔR_{avr} > 1.6 the possibility of visibility of first crescent without optical is strong for both lower and higher latitudes".***

Another way of looking into the details of Lunar Ripeness model is to look into the plots of average ripeness function and the actual ripeness function for both visible and invisible crescents for single latitude. Unfortunately, the data available and considered in this work is restricted in the sense that scientifically recorded observations for a single latitude are not found very frequently except for Athens (latitude 38 degrees north) and Cape Town (latitude 33.9 degrees south). In particular for smaller latitudes, places close to equator observations are not very frequently available. A number of places are selected here and their data plotted for Average Ripeness Function R_{vis} and the Actual Ripeness Function R_{day} for both the reportedly invisible and the visible crescents. These include places with latitudes 31.8N, 33.9S, 6.5N and 38N (Figures 3.3.6 to 3.3.9).

Figure 3.3.6 for latitude 31.8N shows the best success percentage with 5 positive sightings out of 6 (83.3%) in agreement with the Lunar Ripeness law. All negative observations (6 out of 6) are in agreement with the law for this latitude. Next is Cape Town (latitude 33.9 S) with success percentage 65% (13 positive sightings in agreement out of 20). The negative observations for Cape Town are in agreement with law for 93.7% cases. This is followed by Athens (latitude 39N) with 2 out of 3 (66.7%) positive sightings and 11 out of 18 (61.1%) negative sightings agree with the law. For latitude 6.5 degrees, 1 out of 2 (50%) positive sightings and 4 out of 6 (66.7%) agree with the law.

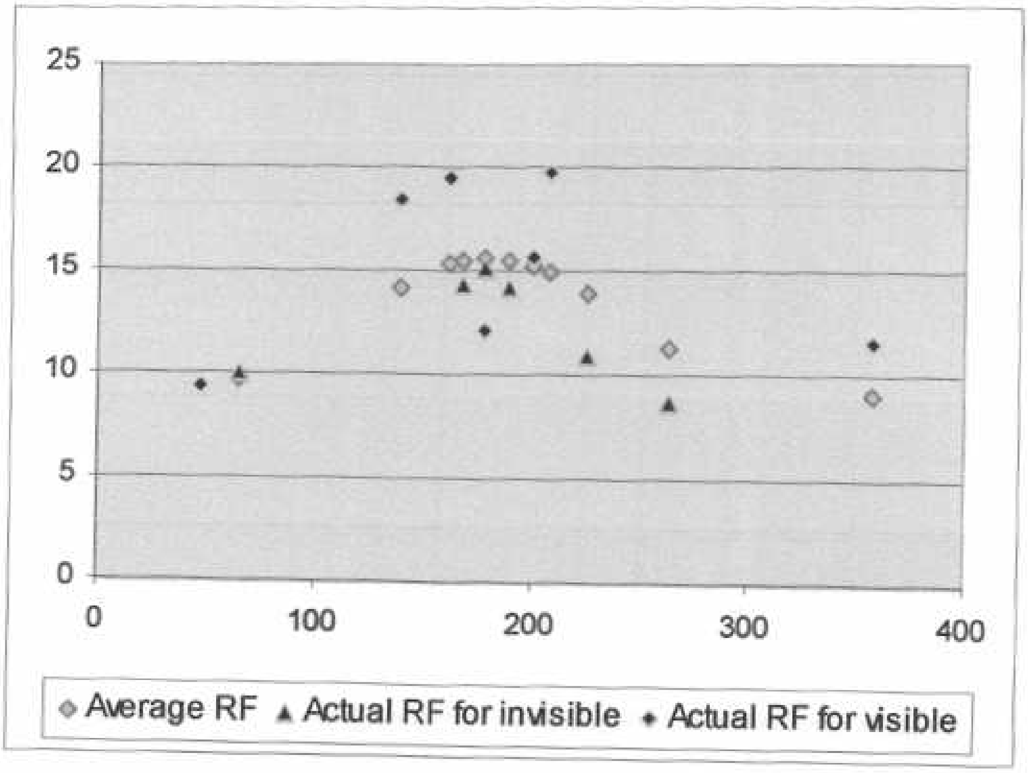


Fig. No. 3.3.6: Ripeness Function for Latitude 31.8 degrees North,

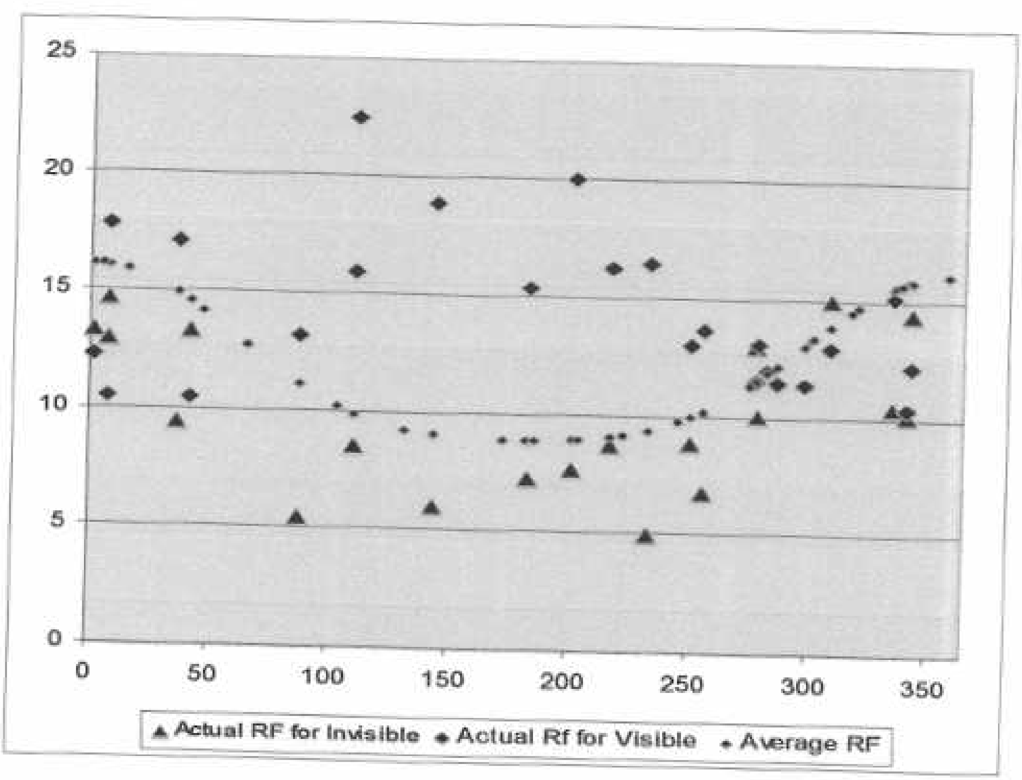


Fig. No. 3.3.7: Ripeness Function for Cape Town, South Africa, Latitude 33.9 degrees

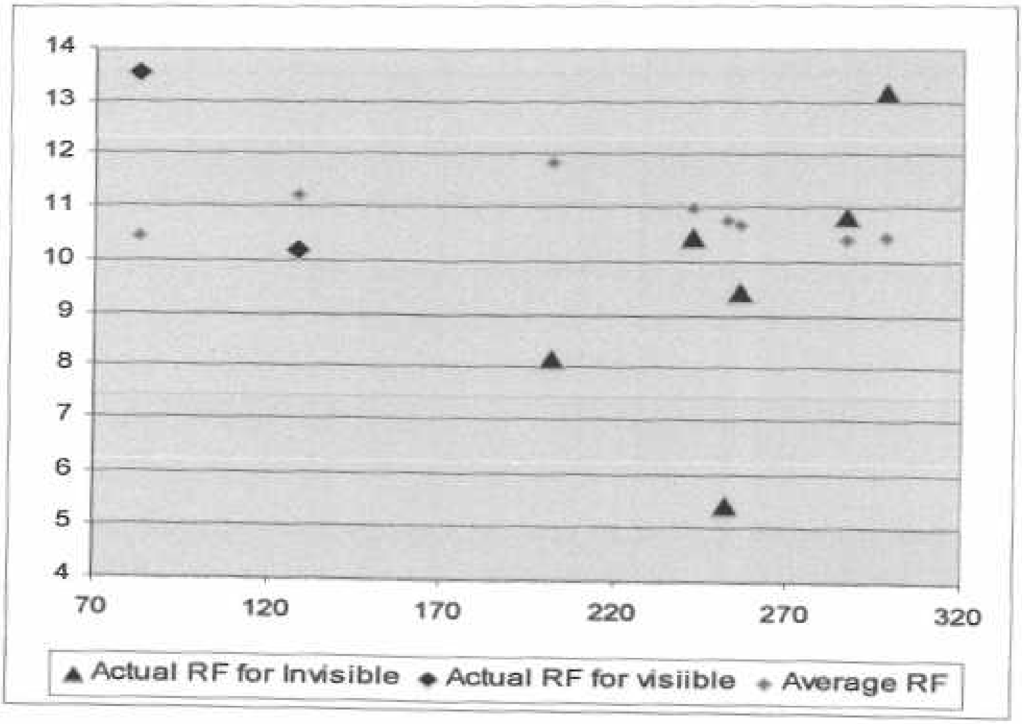


Fig. No. 3.3.8: Ripeness Function for Latitude 6.5 degrees North,

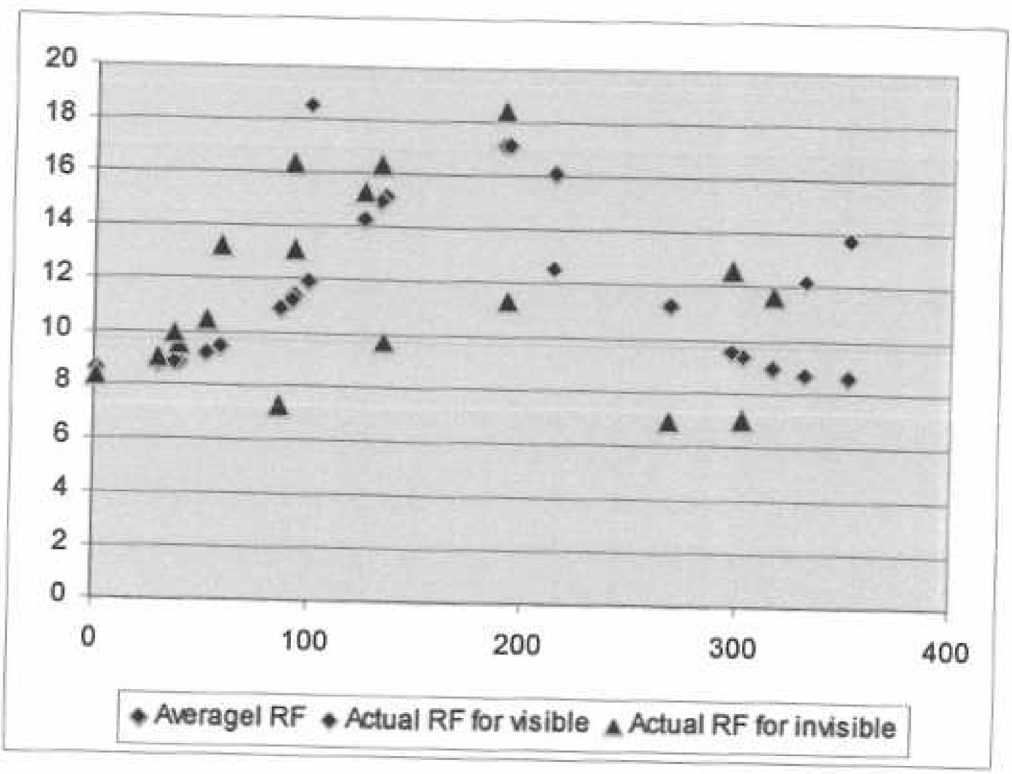


Fig. No. 3.3.9: Ripeness Function for Athens, Latitude 38 degrees North, Longitude 23.7 degrees East

All these figures show a trend for the Average Ripeness Function that is indicated by theoretical considerations and demonstrated in figures 3.3.6 to 3.3.9. Generally the invisible crescents have Actual Ripeness Function values below the Average Ripeness "Curve" and the visible crescents have values that are above the curve. Deviations of both forms are present and are discussed above. A theoretically visible crescents (actual Ripeness Function value more than the average) is reported invisible that generally counts as "Positive Error" and theoretically invisible crescent (actual Ripeness Function value less than the average) reported to be seen, a "Negative Error". The positive errors reported have no affect on the model as these errors may occur due to many uncontrolled factors (like weather conditions and observer's ability to sense the contrast). The negative errors may either by highly optimistic but incorrect claims or they may indicate lack of authenticity of the model both discussed above in detail for the Lunar Ripeness Law.

An over all comparison of the Lunar Ripeness law with the Babylonian criterion shows that the Babylonian criterion much more successful. The success percentage for positive cases for the ripeness law is 92.8% against 96.4% of Babylonian criterion. For the negative sightings the ripeness law succeeds in 57.7% cases against 59.9% for the Babylonian criterion.

3.4 EMPIRICAL MODELS OF EARLY 20TH CENTURY

After looking into the details of the success and the constraints of the Ripeness Model of the Muslims a closer look into the figure 3.3.1 and the analysis of the observed data in comparison to the Lunar Ripeness Laws, an important aspect is revealed. The following figure 3.4.1 which an extension of fig 3.3.1, in addition to Moon being north of the ecliptic, also shows the Moon to be south of ecliptic. In this case though sign of latitude, β_M of the Moon takes care of whether the length HT is to be added or subtracted from $\lambda_M - \lambda_S$ in equation (3.3.11) or (3.3.6) but another question becomes relevant. In the case shown in the figure for same a_S the Moon is much further away from the Sun when Moon is south of the Sun (in northern hemisphere), the crescent has larger arc of light

and is much thicker for same Earth-Moon distance. So it is possible that the crescent with smaller a_S and consequently smaller a_D may be visible. This is precisely the case for the observations no. 286, 2 and 272, when crescent was claimed to be seen with naked eye but the Lunar Ripeness Law indicates the observations were impossible (very low ΔR_{avr} values) whereas the Babylonian criterion allows them. So the question arises, for places with higher latitudes, ecliptic much inclined towards horizon and the crescent being on horizon side of the ecliptic should we give more allowance for a_S? Should the peaks of the Ripeness Function R_{vis} be lower than they are according to figures 3.3.2 to 3.3.5 close to the autumnal equinox? The answer is already present in the discussion of the constraints of the Lunar Ripeness function when it is pointed out that for very low ΔR_{vis} values the crescent of low LAG but old age and high DAZ is reported to been seen. This may be the possible reason for the modern astronomers like Maunder and Fotheringham for exploring relations between ARCV and DAZ for the first visibility of lunar crescent.

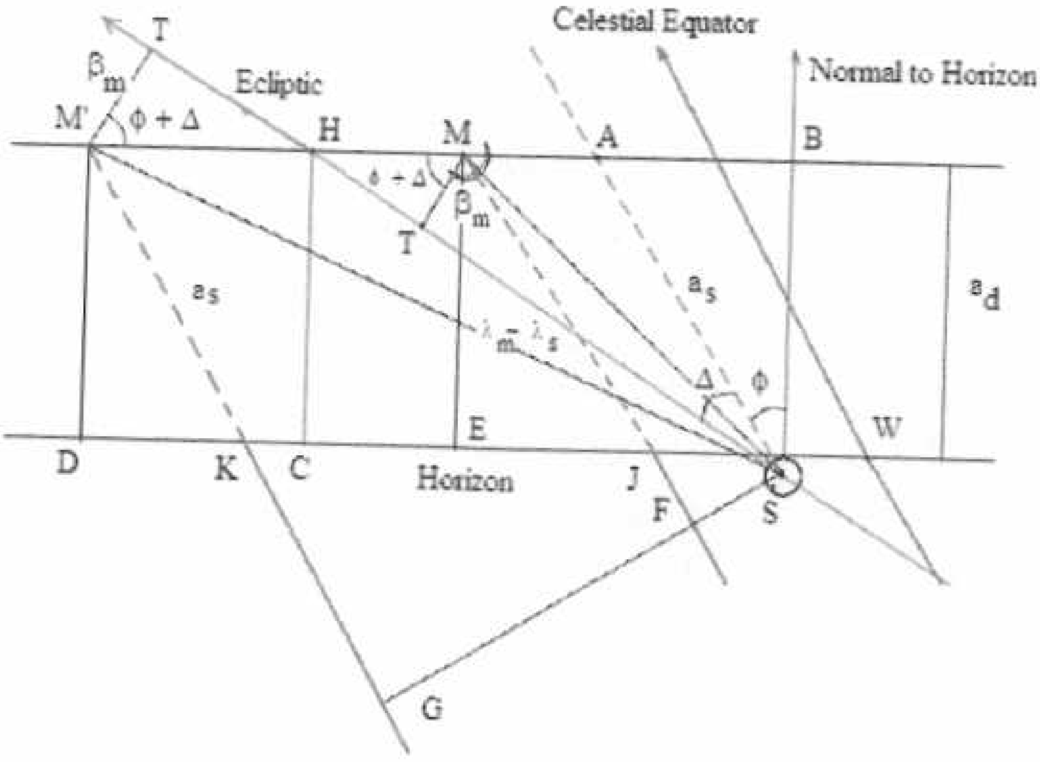


Fig No 3.4.1: Spherical Trigonometric description of conditions of new lunar crescent

In case of Moon north of ecliptic:

$$DAZ = ES = EJ + JS = a_S \sin\varphi + \frac{\delta_S - \delta_M}{\cos\varphi} \tag{3.4.1}$$

and in case of Moon south of ecliptic:

$$DAZ = DS = DK + KS = a_S \sin\varphi + \frac{\delta_S - \delta_M}{\cos\varphi} \tag{3.4.2}$$

DAZ given by (3.4.1) is much smaller than that given by (3.4.2) for the reason that in (3.4.1) the difference of declination of the Sun and the Moon ($\delta_S - \delta_M$) is smaller than that in (3.4.2). With larger ARCL (SM') for M' (3.4.2) as compared to M (3.4.1) R_{day} is larger but R_{vis} or R_{avr} is same. Although ΔR_{avr} ($R_{avr} - R_{day}$) is same for the two cases but crescent at M' is older, thicker and brighter. much smaller than for M. results into different of arcs of light (ARCL) or older crescent with larger phase. Thus in case of in case of M it must be difficult to see the crescent as compared to the case of M'.

Whenever a_S is vertical (perpendicular to the horizon) DAZ vanishes and optimum conditions of 10^0 (when Moon is closest to the Earth) to 12^0 (when the Moon is farthest from the Earth) occur. As and when a_S is not vertical the role DAZ comes into play and the optimum conditions for a_S can be relaxed. For much larger values of DAZ and older and wider crescent may be visible with smaller values of ARCV or a_D. So the models involving ARCV-DAZ relations come into play. In these model ARCV is a function of DAZ so a_S should also be a function of DAZ:

as

$$a_D = a_S \cos\varphi \text{ and } ARCV = f(DAZ) = a_D$$

$$\Rightarrow \qquad a_S = \frac{f(DAZ)}{\cos\varphi} \tag{3.4.3}$$

This means that for constant LAG ($\approx a_s$) ARCV decreases with increasing φ. For constant φ, LAG and ARCV are directly proportional to each other. So for the same latitude φ, large DAZ and large ARCL means smaller ARCV and that means smaller LAG.

During relatively recent times the explorations of the earliest visibility of new lunar crescent or the last visibility of the old crescent began with the observations made in Athens and its vicinity by Schmidt and others. Theoretical exploration was initiated more due to calendarical reasons than any particular astronomical question (Fotheringham 1903).

In the beginning of the twentieth century it was realized that methods of verifying dates, particularly lunar dates, were not available and people were concerned about the astronomical conditions that govern the naked eye visibility of the new lunar crescent (Fotheringham, 1903). To evaluate astronomical conditions for the earliest visibility of the new crescent Moon a number of studies appeared. The work of J. K. Fotheringham (1910) himself and that of E. W. Maunder (1911) is of vital importance. Both these contributions were based on the naked eye observations of new lunar crescent made by August Mommsen, Julius Schmidt and Friedrich Schmidt mostly from Athens. These contributions were empirical in nature and produced initial frame work for further exploration.

Fortheringham claims to have suggested (in his article appeared in the Journal of Philosophy, 1903) that in order to calculate the true date of phasis one ought to have a table of the requisite depression of the Sun below horizon at the moonset, or of the altitude of the Moon at the sunset for different angular distances of the Moon from the Sun. He has also admitted about his unawareness of the contribution of August Mommsen in this regard. Mommsen recorded observations of the first sight of the new moon made in the later half of the nineteenth century by Julius Schmidt, Friedrich Schmidt and Mommsen himself. The tables mentioned earlier could be constructed on the basis of such observations that were provided in Mommsen's *Chronologie* (1883), pp.

69-80. Fotheringham (1910) has reproduced these observations giving civil dates of observation and its results as provided by Mommsen and the true altitude and the azimuth of the Moon relative to the Sun at the time of sunset (or sunrise) calculated by Fotheringham himself. These observations are rearranged in the Table No. 3.4.2 with some additional calculations. The results are also presented on an ARCV-DAZ chart in Figure No. 3.4.2. Fotheringham (1910) also considered a summary table that is reproduced as Table 3.4.1 below, that gives the minimum altitude for the visible crescent for various values of relative azimuths.

DAZ	0	5	10	15	20	25
ARCV	12	11.9	11.4	11	10	7.7

Table No. 3.4.1: Fotheringham's Summary Table

He also developed a mathematical relation to describe the same:

$$\text{Minimum Altitude} = 12.^{\circ}0 - 0''.008Z^2 \qquad (3.4.4)$$

where Z is the relative azimuth. This curve defines a region of sky around the point of sunset as shown in the figure 3.4.2 below. If, at the time of sunset, the crescent is above the curve it should be visible otherwise not. The curve is referred to as Fotheringham's crescent visibility curve.

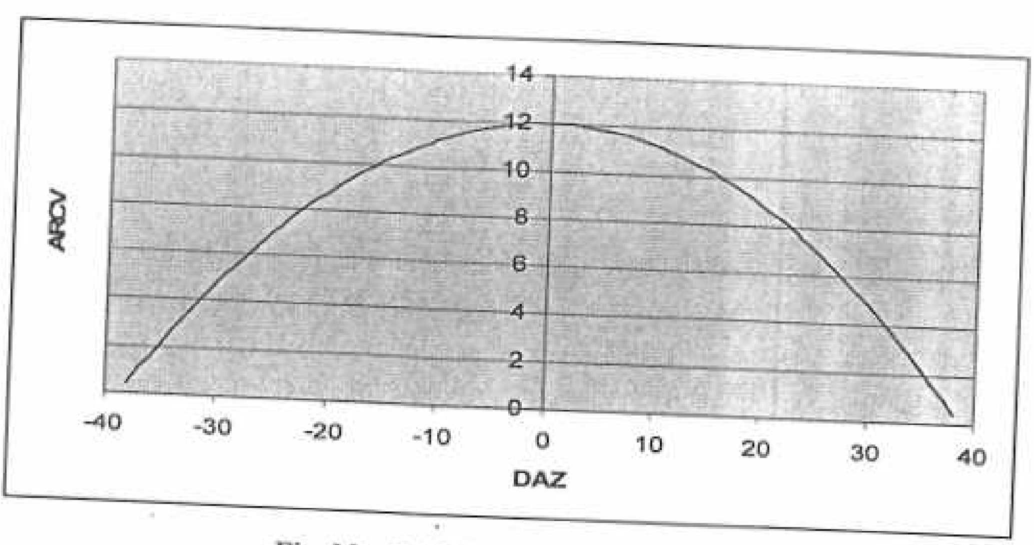


Fig. No. 3.4.2 Fotheringham's curve

Table No. 3.4.2 Fotheringham's Rule											
OB. NO.	DATE	ARCV	DAZ	Vf	RESULT	OB. NO.	DATE	ARCV	DAZ	Vf	RESULT
53	20.12.1873	4.9	10.4	-0.623		29	21.10.1865	12.8	17.3	0.319	V
40	18.06.1871	5.8	4.6	-0.603		54	17.03.1874	15.3	3.2	0.344	V
3	23.01.1860	6	3.2	-0.592		52	27.05.1873	15.2	6.5	0.354	V
7	07.08.1861	5.2	15.2	-0.495		12	03.12.1861	13.6	16.6	0.38	V
19	06.05.1864	7.9	4.6	-0.393		14	31.03.1862	15.9	0.4	0.39	V
10	05.10.1861	5.3	19.4	-0.369		44	07.06.1872	15.3	9	0.395	V
15	29.03.1862	8.6	0.1	-0.34		66	31.07.1878	12.2	21.7	0.397	V
38	20.03.1871	8.4	7	-0.321		56	26.02.1876	16.1	5.5	0.434	V
43	14.09.1871	8.8	2.8	-0.314	V	76	10.01.1880	13.4	19.4	0.441	V
51	27.03.1873	9.1	3.2	-0.276		23	01.11.1864	13.7	18.7	0.45	V
2	27.10.1859	6.1	20.5	-0.254	V	58	22.07.1876	14.6	15.9	0.462	V
48	03.10.1872	8.9	9	-0.245		5	20.06.1860	15.4	12.5	0.465	V
35	12.05.1869	9.1	9.4	-0.219		22	03.09.1864	11.4	25.8	0.473	V
45	06.07.1872	9.8	5.6	-0.195		49	03.10.1872	13.6	20.3	0.49	V
32	05.02.1867	10.5	1.8	-0.147		61	07.11.1877	11.1	27.2	0.502	V
30	17.01.1866	10.7	0.8	-0.129		8	08.08.1861	11.2	27.1	0.508	V
27	24.06.1865	9.2	16	-0.075		11	03.11.1861	12.7	25.2	0.578	V
39	20.05.1871	11.2	8	-0.029		24	28.01.1865	17.8	3.4	0.589	V
16	28.07.1862	8.6	20.5	-0.004		59	16.03.1877	18	5.3	0.622	V
74	13.12.1879	10.8	12.1	-0.003		55	04.07.1875	17.4	12.2	0.659	V
37	20.02.1871	11.4	8.9	0.0034	V	31	16.03.1866	18	9.2	0.668	V
57	22.06.1876	11.9	4.6	0.0069		17	10.01.1864	18.4	6.5	0.674	V
67	27.10.1878	7.9	23	0.0132	V	64	02.06.1878	18.6	8.7	0.721	V
21	04.08.1864	8.7	22.1	0.0607		63	05.01.1878	17.8	14.9	0.758	V
73	12.12.1879	11.2	13.3	0.0615		25	28.03.1865	20	5.4	0.823	V
13	01.01.1862	12.3	7.1	0.0703	V	4	23.02.1860	20.2	2.4	0.825	V
6	12.03.1861	12.8	1.8	0.0826	V	18	09.03.1864	21.1	2.3	0.914	V
65	01.07.1878	12.2	10.4	0.1065	V	68	26.11.1878	15.8	26.2	0.929	V
62	06.12.1877	10.5	18.1	0.1121	V	20	06.06.1864	18.5	20	0.97	V
28	24.07.1865	9.7	21.3	0.133	V	9	07.09.1861	12.9	36.4	1.15	V
1	01.07.1859	12.7	10	0.15	V	34	22.06.1868	19.7	22.8	1.186	V
26	26.03.1865	13.6	7.2	0.2015	V	75	15.12.1879	21.1	21.6	1.283	V
42	17.08.1871	12.9	12.1	0.2071	V	69	23.05.1879	24.9	12.3	1.411	V
50	31.12.1872	12.2	15.3	0.2073	V	72	11.12.1879	21.6	25.9	1.497	V
33	27.11.1867	13.7	9.6	0.2437	V	70	22.06.1879	23.5	24.9	1.646	V
46	04.09.1872	13.1	13.3	0.2736	V	47	30.09.1872	28.7	2.1	1.674	V
60	12.06.1877	14.7	6.4	0.3028	V	71	22.07.1879	20	37.1	1.901	V
41	19.06.1871	13.2	10.9	0.315	V	36	25.07.1870	32.8	23.3	2.514	V

We define a parameter v_F, "visibility parameter" according to Fotheringham as:

$$v_F = (ARCV - 12 + 0.008Z^2)/10 \qquad (3.4.5)$$

In the Table No. 3.4.2 the last column contains the value of v_F for each observation reported by Fotheringham and then the table is sorted in the increasing order of v_F. In view of (3.4.4) if the lunar altitude is less than $12.^\circ0 - 0^\circ.008Z^2$ the crescent should not be visible. Alternately in view of (3.4.5) if the value of v_F is negative the crescent should not be visible. The values of relative altitude of the Moon and its relative azimuths are those calculated by Fotheringham. It is easily noted that out of 20 observations for which the value of v_F is negative two observations are positive: One, on Oct. 27, 1859 (observation no. 2) and the other on the morning of Sept. 14, 1871 (observation no. 43). However he himself admits that his mathematical relation (3.4.4) is not as reliable as the summary table data. The same is exhibited in the Figure No. 3.4.3. All the rest of the positive observations are above Fotheringham's visibility curve.

Instead of relying on the "rough" approximate mathematical relation (3.4.4) one may consider the summary table data and fit a quadratic curve using least square approximation. Following this we obtained the following relation between minimum ARCV and DAZ for the summary table 3.4.1 data:

$$ARCV = -0.00929DAZ^2 + 0.074429DAZ + 11.86429 \qquad (3.4.6)$$

and define an alternate visibility parameter v_{LS} based on Second degree Least square approximation as follows:

$$v_{LS} = (ARCV + 0.00929DAZ^2 - 0.074429DAZ - 11.86429)/10 \qquad (3.4.7)$$

Table No. 3.4.3

OB. NO.	DATE	ARCV	DAZ	Vf	RESULT	OB. NO.	DATE	ARCV	DAZ	Vf	RESULT
53	20.12.1873	4.9	10.4	-0.672		60	12.06.1877	14.7	6.4	0.274	V
40	18.06.1871	5.8	4.6	-0.621		12	03.12.1861	13.6	16.6	0.309	V
3	23.01.1860	6	3.2	-0.601		66	31.07.1878	12.2	21.7	0.314	V
7	07.08.1861	5.2	15.2	-0.563		52	27.05.1873	15.2	6.5	0.325	V
10	05.10.1861	5.3	19.4	-0.447		54	17.03.1874	15.3	3.2	0.329	V
19	06.05.1864	7.9	4.6	-0.411		44	07.06.1872	15.3	9	0.353	V
38	20.03.1871	8.4	7	-0.352		76	10.01.1880	13.4	19.4	0.363	V
2	27.10.1859	6.1	20.5	-0.334	V	23	01.11.1864	13.7	18.7	0.373	V
15	29.03.1862	8.6	0.1	-0.327		22	03.09.1864	11.4	25.8	0.387	V
43	14.09.1871	8.8	2.8	-0.32	V	58	22.07.1876	14.6	15.9	0.393	V
51	27.03.1873	9.1	3.2	-0.291		14	31.03.1862	15.9	0.4	0.401	V
48	03.10.1872	8.9	9	-0.287		5	20.06.1860	15.4	12.5	0.407	V
35	12.05.1869	9.1	9.4	-0.263		49	03.10.1872	13.6	20.3	0.41	V
45	06.07.1872	9.8	5.6	-0.219		56	26.02.1876	16.1	5.5	0.411	V
32	05.02.1867	10.5	1.8	-0.147		61	07.11.1877	11.1	27.2	0.416	V
27	24.06.1865	9.2	16	-0.145		8	08.08.1861	11.2	27.1	0.422	V
30	17.01.1866	10.7	0.8	-0.122		11	03.11.1861	12.7	25.2	0.492	V
16	28.07.1862	8.6	20.5	-0.084		24	28.01.1865	17.8	3.4	0.579	V
67	27.10.1878	7.9	23	-0.071	V	59	16.03.1877	18	5.3	0.601	V
39	20.05.1871	11.2	8	-0.066		55	04.07.1875	17.4	12.2	0.603	V
74	13.12.1879	10.8	12.1	-0.059		31	16.03.1866	18	9.2	0.625	V
37	20.02.1871	11.4	8.9	-0.038	V	17	10.01.1864	18.4	6.5	0.645	V
21	04.08.1864	8.7	22.1	-0.022		64	02.06.1878	18.6	8.7	0.68	V
57	22.06.1876	11.9	4.6	-0.011		63	05.01.1878	17.8	14.9	0.691	V
73	12.12.1879	11.2	13.3	0.0007		25	28.03.1865	20	5.4	0.801	V
62	06.12.1877	10.5	18.1	0.0366	V	4	23.02.1860	20.2	2.4	0.821	V
13	01.01.1862	12.3	7.1	0.0381	V	68	26.11.1878	15.8	26.2	0.843	V
28	24.07.1865	9.7	21.3	0.0511	V	20	06.06.1864	18.5	20	0.89	V
65	01.07.1878	12.2	10.4	0.0578	V	18	09.03.1864	21.1	2.3	0.911	V
6	12.03.1861	12.8	1.8	0.0833	V	9	07.09.1861	12.9	36.4	1.077	V
1	01.07.1859	12.7	10	0.1031	V	34	22.06.1868	19.7	22.8	1.102	V
50	31.12.1872	12.2	15.3	0.1396	V	75	15.12.1879	21.1	21.6	1.201	V
42	17.08.1871	12.9	12.1	0.1511	V	69	23.05.1879	24.9	12.3	1.354	V
26	26.03.1865	13.6	7.2	0.1687	V	72	11.12.1879	21.6	25.9	1.411	V
33	27.11.1867	13.7	9.6	0.1987	V	70	22.06.1879	23.5	24.9	1.561	V
46	04.09.1872	13.1	13.3	0.2092	V	47	30.09.1872	28.7	2.1	1.672	V
29	21.10.1865	12.8	17.3	0.2459	V	71	22.07.1879	20	37.1	1.83	V
41	19.06.1871	13.2	10.9	0.2641	V	36	25.07.1870	32.8	23.3	2.43	V

This modification in the rule by Fotheringham is applied to the observations reported by Fotheringham and results are presented in Table no. 3.4.3. The data of the table is also presented in Fig. No. 3.4.3. It is easily seen that the least square fitting to a second degree polynomial has not improved anything rather two more positive observation have fallen into the range of negative values of v_{LS}. These are numbered 67 (of Oct. 27, 1878) and 37 (Feb 20, 1871).

During the same era Maunder (1911) considered another basic data set given in the table no. 3.4.4 to fit the observational data.

DAZ	0	5	10	15	20
ARCV	11	10.5	9.5	8	6

Table No. 3.4.4

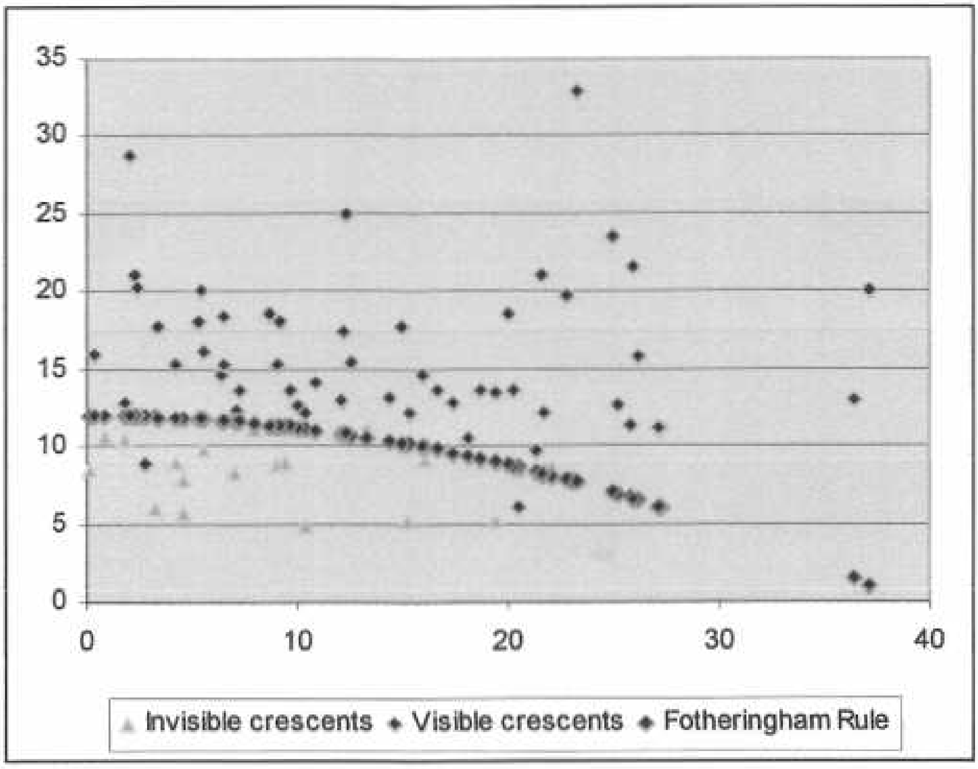


Fig. No. 3.4.3 Fotheringham's Rule

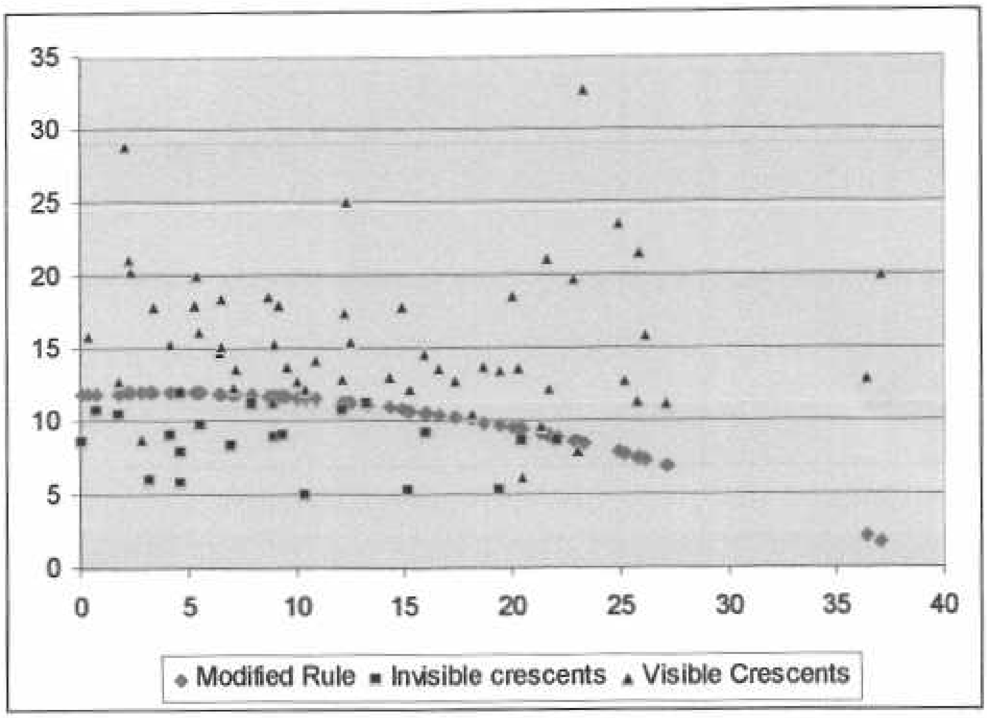


Fig. No. 3.4.4

A least square quadratic polynomial fitted to this data yields the following relation:

$$ARCV = -\frac{DAZ^2}{100} - \frac{|DAZ|}{20} + 11 \qquad (3.4.8)$$

Using this polynomial and the condition that the crescent would be visible if:

$$ARCV > -\frac{DAZ^2}{100} - \frac{|DAZ|}{20} + 11 \qquad (3.4.9)$$

and applied it to data used by Fotheringham the results obtained are presented in table no. 3.4.4. and fig. no. 3.4.4. In the table V_f is the "visibility parameter" defined as:

$$V_f = \left(ARCV - \left\{ -\frac{DAZ^2}{100} - \frac{|DAZ|}{20} + 11 \right\} \right) \qquad (3.4.10)$$

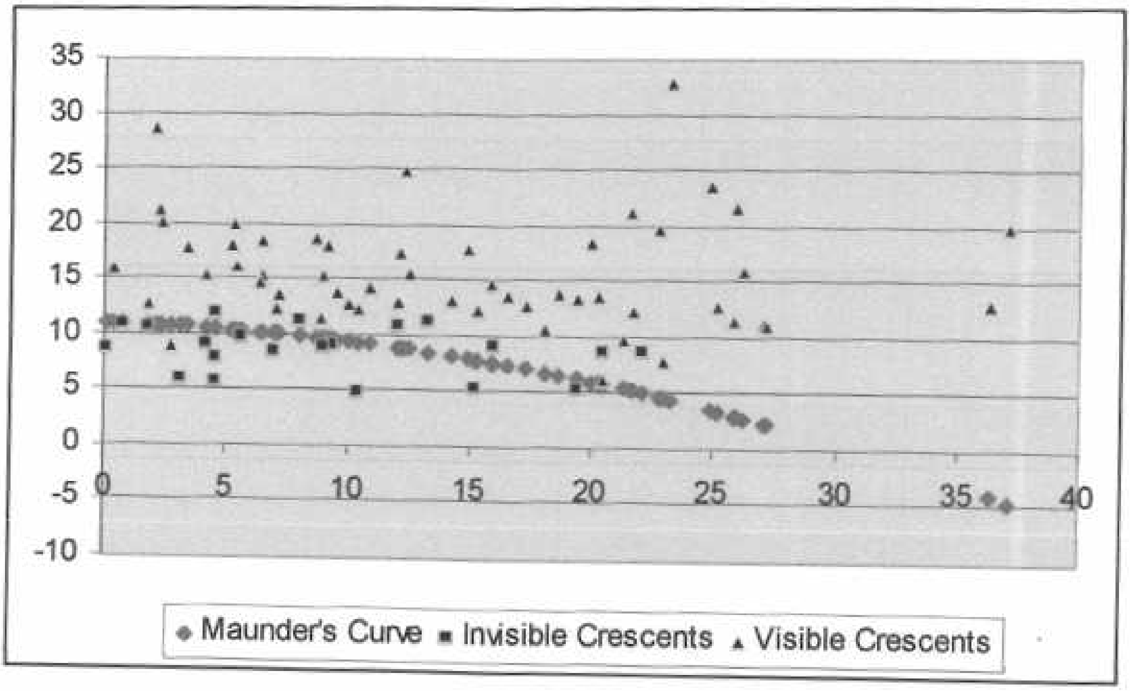


Fig. No. 3.4.5

The table 3.4.5 shows that the results based on Maunder's method are much improved and there is only one observation i.e. no. 43 on Sept. 14, 1871, that deviates from the condition (3.4.9). The table 3.4.4 and the figure both show that model due to Maunder is much improved as compared to the model due to Fotheringham.

Finally in this work both the models, due Fotheringham and that due to Maunder are applied to the selected 463 observations from recent literature (Odeh, 2004). The results are presented in figures no. 3.4.6 (Fotheringham's model) and 3.4.7 (Maunder's model) and the appendix-II (Columns C and D, respectively).

The figure no. 3.4.6 and the table in appendix-II (column C) show that there are a large number (90 out of 196 positive sightings) of observations when the crescent was reported visible that deviate from Fotheringham's model and lie below the limit provided by the rule (3.4.4) due to Fotheringham. This suggests that the success of the rule is highly restricted to the data on which Fotheringham worked on and required serious modification.

Table No. 3.4.5											
OB. NO.	DATE	ARCV	DAZ	Vf	RESULT	OB. NO.	DATE	ARCV	DAZ	Vf	RESULT
3	23.01.1860	6	3.2	-4.74		25	28.03.1865	20	5.4	9.56	V
7	07.08.1861	5.2	15.2	-2.73		26	26.03.1865	13.6	7.2	3.48	V
10	05.10.1861	5.3	19.4	-0.97		28	24.07.1865	9.7	21.3	3.3	V
15	29.03.1862	8.6	0.1	-2.39		29	21.10.1865	12.8	17.3	5.66	V
16	28.07.1862	8.6	20.5	2.828		31	16.03.1866	18	9.2	8.31	V
19	06.05.1864	7.9	4.6	-2.66		33	27.11.1867	13.7	9.6	3.1	V
21	04.08.1864	8.7	22.1	3.689		34	22.06.1868	19.7	22.8	15	V
27	24.06.1865	9.2	16	1.56		36	25.07.1870	32.8	23.3	28.4	V
30	17.01.1866	10.7	0.8	-0.25		37	20.02.1871	11.4	8.9	1.64	V
32	05.02.1867	10.5	1.8	-0.38		41	19.06.1871	13.2	10.9	4.93	V
35	12.05.1869	9.1	9.4	-0.55		42	17.08.1871	12.9	12.1	3.97	V
38	20.03.1871	8.4	7	-1.76		43	14.09.1871	8.8	2.8	-2	V
39	20.05.1871	11.2	8	1.24		44	07.06.1872	15.3	9	5.56	V
40	18.06.1871	5.8	4.6	-4.76		46	04.09.1872	13.1	13.3	4.86	V
45	06.07.1872	9.8	5.6	-0.61		47	30.09.1872	28.7	2.1	17.8	V
48	03.10.1872	8.9	9	-0.84		49	03.10.1872	13.6	20.3	7.74	V
51	27.03.1873	9.1	3.2	-1.51		50	31.12.1872	12.2	15.3	3.31	V
53	20.12.1873	4.9	10.4	-4.5		52	27.05.1873	15.2	6.5	4.95	V
57	22.06.1876	11.9	4.6	1.342		54	17.03.1874	15.3	3.2	4.69	V
73	12.12.1879	11.2	13.3	2.634		55	04.07.1875	17.4	12.2	8.5	V
74	13.12.1879	10.8	12.1	1.869		56	26.02.1876	16.1	5.5	5.68	V
1	01.07.1859	12.7	10	3.2	V	58	22.07.1876	14.6	15.9	6.92	V
2	27.10.1859	6.1	20.5	0.328	V	59	16.03.1877	18	5.3	7.55	V
4	23.02.1860	20.2	2.4	9.378	V	60	12.06.1877	14.7	6.4	4.43	V
5	20.06.1860	15.4	12.5	6.588	V	61	07.11.1877	11.1	27.2	8.86	V
6	12.03.1861	12.8	1.8	1.922	V	62	06.12.1877	10.5	18.1	3.68	V
8	08.08.1861	11.2	27.1	8.899	V	63	05.01.1878	17.8	14.9	9.77	V
9	07.09.1861	12.9	36.4	16.97	V	64	02.06.1878	18.6	8.7	8.79	V
11	03.11.1861	12.7	25.2	9.31	V	65	01.07.1878	12.2	10.4	2.8	V
12	03.12.1861	13.6	16.6	6.186	V	66	31.07.1878	12.2	21.7	6.99	V
13	01.01.1862	12.3	7.1	2.159	V	67	27.10.1878	7.9	23	3.34	V
14	31.03.1862	15.9	0.4	4.922	V	68	26.11.1878	15.8	26.2	13	V
17	10.01.1864	18.4	6.5	8.148	V	69	23.05.1879	24.9	12.3	16	V
18	09.03.1864	21.1	2.3	10.27	V	70	22.06.1879	23.5	24.9	19.9	V
20	06.06.1864	18.5	20	12.5	V	71	22.07.1879	20	37.1	24.6	V
22	03.09.1864	11.4	25.8	8.346	V	72	11.12.1879	21.6	25.9	18.6	V
23	01.11.1864	13.7	18.7	7.132	V	75	15.12.1879	21.1	21.6	15.8	V
24	28.01.1865	17.8	3.4	7.086	V	76	10.01.1880	13.4	19.4	7.13	V

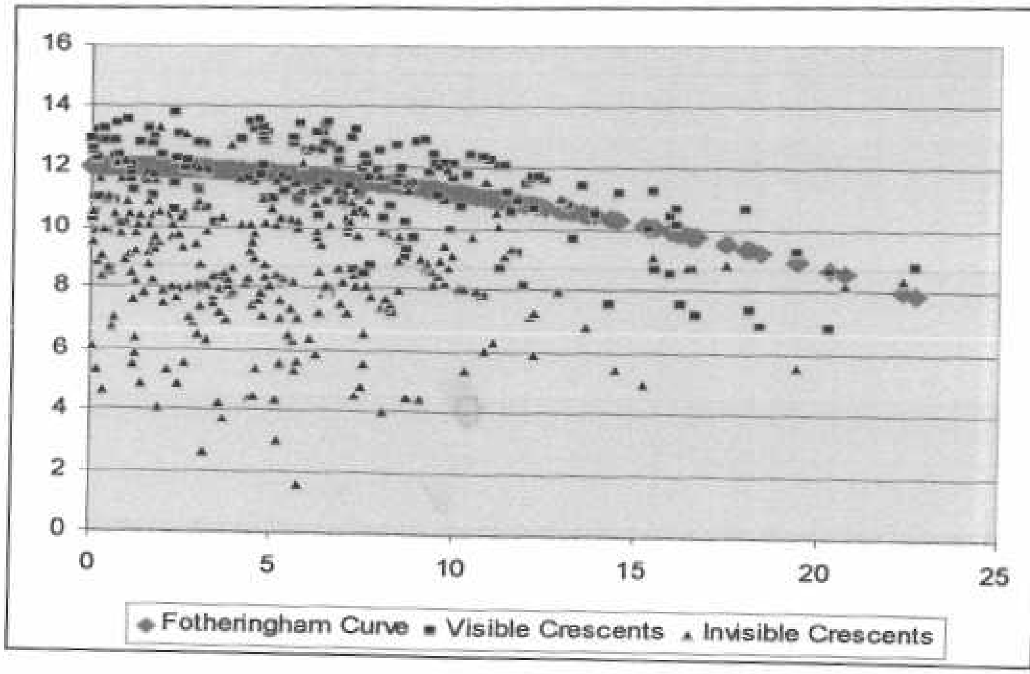


Fig. No. 3.4.6

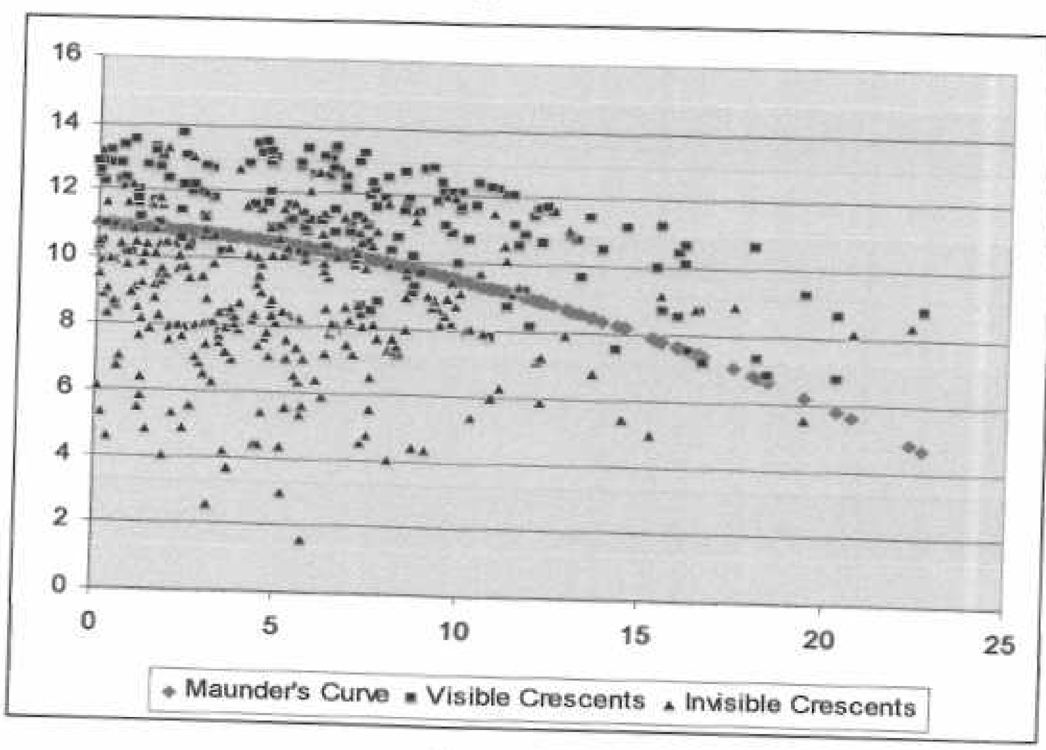


Fig. No. 3.4.7

On the other hand, figure 3.4.7 and the table in appendix II (column D) show that out of 196 positive sightings only 30 observations deviate from the Maunder's model. Thus Maunder's effort is much better than that of Fotheringham's work. However, both

the efforts are not as successful as the Babylonian criterion and Lunar Ripeness Law considered in the previous two articles as far as the number of deviation from the law in the observed crescent are concerned.

Another effort of great significance that is found in literature is based on the work of Schoch (1930) and is known as Indian Method given in the Explanation to *The Indian Astronomical Ephemeris*". In this method the basic data used is given here in table no. 3.4.6.

DAZ	0	5	10	15	20
ARCV	10.4	10.0	9.3	8.0	6.2

Table No. 3.4.6

A least square quadratic polynomial fitted to this data yields the following relation:

$$ARCV = 10.3743 - 0.0137|DAZ| - 0.0097 DAZ^2 \qquad (3.4.11)$$

Using this polynomial and the condition that the crescent would be visible if:

$$ARCV > 10.3743 - 0.0137|DAZ| - 0.0097 DAZ^2 \qquad (3.4.12)$$

and applied it to data set of 463 observation used in this work the results obtained are presented in fig. no. 3.3.8. For this figure V_f used is the "visibility parameter" defined as:

$$V_f = (ARCV - (10.3743 - 0.0137|DAZ| - 0.0097 DAZ^2)) \qquad (3.4.13)$$

There are only 19 out 196 positive sightings that deviate from the condition (3.4.12). These are presented in the table of Appendix II (column E) and Fig no. 3.4.8. Thus till the later half of the 20th century the Indian method was considered to be the best. However in view of the analysis of Babylonian criterion and the Lunar Ripeness Law explored and presented in this work, not even the Indian method is as successful in terms

of positive sightings deviating from model. During the modern times the Babylonian criterion and Ripeness function has not been explored as thoroughly as is done in the work. This exploration has lead to a significant finding that the ancient and the medieval models for the earliest visibility of new lunar crescent are as useful as some of the modern day methods.

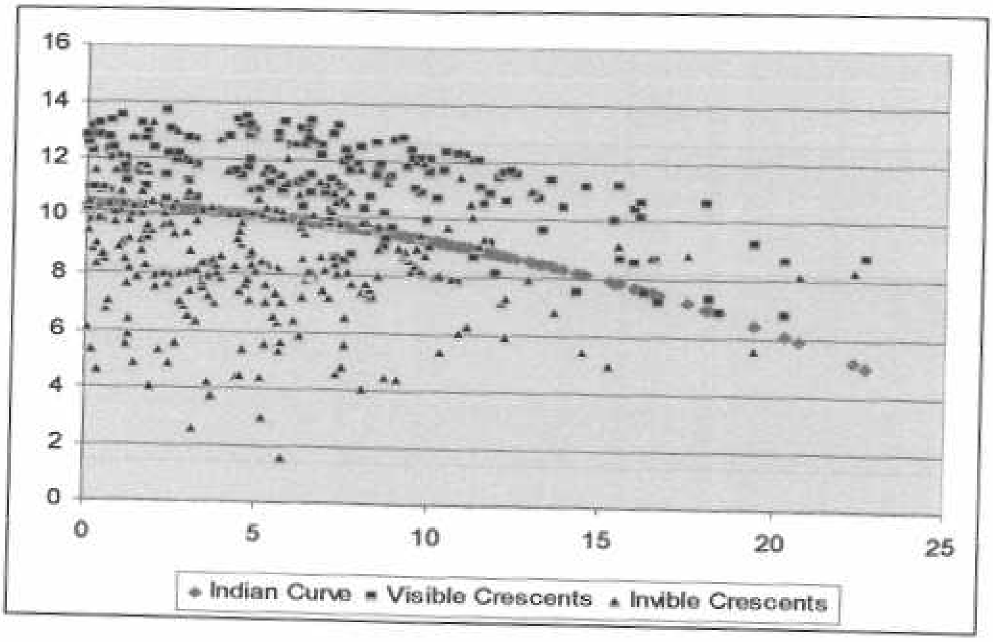


Fig. 3.4.8

3.5 COMPARISON AND DISCUSSION

The work presented in this chapter can be summarised as follows:

- For an observational lunar calendar it is important to realize that the condition "birth of new Moon or conjunction before local sunset" is not at all a reliable condition for visibility of new lunar crescent. This is particularly important in cases when the Moon can set before sunset even after conjunction. Thus a lunar calendar based on conjunction of Moon with the sun before the local sunset and an observational lunar calendar that requires actual sighting of the new crescent have to be essentially different.

- The angle of ecliptic with the horizon plays an important role for the conditions of earliest visibility of new lunar crescent. For northern hemisphere if a conjunction falls near autumnal equinox this angle is small for both middle and higher latitudes and therefore crescent is either close to the horizon or even below the horizon at the time of sunset. Therefore older crescents may escape sighting.

- The ancient Babylonian criterion for the earliest visibility of new lunar crescent has the highest success percentage (96.4%) amongst all the models considered in this chapter for positive sightings considered in this work. However, the success percentage for negative sightings is not good enough (59.9%). Thus the overall success percentage of the Babylonian criterion is 75.4%.

- The ideas related to the Lunar Ripeness function that developed during the medieval era are thoroughly investigated. With modern techniques of computations this has resulted into a useful method for determining the day of the first sighting of new lunar crescent. The only problem that surfaced against this method is the sightings that deviated from the model in higher latitudes. These are the cases when older crescents and brighter crescents have lower Ripeness function values. The success percentage of Lunar Ripeness law for positive sightings is 92.8% (better than all models considered except Babylonian criterion) but that for negative sightings is only 57.7% (worse than all other criteria considered in this chapter). The overall success percentage is 72.1%.

- The advantage of methods that are based on relation between arc of vision and relative azimuths and that are more thoroughly investigated during modern era is also explored. It is found that the Indian method based on the basic data of Schoch is the best amongst the ARCV-DAZ based methods.

- Amongst the empirical models of the early 20th century the Indian method based on data of Schoch has the best success percentage for positive sightings (90.3%) follwed by the Maunder's method (84.7%) and the method due to Maunder

(54.1%). However, in terms of the success percentage for negative sightings Fotheringham's criterion is the best (amongst all methods considered in the chapter) with 94.7% follwed by Maunder's (82%) and then the Indian method (67.8%).

- The overall success percentage of the Indian method is 79.5%, of the Maunder's method is 83.1% and that of Fotheringham's method is 79%.

The authenticity or success of each is measured in terms of number of crescent sightings without optical aid (positive sightings) that are in agreement with the criterion. Some authors have stressed on testing criteria on the basis of number of cases when the criterion predicts sighting and the crescent is not seen (negative sightings) (Fatoohi et al, 1999) as well. However others have indicated that chances of crescents sightings may increase with increase in more observers that are trained and experienced (Schaefer, 1988a). Therefore our emphasis is on exploring conditions under which crescent can be seen and not on whether it is actually seen or not so that boundaries may be sketched for judging the reliability of the claims of sightings. As mentioned earlier there can be a number of factors that may lead to negative sightings. If a criterion predicts sighting and the crescent is not actually seen does not at all mean that the criterion is not reliable. The weather conditions greatly affect visibility even if the sky is not overcast as shall be seen in the next chapter. Before authenticity of a criterion is to be judged one must explore further conditions on weather and the ability of the eyes of a person to sense the contrast between the dimly illuminated thin crescent and the brightness of twilight sky.

Chapter No. 4

PHYSICAL MODELS & THEIR EVOLUTION

All through the twentieth century and into the twenty first century a lot of work has been done on various aspects of the problem of visibility of new lunar crescent. These other issues include (i) length of lunar crescent (Danjon 1932, 1936, Ilyas 1983b, 1984a, McNally, Schaefer, 1991b, McNally, 1983, Sultan, 2005, Qureshi & Khan, 2006, etc.) (ii) the minimum or limiting elongation of new visible lunar crescent (Danjon, 1932, Ilyas 1983b etc.), (iii) seasonal variations in the earliest visibility of new lunar crescent (Ilyas, 1985, Caldwell & Laney, 2000 etc.). One of the most significant of these and other efforts is the introduction of the "International Lunar Date Line" or ILDL (contrasting the international date line (solar)) by Ilyas (Ilyas, 1986b). Though the idea has never been used in practice of lunar calendars but the same has been extensively used in software (for instance MoonCal by Manzur and Accurate Time by Odeh) as a guide for the regions of visibility or invisibility of the new lunar crescent. However, in this work the main emphasis is on the models that deal with the problem of earliest visibility of new lunar crescent so that other issues are not considered.

The first astrophysical model for solving this problem was that of Bruin (Bruin, 1977). This was based on the average brightness model for full Moon, the average brightness of the twilight sky and the theory of extinction (Kooman, 1952, Bemporad, 1904, Siedentopf, 1940). Bruin was also the first in modern times to exploit the variations of lunar semi-diameter with the Earth-Moon distance. Afterwards, appeared the extensive use of the physics and science of visibility by Schaefer during the last quarter of the twentieth century (Schaeffer, 1986, 1988a, 1988b, 1989, 1990, 1991a, 1993) based on various factors like atmospheric

extinction and sky brightness due to various objects leading to the limiting magnitude of the sky. He also the realized the importance of (a) lack of information about weather prediction systems and (b) need of further exploration of the physiology of human vision capabilities. Thus, since the theoretical model leading to Lunar Ripeness law by medieval Muslims the only theoretical models are due to Bruin and Schaefer. In this work Schaefer's techniques are applied to the recent observational data and are found to be in good agreement with the observational results.

The exploits of Yallop (Yallop, 1998) which was again more of empirical in nature is based on the observational data and part of Bruin's model but with the simplicity of a single parameter criterion for the new crescent visibility. Thus Yallop's model can be termed as a semi-empirical model. One of the most significant contributions of Yallop is his concept of best time of visibility. The software Hilal01 computes both the *q*-values (Yallop, 1998) and the *magnitude contrast* (the term coined in this work) that is the difference of the Magnitude of the Moon and the limiting magnitude of the sky close to crescent. The comparison of the two is discussed and some of the extra ordinary observations are critically analysed. The most significant part of this chapter is the development of a new single parameter criterion for the first visibility of new lunar crescent. We have considered the actual brightness of the crescent that is phase dependent (instead of average brightness of the full Moon close to horizon used by Bruin) and the actual brightness of the twilight sky close to the point where the crescent is present. For the brightness of both (the crescent and the sky) the tools developed Schaefer and others have been used. This has resulted into new visibility and limiting visibility curves. This leads to a new set of basic data which in turn is converted into a new single parameter criterion based on a relation between ARCV and width of crescent. Thus our model is another semi-empirical model. Our criterion is found to have better success percentage than any other criterion developed during the 20th century.

4.1 BRUIN'S PHYSICAL MODEL

Bruin based his work (Bruin, 1977) on the observed average brightness of sky against the position of the sun below horizon after sunset (that matched the results of Kooman et. al. (1952)) and the brightness of the Moon as a function of altitude, based on the theory of extinction due Bemporad (Bemporad, 1904). The figures given by him, Fig. 7 and 8 (Bruin, 1977, pp. 339) are reproduced here in Fig No. 4.1.1. On the basis of these studies Bruin developed the Lunar Visibility curves (relating altitude, h of crescent plotted against s, the solar depression below horizon) and the Limiting Visibility curves (relating $h + s$ against s) and presented in fig. no. 9 (included in the same Fig No. 4.1.1) (Bruin,, 1977 pp. 339).

Fig 7 shows how the average brightness of the sky B_S diminishes after sunset as a function of the altitude of the sun, the solar depression or dip s, as the sun goes below horizon. Fig 8 (Bruin, 1977) shows the variation of the average brightness of full Moon B_M as a function of the altitude of the Moon h above horizon. The Lunar Visibility curves developed by Bruin shown in fig 9 (Bruin, 1977) are developed using the two functions B_S and B_M as follows:

Assume a particular brightness of sky after the sunset, say 10^{-3} stilb, read out the corresponding solar depth below horizon from fig 7, $s = 4.4$ degrees in this case. In order that the crescent is visible in such a bright sky the crescent should also be at least as bright (10^{-3} stilb). Then from fig 8 read out the altitude of the Moon with the corresponding brightness that comes out to be $h = 1.9$ degrees. This produces a point on the visibility curve in fig 9 that shows a relation between the altitude h of the crescent and the solar dip s below horizon for a particular brightness of crescent and of sky. Thus a visibility curve is a collection of points (s, $h = f(s)$), where the brightness of crescent and that of sky match for constant width of crescent. The question is that fig 8 gives the brightness of the full Moon (around 30 arc minutes wide) and the crescent is generally less than 1 arc minute in width. Bruin realizes this problem but leaves it as it is by stating that any discrepancies shall be accounted for by some kind of "Gestalt" factor. As

exhibited in fig 9 it is noted that at solar dip $s = 0$ degrees the minimum altitudes for different width crescents are given in table 4.1.1.

W	0.5	0.7	1	2	3
H	14.5	10.2	8.2	4.4	4.3

Table 4.1.1

This means that the brightness of the sky at these altitudes (h) is same as the brightness of the crescent of corresponding width when the sun has just set. In order that the crescent is at least as bright as the sky with decreasing altitude the crescent must be wider and wider. These are the starting points of the visibility curves that are all sharply decreasing functions of the solar dip. That means that not only the brightness of crescent but the brightness of sky (that equal along these curves) both diminish sharply with the increasing solar depression.

Therefore, for larger values of s the crescent of same brightness can be seen at lower and lower altitude h. These are also the starting points of the curves that show the behaviour of $h + s$ (sum of altitude of Moon and the solar depth) against the solar depth s. Although the sum of the altitude h of the crescent and the s the solar dip remains almost constant as the crescent goes down these curves corresponding to fixed crescent width and thereby to fixed brightness. As the altitude of crescent decreases the sky brightness first decreases but then closer to the horizon the "visibility" starts decreasing. Thus these curves first start decreasing with increasing s, reach a minimum and then start increasing. This in fact shows the varying contrast of the brightness of the crescent and that of the sky. With smaller s and larger h the contrast is against the visibility. As s increases and h decreases the contrast becomes favourable for visibility of crescent. However, as h further decreases the contrast again becomes unfavourable for visibility.

Fig. No. 4.1.1 the figures from Bruin's Paper (1977)

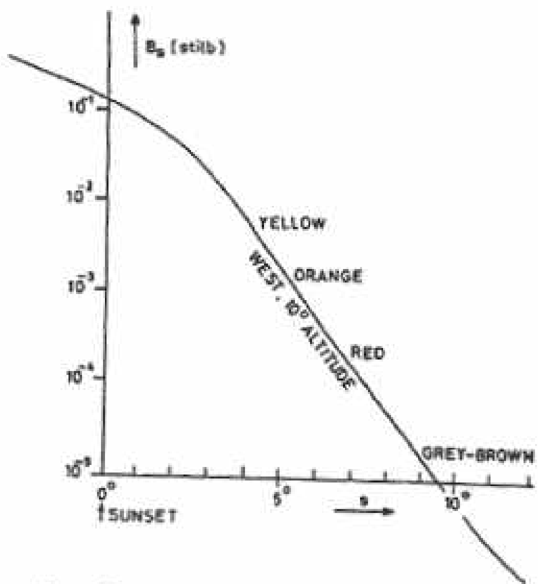


Fig. 7. Mean brightness B_s of the western sky after sunset, as a function of the solar dip angle s.

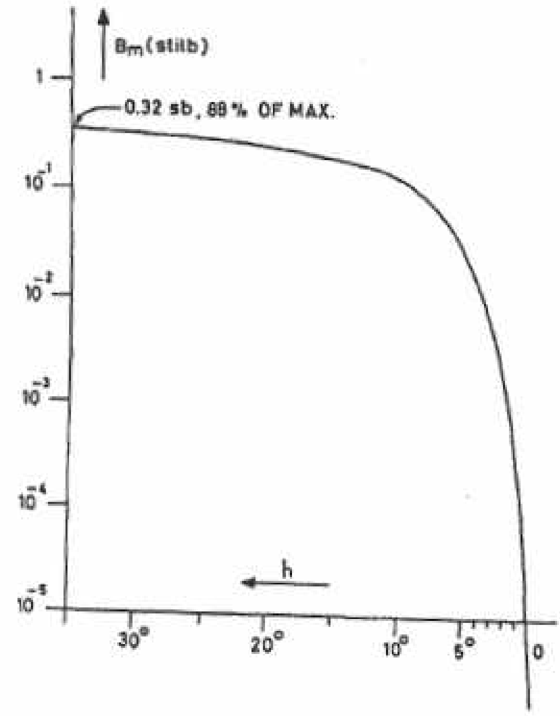


Fig. 8. Brightness B_m of the full moon at night, as a function of the altitude h in degrees.

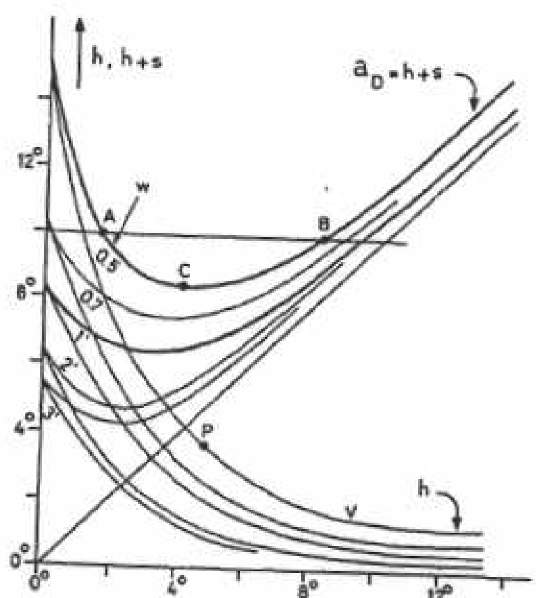


Fig. 9. Lunar visibility curves.

For thinner crescent the $h + s$ against s plots give the best time of visibility as the point which is the minimum of the curve. It also suggests a range of time for which the crescent may remain visible. These are excellent ideas that could help crescent-hunters but unfortunately Bruin has not presented a clear cut computational technique. Based on similar ideas Schaeffer has worked out another scheme of computation to be discussed later. The scheme for computations is deduced from the visibility curves of Bruin and is based on the minimum point of the $s + h$ plot against s. A relation between the crescent width corresponding to the $s + h$ curve and the value of $h + s$ at the minimum of the curve may be deduced from tabulating these values. For this purpose the data deduced from fig 9 by Yallop (Yallop, 1998) is as follows:

Table 4.1.2

W	0'·3	0'·5	0'·7	1'	2'	3'
$ARCV$	10·0	8·4	7·5	6·4	4·7	4·3

The values of ARCV are the values of $h + s$ picked from the minimum of the $h + s$ curves against s which corresponds to the best "time" of visibility of crescent. This data fitted to a third degree polynomial using least square approximation leads to the following relation between ARCV and W:

$$ARCV = 12.4023 - 9.4878W + 3.9512W^2 - 0.5632W^3 \qquad (4.1.1)$$

This can be transformed to the "visibility parameter" v_p function as follows:

$$v_p = (ARCV - (12.4023 - 9.4878W + 3.9512W^2 - 0.5632W^3))/10$$

(4.1.2)

This can be tested on the observational data with the visibility condition that a crescent should be visible if $v_p > 0$ otherwise it should remain invisible. Using condition (4.1.2) on the data set used to evaluate methods in chapter 3 the plot we obtained is shown in figure no. 4.1.2. In this figure the curve named "Bruin's Limit" is the plot of the equation 4.1.2.

The rest of the plot shows the relative altitude (in degrees) against the crescent width (in arc seconds) calculated at the best time of visibility for each observation of the data set considered in the previous chapter. All the positive sighting cases and the results of calculations based on (4.1.2) are shown in Table 4.1.3. The complete data set is shown in appendix-III.

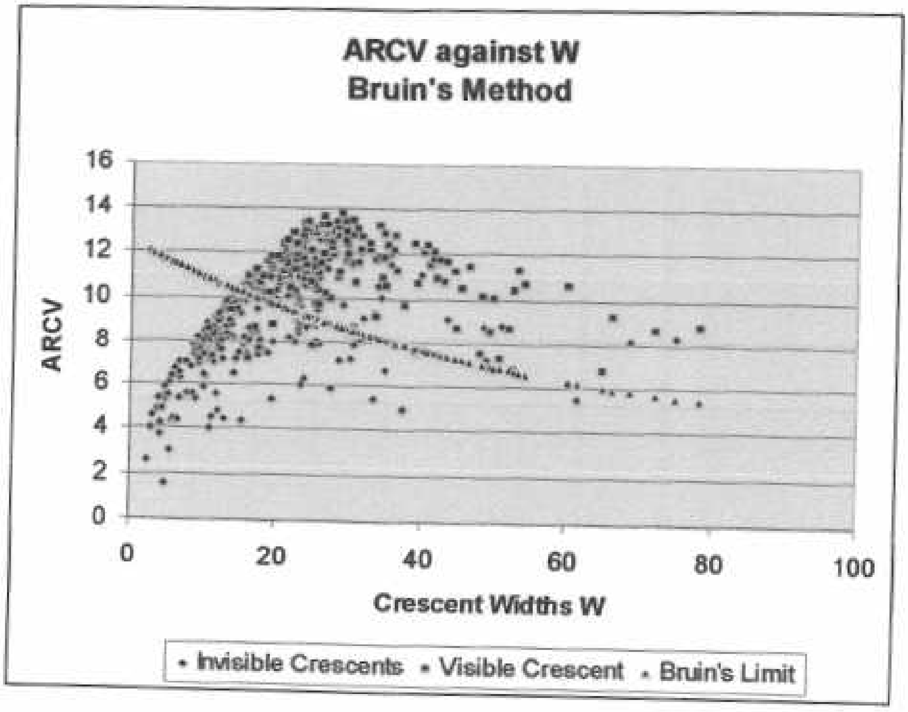


Fig. No. 4.1.2

Appendix-III shows the results of applying Bruin's limit, Yallop's (Indian) criterion (to be discussed in next article) and the new criterion developed in this work (to be reported in article 4.4 below). Under the heading "Model" the first column contains the values of visibility parameter defined by (4.1.2). The next two columns are for the other two models.

Table No. 4.1.3

S. No.	Date	Lat	Longit	Age	Lag	ARCL	ARCV	DAZ	Width	V_P
389	7/1/2000	-33.9	18.4	24.05	35.12	10.96	7.19	-8.28	16.23	-0.292
455	25/3/2001	-33.9	18.4	15.77	36.93	9.06	8.3	-3.63	11.33	-0.245
274	25/2/1990	35.6	-83.5	14.81	39.26	8.53	8.51	-0.55	10.75	-0.232
433	26/12/2000	-33.9	18.4	24.94	43.04	11.34	8.49	-7.52	17.28	-0.149
341	18/1/1999	-33.9	18.4	26.5	37.42	13.31	7.79	-10.8	25.04	-0.13
434	26/12/2000	-32.4	20.8	24.72	42.88	11.23	8.62	-7.21	16.96	-0.14
319	7/5/1997	31.8	34.9	19.94	40.88	11.64	8.74	7.68	19.57	-0.097
416	31/7/2000	6.5	3.4	15.94	37.68	9.58	9.41	1.8	13.86	-0.1
316	8/2/1997	-33.9	18.4	26.91	34.27	16.1	7.51	-14.3	39.31	-0.021
727	12/12/2004	32.4	-111	23.15	44.24	14.43	8.13	11.93	31.59	-0.029
639	22/1/2004	32.5	3.7	20.38	44.59	12.49	9.03	8.63	22.97	-0.029
315	13/10/1996	31.8	34.9	25.24	41.59	12.72	9.27	8.73	22.77	-0.007
285	24/5/1990	34.2	-118	15.57	52	10.17	10.14	0.81	15.72	-0.004
290	15/2/1991	33.4	73.1	19.68	46.87	10.17	10.12	-0.94	14.46	-0.022
334	27/2/1998	-33.9	18.4	24.29	39.37	14.23	8.69	-11.3	30.63	0.018
88	28/11/1913	-33.9	18.5	16.37	53.6	10.25	10.25	-0.04	15.34	0.002
162	9/3/1978	45.1	-64.2	20.04	54.55	10.73	10.16	3.46	16.65	0.01
173	28/1/1979	29.9	-81.3	17.01	48.07	10.42	10.42	0.09	16.57	0.035
286	20/9/1990	31.8	34.7	39.13	29.5	19.64	6.85	18.42	52.68	0.011
176	28/1/1979	42	-93.6	17.5	58.52	10.7	10.39	2.54	17.46	0.043
633	24/12/2003	49.6	8.7	30.16	53.8	18.11	7.21	16.62	48.95	0.022
320	7/5/1997	-33.9	18.4	19.59	49.11	11.46	10.18	-5.27	18.99	0.04
314	21/1/1996	-33.9	18.4	29.39	35.22	17.89	7.51	-16.3	48.12	0.047
616	26/9/2003	32.4	-111	22.5	42	13.14	9.68	8.89	25.79	0.067
218	12/12/1985	-32.4	20.8	17.13	55.16	10.78	10.53	-2.29	17.51	0.057
253	26/6/1987	33.5	-112	21.55	56	11.01	10.6	2.97	16.25	0.049
163	9/3/1978	42.7	-73.8	20.71	54.95	11.08	10.62	3.18	17.74	0.069
272	1/10/1989	31.3	34.6	41.91	32.07	19.52	7.34	18.1	50.67	0.047
417	28/9/2000	-33.9	18.4	21.25	47.16	12.48	10.26	-7.13	22.48	0.089
418	28/9/2000	-33.9	18.4	21.25	47.16	12.48	10.26	-7.13	22.48	0.089
142	18/2/1977	43.8	-87.7	20.23	57.95	10.88	10.85	0.66	16.65	0.079
347	18/3/1999	36	50.8	20.33	49.01	12.12	10.38	6.26	21.91	0.094
471	21/7/2001	4.1	73.3	17.97	44.8	10.93	10.92	0.56	18.12	0.104
622	26/10/2003	33.3	50.1	25.39	43.25	14.9	9.19	11.75	33.63	0.096
96	8/2/1921	42.3	-71.1	21.94	57.93	10.98	10.98	0.13	16.31	0.088
306	28/6/1995	-30.1	-71	21.45	52.52	10.94	10.94	-0.24	16.11	0.081
396	6/2/2000	-33.9	18.4	29.1	47.46	14.06	9.99	-9.91	27.26	0.114
348	18/3/1999	-34	18.4	22.56	46.24	13.35	10.16	-8.67	26.59	0.124
349	18/3/1999	-33.9	18.4	22.56	46.26	13.35	10.18	-8.66	26.59	0.126
89	16/3/1915	49.4	8.7	22.28	64.14	11.16	11.03	1.69	17.06	0.102
544	9/8/2002	-34	18.4	21.32	50.4	13.09	10.31	-8.08	25.59	0.128
419	28/9/2000	26.2	32.7	20.1	45.95	11.93	10.86	4.93	20.53	0.126

Table 4.1.3 Continued										
S. No.	Date	Lat	Longit	Age	Lag	ARCL	ARCV	DAZ	Width	V_P
323	5/7/1997	-33.9	18.5	21.58	56.2	11.24	11.18	-1.21	17.34	0.12
473	21/7/2001	31.9	35.8	21.33	51.76	12.88	10.56	7.38	25.13	0.148
574	3/1/2003	-33.9	18.4	22.06	56.88	12.26	10.84	-5.73	21.58	0.137
93	19/4/1920	43.5	7	21.06	60.27	11.94	10.93	4.81	21.3	0.142
2	27/10/1859	38	23.7	39.24	33.62	21.43	6.8	20.34	65.04	0.076
350	18/3/1999	29.6	52.5	20.22	48.29	12.06	10.95	5.05	21.69	0.149
456	24/4/2001	32.6	51.7	24.1	50.58	12.89	10.75	7.13	23.57	0.15
545	9/8/2002	32.6	51.7	20.54	51.76	12.67	10.86	6.53	23.97	0.165
167	9/3/1978	40.5	-89	21.74	55.53	11.63	11.24	2.98	19.51	0.152
414	2/7/2000	-32.4	20.8	20.79	55.32	12.33	10.93	-5.71	23.09	0.163
503	16/11/2001	49.6	8.7	33.45	58.35	17.76	8.7	15.5	44.97	0.143
420	28/9/2000	32.5	3.7	22.04	48.73	12.87	10.85	6.93	23.87	0.163
553	7/10/2002	49.6	8.7	29.94	49.46	18.07	8.53	15.95	49.37	0.157
79	7/12/1885	50.6	5.7	26.83	75.1	13.51	10.7	8.26	24.73	0.158
342	18/1/1999	28.8	43.7	23.16	53.51	11.64	11.41	2.32	19.16	0.165
397	6/2/2000	32.6	51.7	25.5	53.03	12.33	11.14	5.29	20.97	0.16
561	5/11/2002	-33.9	18.4	21.14	55.32	12.4	11.18	-5.36	23.16	0.188
122	8/12/1942	40.7	-74	19.95	62.85	12.56	11.15	5.8	23.96	0.194
222	28/4/1987	38.9	-77	22.81	58.01	11.64	11.54	1.52	18.76	0.174
223	28/4/1987	26.7	-81.1	22.72	50.44	11.6	11.54	-1.13	18.62	0.172
227	28/4/1987	38.9	-77.1	22.82	58.02	11.64	11.54	1.52	18.77	0.174
85	31/1/1911	51	-0.9	31.6	65.5	16.4	9.68	13.26	37.66	0.182
398	6/2/2000	36.2	37.2	26.39	55.72	12.76	11.17	6.17	22.45	0.179
595	2/5/2003	38.2	46	27.99	59.35	12.63	11.32	5.6	21.37	0.182
490	17/10/2001	-33.9	18.4	22.02	52.4	13.09	11.1	-6.95	25.32	0.204
407	5/4/2000	-33.9	18.4	22.77	50.92	13.29	11.05	-7.38	25.72	0.203
108	27/5/1922	-33.9	18.5	22.14	56.87	12.3	11.45	-4.5	21.52	0.197
76	30/3/1881	51.5	-2.6	20.68	73.32	11.81	11.7	1.62	20.01	0.204
230	28/4/1987	36.2	-81.7	23.16	60	11.8	11.75	1.1	19.29	0.201
640	22/1/2004	-33.9	18.4	21.31	57.29	12.96	11.33	-6.29	24.73	0.221
351	18/3/1999	31.9	35.8	21.36	51.5	12.69	11.37	5.63	24	0.217
596	2/5/2003	32.6	51.7	27.42	56.21	12.37	11.6	4.32	20.52	0.2
352	18/3/1999	31.8	35.2	21.4	51.6	12.71	11.41	5.61	24.09	0.222
83	1/5/1908	44.1	3.1	27.74	60.41	14.76	10.71	10.17	30.78	0.221
716	13/11/2004	36.8	-81.8	32.21	46.05	18.66	8.7	16.53	51.98	0.192
597	2/5/2003	27.7	54.4	27.1	54.12	12.23	11.8	3.22	20.04	0.215
288	18/12/1990	33.4	73.1	32.1	56.75	14.57	10.88	9.72	28.5	0.215
458	24/4/2001	31.9	35.8	25.17	53.28	13.4	11.36	7.12	25.48	0.232
651	21/3/2004	36.8	-81.8	25.39	54.72	12.78	11.64	5.3	22.7	0.229
81	19/4/1901	50.7	-2.8	22.12	74.22	13.14	11.47	6.43	26.24	0.251
598	2/5/2003	51.7	-9.5	32.38	78	14.61	11.09	9.53	28.56	0.237
547	9/8/2002	30.4	35.5	21.57	53.59	13.23	11.5	6.54	26.12	0.253
138	21/12/1976	42.7	-83.6	20.45	69.4	12.6	11.7	4.68	23.6	0.245
421	28/9/2000	43.3	-79.9	27.61	54.45	15.64	10.57	11.54	35.07	0.248
146	9/1/1978	41.9	-87.6	19.14	68.2	12.23	11.95	2.59	22.61	0.259
147	9/1/1978	36	-79.8	18.84	62.08	12.07	12.01	1.14	22.04	0.259

Table 4.1.3 Continued										
S. No.	Date	Lat	Longit	Age	Lag	ARCL	ARCV	DAZ	Width	V_P
148	9/1/1978	36	-79.8	18.84	62.08	12.07	12.01	1.14	22.04	0.259
150	9/1/1978	43	-89.8	19.24	69.62	12.28	11.94	2.88	22.81	0.26
151	9/1/1978	34	-81.1	19	60.95	12.15	12.13	0.72	22.35	0.274
153	9/1/1978	29.9	-81.3	19.15	58.38	12.23	12.23	-0.19	22.64	0.288
445	24/2/2001	51.7	7.2	33.2	69	15.98	10.89	11.72	34.51	0.274
550	7/9/2002	10.7	-61.5	19.38	48.08	12.44	12.21	2.37	23.46	0.295
155	9/1/1978	41.6	-93.6	19.57	71.04	12.46	12.18	2.65	23.48	0.292
55	4/6/1875	51.5	-2.6	22.71	96.96	14.24	11.5	8.41	30.56	0.298
156	9/1/1978	33.9	-84.3	19.22	61.49	12.27	12.25	0.76	22.79	0.291
139	21/12/1976	42	-91.6	21.03	70.39	12.91	12.02	4.71	24.75	0.29
157	9/1/1978	27.7	-82.7	19.32	57.34	12.32	12.3	-0.67	22.97	0.298
377	10/10/1999	5.3	103	23.72	47.88	12.37	12.35	0.78	20.93	0.28
269	4/6/1989	50.8	-1	25	93.93	14.49	11.5	8.83	30.55	0.298
144	11/12/1977	47.8	20	21.82	77.59	13.93	11.64	7.67	29.52	0.302
634	24/12/2003	33.4	73.1	26.79	56.89	16.21	10.68	12.21	39.34	0.296
329	30/12/1997	31.3	34.6	22.31	61.03	12.53	12.37	1.96	22.76	0.303
330	30/12/1997	31.3	35.2	22.27	60.93	12.51	12.35	1.95	22.68	0.3
241	28/4/1987	30.6	-104	24.52	60.9	12.46	12.46	0.04	21.46	0.297
339	21/10/1998	31.8	34.7	29.3	54.83	14.08	11.94	7.47	26.49	0.301
463	22/6/2001	49.6	8.7	32.19	73.52	18.31	10.01	15.35	49.93	0.309
38	20/2/1871	38	23.7	26.76	58.02	14.49	11.77	8.46	29.56	0.315
373	12/8/1999	31.8	34.7	29.72	51.8	15.98	11.16	11.46	36.59	0.32
670	18/6/2004	47.6	-118	32.14	85	15.27	11.58	9.97	31.2	0.312
378	10/10/1999	32	35.9	28	53.92	14.27	11.9	7.9	27.78	0.31
158	9/1/1978	30	-90.2	19.76	60.9	12.56	12.56	-0.05	23.86	0.334
159	9/1/1978	30	-90.2	19.76	60.9	12.56	12.56	-0.05	23.86	0.334
119	13/6/1934	55.6	33.9	40.99	97.17	19.04	10.13	16.14	48.46	0.311
243	28/4/1987	40.7	-112	25.82	70	13.08	12.7	3.15	23.63	0.346
674	18/7/2004	35.7	51.3	28.9	63.91	14.31	12.14	7.59	27.73	0.333
539	11/6/2002	32.4	-111	27.28	63	13.96	12.2	6.79	27.67	0.339
80	29/5/1900	38.7	-0.7	28.96	63.91	15.34	11.67	9.98	33.6	0.344
140	21/12/1976	29.9	-81.3	20.83	61.66	12.8	12.69	1.72	24.36	0.353
580	2/2/2003	32.6	51.7	27.73	58.07	14.66	11.92	8.55	29.85	0.333
732	11/1/2005	43.9	18.4	27.94	65.85	17.35	10.49	13.84	45.56	0.326
307	25/9/1995	-33.9	18.4	24.27	59.48	12.81	12.74	-1.35	23.69	0.35
399	6/2/2000	-4	39.7	27.04	50.79	13.07	12.73	-2.97	23.56	0.348
395	7/1/2000	10	-61.5	28.16	53.97	12.83	12.83	-0.13	22.23	0.343
635	24/12/2003	35.7	51.3	28.16	59.55	16.98	10.76	13.16	43.13	0.335
479	19/8/2001	33.9	-118	24.04	55	15.07	11.85	9.32	34.44	0.37
576	3/1/2003	32.4	-111	28.66	59	15.63	11.74	10.35	34.87	0.363
100	4/8/1921	-33.9	18.5	20.31	61.65	12.83	12.82	0.41	25.02	0.373
278	25/4/1990	41.6	-73.7	19.81	67.33	12.82	12.8	0.63	24.98	0.37
570	5/12/2002	35.7	51.4	30.21	59.21	16.94	10.9	12.99	42.05	0.34
493	17/10/2001	10.3	9.8	22.03	50.74	13.09	12.76	2.94	25.33	0.37
161	9/1/1978	29.7	-98.1	20.31	62.14	12.85	12.85	-0.01	24.98	0.375
215	21/1/1985	19	-155	26.2	54	13.88	12.55	5.94	26.7	0.364

Table 4.1.3 Continued										
S. No.	Date	Lat	Longit	Age	Lag	ARCL	ARCV	DAZ	Width	V_P
530	13/5/2002	32.6	51.7	29.09	62.33	14.02	12.52	6.33	27.18	0.366
675	18/7/2004	32.7	51.7	28.75	63.25	14.25	12.52	6.81	27.48	0.369
459	24/4/2001	32.5	3.7	27.36	58.8	14.47	12.36	7.53	29.69	0.375
441	25/1/2001	40.4	-74.5	33.52	67.01	15.37	12.13	9.45	31.57	0.371
112	25/5/1933	55.6	33.9	32.86	121	15.67	12.11	9.96	32.78	0.38
563	5/11/2002	32.4	-111	28.42	54	16.56	11.47	11.97	41.09	0.389
244	28/4/1987	37	-122	25.94	68	13.13	13.01	1.76	23.83	0.379
526	13/4/2002	26	-80.3	28.78	54.84	13.7	12.83	4.81	25.41	0.378
617	27/9/2003	49.6	8.7	38.44	49.85	22.05	8.65	20.3	72.22	0.293
531	13/5/2002	33.3	44.4	29.61	63.74	14.27	12.67	6.58	28.16	0.391
532	13/5/2002	29.6	52.5	28.94	61.41	13.94	12.78	5.58	26.89	0.389
92	1/4/1919	53.9	-1.6	22.24	87.1	13.51	12.86	4.15	27.67	0.405
87	25/8/1911	49.9	2.3	39.03	53.74	21.5	9.26	19.43	66.22	0.327
129	6/4/1970	48	-122	23.22	78	13.29	13.06	2.42	25.69	0.404
696	15/9/2004	-33.9	18.4	26.6	60.21	13.74	12.91	-4.71	26.84	0.401
446	24/2/2001	36	50.8	30.54	60.59	14.8	12.5	7.93	29.59	0.388
708	15/10/2004	32.9	59.2	35.06	48.8	19.01	10.42	15.92	52.75	0.369
325	3/9/1997	31.8	34.7	40.56	50.19	18.3	11.18	14.5	44.61	0.388
124	5/3/1954	44.5	-88	21.08	69.91	13.15	13.15	0.2	26.14	0.418
564	5/11/2002	32	-117	28.76	55	16.76	11.62	12.1	42.07	0.412
717	13/11/2004	13.7	10.7	26.71	52.08	15.52	12.13	9.69	36	0.412
533	13/5/2002	29.4	48	29.24	62.1	14.09	12.94	5.57	27.45	0.411
354	18/3/1999	34	-6.8	24.26	59.63	14.3	12.76	6.46	30.48	0.423
133	25/4/1971	39.5	-88.2	21.22	71	13.22	13.22	0.33	26.06	0.424
641	22/1/2004	41.8	-123	28.78	66	16.85	11.69	12.15	41.51	0.415
380	10/10/1999	34	-6.8	30.86	57.72	15.55	12.4	9.4	32.91	0.411
6	12/3/1861	38	23.7	27.35	64.33	13.37	13.28	-1.58	23.96	0.407
78	12/3/1899	52.5	13.3	21.83	83.59	13.32	13.22	1.6	26.18	0.425
652	21/3/2004	33.9	-118	27.84	60	13.94	13.08	4.83	26.96	0.419
116	14/5/1934	55.6	33.9	30.13	117.6	15.22	12.68	8.44	31.64	0.427
534	13/5/2002	26.2	50.5	28.97	60.9	13.96	13.13	4.75	26.95	0.424
464	22/6/2001	43.9	18.4	31.12	72.51	17.7	11.44	13.53	46.66	0.429
447	24/2/2001	5.3	103	27.39	52.52	13.4	13.38	0.69	24.27	0.421
465	22/6/2001	38.2	46	28.92	68.14	16.44	12.05	11.21	40.28	0.441
448	24/2/2001	-33.9	18.4	33.55	57.57	16.14	12.35	-10.4	35.2	0.427
535	13/5/2002	25.3	49.7	28.99	60.82	13.97	13.22	4.53	27	0.434
636	24/12/2003	32.7	51.7	28.28	61.51	17.05	11.59	12.52	43.46	0.42
536	13/5/2002	31.9	35.8	30.15	64.78	14.53	13.08	6.33	29.19	0.442
710	15/10/2004	32.6	51.7	35.57	49.83	19.29	10.66	16.1	54.32	0.403
709	15/10/2004	32.6	51.6	35.58	49.84	19.3	10.66	16.11	54.34	0.403
454	24/2/2001	32.6	51.7	30.53	60.33	14.79	12.95	7.16	29.57	0.433
568	5/12/2002	32.6	51.7	30.33	61.08	17.01	11.73	12.34	42.38	0.426
466	22/6/2001	35.7	51.3	28.44	67.26	16.16	12.39	10.4	38.94	0.463
366	14/6/1999	6.5	3.4	23.39	55.77	14.08	13.25	4.76	29.83	0.466
554	7/10/2002	32.5	51.7	27.3	55.23	16.56	12.06	11.37	41.54	0.452
1	1/7/1859	38	23.7	27.67	67.54	16.42	12.25	10.95	40.71	0.464

Table 4.1.3 Continued										
S. No.	Date	Lat	Longit	Age	Lag	ARCL	ARCV	DAZ	Width	V_P
13	1/1/1862	37.9	22.9	25.95	71.23	14.57	13.08	6.42	31.15	0.462
136	5/3/1973	40	-85	23.98	66.99	13.56	13.53	-0.98	26.57	0.46
527	13/4/2002	32.4	-111	30.56	62	14.49	13.11	6.21	28.4	0.437
588	2/4/2003	33.8	-118	31.37	62	14.54	13.37	5.72	28.29	0.462
297	23/2/1993	-34	18.4	52.71	39.98	24.33	8.81	-22.7	78.37	0.331
141	21/12/1976	37.6	-123	23.33	72	14.14	13.45	4.36	29.63	0.484
577	3/1/2003	10.4	-61.5	25.99	58.51	14.27	13.51	4.6	29.13	0.485
718	13/11/2004	10.3	9.8	26.87	54.38	15.6	12.8	8.94	36.41	0.483
449	24/2/2001	29.6	52.5	30.52	60.09	14.79	13.31	6.46	29.55	0.469
733	11/1/2005	32.6	51.6	26.21	63.76	16.38	12.32	10.81	40.65	0.471
423	28/10/2000	32.6	51.7	30.25	60.9	15.81	12.86	9.21	34.81	0.474
180	13/7/1980	41.4	-70.7	41.93	59	20.89	10.65	18	60.12	0.435
117	14/5/1934	50	36.2	29.44	99.97	14.91	13.43	6.49	30.38	0.489
548	9/8/2002	10.3	9.8	22.86	56.26	13.92	13.73	2.33	28.93	0.505
711	15/10/2004	30.2	57.1	35.25	51.37	19.12	11.24	15.49	53.35	0.455
126	5/4/1962	-25.8	-28.2	24.46	56.59	15.1	13.23	-7.31	34.2	0.506

Out of the 196 positive sighting in the sample 14 cases deviate from the Limit due to Bruin. Therefore the model due to Bruin is the best amongst all the other 20th century models and is as good as the Lunar Ripeness Law. All six cases that deviate from Babylonian criterion also deviate from Bruin's model. A closer look in to the details of 14 cases that deviate from Bruin's model reveals that the 14 cases deviating from the Lunar Ripeness Law and these 14 cases have 8 common cases. The first 3 cases of visible crescent in table 3.3.1 (observation no. 286, 2 and 272) of old crescent close to autumnal equinox have values of v_p well above Bruin's limit given by (4.1.2). As it was discussed in article 3.3 the crescents in these cases were older and must be brighter making visibility possible. The Bruin's limit is based on the brightness models therefore all these cases are satisfying the Bruin's limit despite having small lag.

Out of the six cases that deviate from Bruin's limit but not from Lunar Ripeness law (obs. no. 433, 434, 319, 315, 285 and 290) first three have small ΔR_{awr} values (0.01, 0.061 and 0.07 respectively) so are marginal for Lunar Ripeness Law. Similarly six of the cases that deviate from the Lunar Ripeness law but not from Bruin's limit (observation numbers 286, 2, 272, 314, 633 and 716) first three have been discussed before. The next

two 314 and 633 have small v_p values so are marginal cases in Bruin's Model. The last one (obs. No.716) is again an old age, wide and bright crescent.

Thus we observe that the Bruin's Limit and the Lunar Ripeness Law are not exactly supplementing each other but they are numerically equivalent. However as Bruin's limit is taking brightness into consideration therefore Bruin's limit is more logical and more successful.

4.2 YALLOP'S SINGLE PARAMETER MODEL

To develop his one parameter model of first visibility of lunar crescent Yallop used Bruin's work in order to extract optimum crescents width for various relative altitudes or arcs of vision ARCV, mentioned in previous article. Like Fotheringham developed a formula relating ARCV with DAZ on the basis of his summary table data, Yallop considered the basic data for developing a relation between ARCV and the width of crescent W. His data (from page 2 of NAO Technical Note No. 69 (1998)) is reproduced in the table no. 4.1.2.

However, as stated by Yallop, from 1996 March HM Nautical Almanac Office decided to abandon its test based on the Bruin method (4.2.1) for the one based on the "Indian" method as the Indian method produced more sensible results for old age sightings at high altitudes that occurs at least once a year for latitudes around 55 degrees. Based on the work of Schoch (1930) the basic data used in the Indian method is given in the table 4.2.1.

DAZ	0°	5°	10°	15°	20°
ARCV	10·4	10·0	9·3	8·0	6·2

Table No. 4.2.1

As the width of the crescent is given by (2.8.9) and that can also be written as:

$$W = 15(1 - \cos(ARCV)\cos(DAZ)) \quad (4.2.1)$$

assuming the semi-diameter of the Moon to be a constant 15 arc minutes. This basic Data can be transformed into data relating the width of the crescent and arc of vision ARCV like that in table 4.1.2 of Yallop based on Bruin. Yallop transformed that data fitting it to a cubic polynomial using least square approximation and obtained the following relation:

$$ARCV = 11.8371 - 6.3226W + 0.7319W^2 - 0.1018W^3 \quad (4.2.2)$$

While applying Bruin's method (equation 4.1.2) or the Indian method (4.2.2) the actual width W should be considered (not a constant 15 arc minutes). The method says that if $ARCV$ for the place of observation is more than the value of the right hand side of (4.2.2) the crescent should be visible. Thus a visibility parameter may be defined as:

$$V_p = (ARCV - (11.8371 - 6.3226W + 0.7319W^2 - 0.1018W^3))/10 \quad (4.2.3)$$

Yallop calls this V_p as q-value. In general if the visibility parameter V_p is positive the crescent may be seen. Applying this visibility condition on the data set of chapter 3 it is found that 16 positive sightings out 196 deviate from this "Indian" visibility condition. These deviating cases are shown in figure 4.2.1 as visible crescents below the Indian "Limit" and are tabulated in table 4.2.2 and the results for complete data set are presented in appendix-III in the second column under the heading "Models". In view of these results we find that the Bruin's Model is marginally better than Yallop's criterion for this data set.

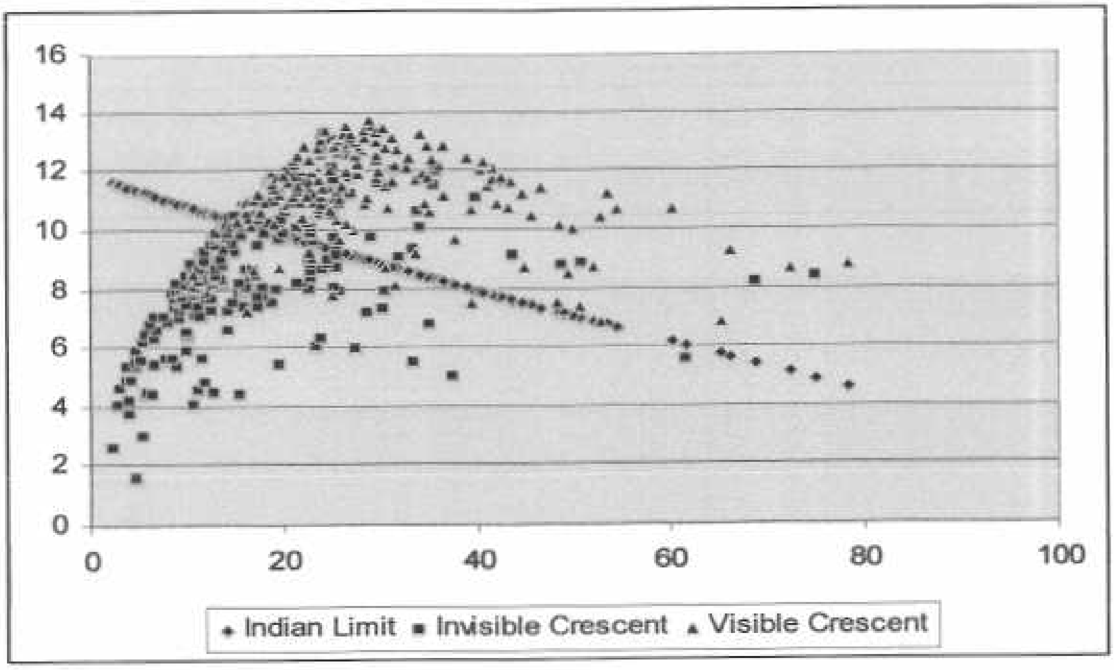
Yallop also converted Maunder's condition from DAZ-based to the width-based and obtained the following cubic relation between ARCV and W:

$$ARCV = 13.1783 - 9.0812W + 2.0709W^2 - 0.3360W^3 \quad (4.2.4)$$

which leads to the visibility parameter:

$$V_P = (ARCV - (13.1783 - 9.0812W + 2.0709W^2 - 0.3360W^3))/10 \tag{4.2.5}$$

Fig. No. 4.2.1



Applying Maunder's modified condition to the data set of Chapter 3 it is found that out of 196 positive sightings are there 27 cases that deviate from the condition defined by (4.2.5). The same are shown in figure 4.2.2. Thus Maunder's modified condition is still not better than the Indian and the Bruin's conditions. All the 14 cases that deviate from Bruin's limit also deviate from the Indian limit. The two additional cases (obs. no. 334 and 88) that deviate from Indian limit are found to be marginal cases (V_p = 0.018 and 0.002 respectively) in Yallop's Indian model. Thus the two models are in close agreement as they are both based on similar approaches and only slightly differ in their basic data sets.

Table No. 4.2.2

S. No.	Date	Lat	Longit	VISIBILITY			Age	Lag	ARCL	ARCV	DAZ	Width	Bruin	Indian
		Deg	Deg	N	B	T	Hrs	Min	Deg	Deg	Deg			
389	7/1/2000	-33.9	18.4	V			24.1	35.1	11	7.19	-8.28	16.2	-0.292	-0.299
455	25/3/2001	-33.9	18.4	V			15.8	36.9	9.06	8.3	-3.63	11.3	-0.245	-0.237
274	25/2/1990	35.6	-83.5	V		V	14.8	39.3	8.53	8.51	-0.55	10.7	-0.232	-0.222
433	26/12/2000	-33.9	18.4	V			24.9	43	11.3	8.49	-7.52	17.3	-0.149	-0.158
341	18/1/1999	-33.9	18.4	V			26.5	37.4	13.3	7.79	-10.8	25	-0.13	-0.153
434	26/12/2000	-32.4	20.8	V			24.7	42.9	11.2	8.62	-7.21	17	-0.14	-0.149
319	7/5/1997	31.8	34.9	V			19.9	40.9	11.6	8.74	7.68	19.6	-0.097	-0.111
416	31/7/2000	6.5	3.4	V			15.9	37.7	9.58	9.41	1.8	13.9	-0.1	-0.1
727	12/12/2004	32.4	-111	V	V	V	23.2	44.2	14.4	8.13	11.9	31.6	-0.029	-0.057
639	22/1/2004	32.5	3.7	V	V		20.4	44.6	12.5	9.03	8.63	23	-0.029	-0.049
316	8/2/1997	-33.9	18.4	V			26.9	34.3	16.1	7.51	-14.3	39.3	-0.021	-0.047
315	13/10/1996	31.8	34.9	V			25.2	41.6	12.7	9.27	8.73	22.8	-0.007	-0.027
290	15/2/1991	33.4	73.1	V			19.7	46.9	10.2	10.1	-0.94	14.5	-0.022	-0.023
334	27/2/1998	-33.9	18.4	V			24.3	39.4	14.2	8.69	-11.3	30.6	0.018	-0.01
285	24/5/1990	34.2	-118	V		V	15.6	52	10.2	10.1	0.81	15.7	-0.004	-0.009
88	28/11/1913	-33.9	18.5	V			16.4	53.6	10.3	10.3	-0.04	15.3	0.002	-0.002

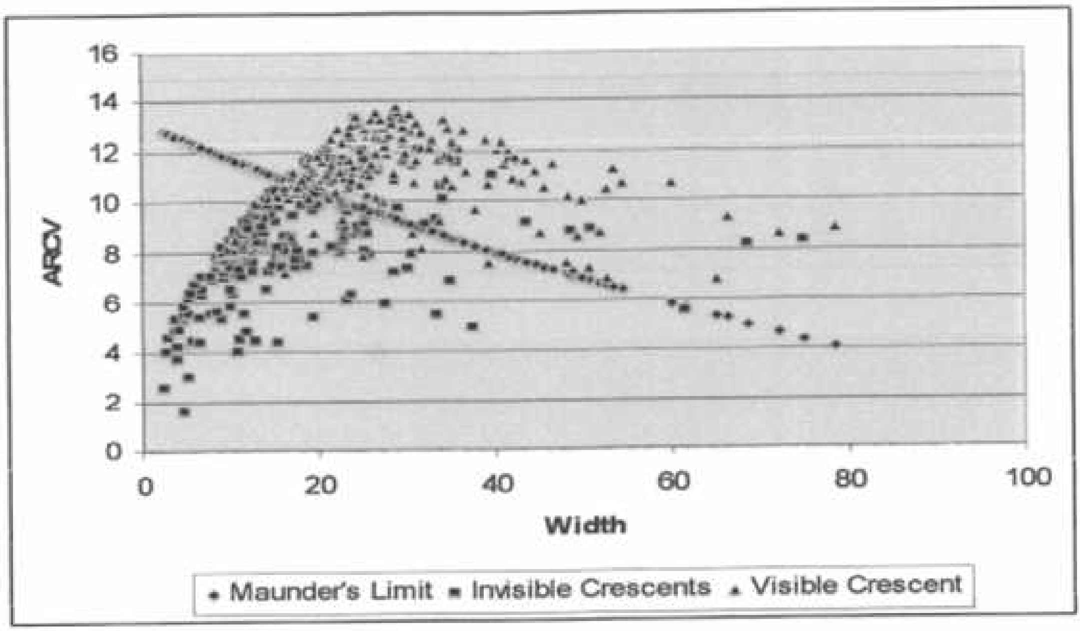


Fig. No. 4.2.2

In addition to converting all visibility conditions from DAZ-based to the Width-based Yallop achieved two other remarkable tasks. One of them is his deduction of "Best time of Visibility". The other remarkable contribution from Yallop is to draw lines

between "regions" of various visibility conditions on the bases of visibility parameter V_P used in the Indian method that he calls *q-value*.

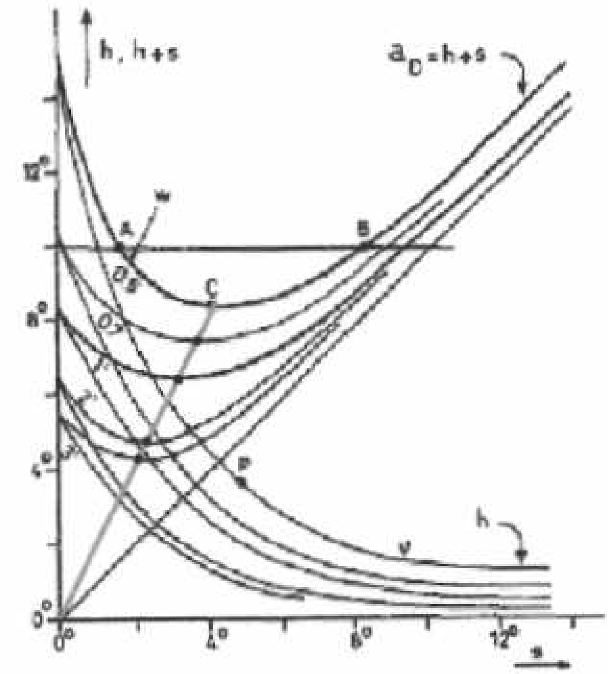


Fig. No. 4.2.3

For the "best time of crescent visibility" he notes that in fig 9 of Bruin the minima of visibility curves of $h + s$ against s for different crescent widths, form a straight line which when projected meets the origin of the coordinate system (h, s). The same is shown here in figure 4.2.3 as a red line passing through minimum of each curve. The slope of this straight line is $((h + s)/s)$ 9/4 or $h/s = 5/4$. This minimum corresponds to the best visibility condition, the best contrast between the average brightness of the sky and the brightness of the crescent according to Bruin. Yallop interprets it as "point in time" that divides "the time line" between the Sunset T_S and the Moonset T_M in a ratio 5:4. Thus this "point in time" is the best time T_B of crescent visibility given by:

$$T_B = \frac{5T_S + 4T_M}{9}$$

or

$$T_B = \frac{5T_S + 4(T_S + LAG)}{9} = T_S + \frac{4}{9}LAG \qquad (4.2.7)$$

In all calculations in this work the computations have been done for this best time given by (4.2.7). Especially the calculations for the table no. 4.2.2 in which the Indian condition (4.2.4) is used the computations are done for this best time.

Finally, Yallop deduces the visibility conditions for different ranges of *q-value* after a detailed analysis of the data set of around 256 observations available in his time. Our results in the second last column of table in appendix-III, are application of the Yallop's condition (based on basic data of Schoch, 1930 or the Indian method) for the data selected in chapter 3 which is taken mostly from Odeh (Odeh, 2004). The conditions deduced by Yallop are reproduced here in table 4.2.4. These conditions are indicators for visibility with or without optical aid.

Table 4.2.4: The q-test criteria.

	Range	Visibility circumstances	Vis.Code
(A)	$q > 0{\cdot}216$	Easily visible ($ARCL \geq 12°$) (EV)	V
(B)	$0{\cdot}216 \geq q > -0{\cdot}014$	Visible under perfect conditions (VUPC)	V(V)
(C)	$-0{\cdot}014 \geq q > -0{\cdot}160$	May need optical aid to find crescent (MNOA)	V(F)
(D)	$-0{\cdot}160 \geq q > -0{\cdot}232$	Will need optical aid to find crescent (ROA)	I(V)
(E)	$-0{\cdot}232 \geq q > -0{\cdot}299$	Not visible with a telescope $ARCL \leq 8{\cdot}5°$ (I)	I(I)
(F)	$-0{\cdot}299 \geq q$	Not visible, below Danjon limit, $ARCL \leq 8°$	I

The limiting values of q were chosen for the six criteria A to F for the following reasons (Yallop, 1998):

(A) A lower limit is required to separate observations that are trivial from those that have some element of difficulty. According to Yallop it was found that the ideal situation $ARCL = 12°$ and $DAZ = 0°$ produces a sensible cut-off point, for which $q = +0{\cdot}216$. There are 131 examples in Table in appendix-III (second last column) when q exceeds this value, and in general it should be very easy to see the new

crescent in these cases, provided there is no obscuring cloud in the sky. The reported positive sightings in these cases are 114. Thus it is reasonable to consider cases with $q > +0.216$ to be those when the crescent is easily visible.

(B) From observers reports it has been found that, in general, $q = 0$ is close to the lower limit for first visibility under perfect atmospheric conditions at sea level, without requiring optical aid. Yallop used his Table 4 to set this lower limit for visibility more preciselyand he says that from inspection of Table 4, the significance of $q = 0$ can be seen, but $q = -0{\cdot}014$ is another possible cut-off value. There are 68 cases in Table 4 with q in this range in the data used by Yallop with 48 positive sightings. The data used in this work (in appendix-III second last column) there are 133 cases with $q > -0.014$ but less than $+0.216$. Out of these 133 cases there are 69 positive sightings without optical aid.

(C) Yallop used his table 4 to find the cut-off point when optical aid is always needed to find the crescent moon by matching the q-test visibility code with Schaefer's code. The rounded value of $q = -0{\cdot}160$ was chosen for the cut-off criterion. In Table 4 (Yallop, 1998), there were 26 cases that satisfy this criterion ($-0.16 < q < -0.014$), only three cases out of these were positive unaided sightings the rest were seen with binoculars or telescopes. In Table in appendix-III, second last column, used in this work there 62 cases in this range of q-values out of which 10 were unaided sightings and 29 times the crescent was seen either with binocular or with telescopes.

(D) In this case ($-0{\cdot}160 \geq q > -0{\cdot}232$) Yallop's Table 4 has too few entries from which to estimate a lower limit for q. The situation is made worse by the fact that where there is an entry, in most cases, the Moon was not seen even with optical aid. In fact it is rare for the crescent to be observed below an apparent elongation of about $7^{\circ}{\cdot}5$ (Fatoohi et al, 1998). Yallop's Table 4 has 14 cases in this range out which 6 are positive sightings through binoculars or telescope and one extra ordinary case of unaided sighting. In the table in appendix-III (second last

column) of this work there are 33 cases out of which 18 are positive sightings with binocular or telescopes and three extra ordinary cases of unaided sighting (observation no. 389, 455 and 274) that is different from the case considered by Yallop. At the time of Yallop (1998) this was the limit below which it was assumed that it is not possible to see the thin crescent moon even with a telescope. Allowing 1° for horizontal parallax of the Moon, and ignoring the effect of refraction, for an apparent elongation of 7°·5, $ARCL$ = 8°·5. If DAZ = 0° this corresponds to a lower limit of $q = -0{\cdot}232$. Without good finding telescopes and positional information, observers are unlikely to see the crescent below this limit.

(E) There is a theoretical cut-off point when the apparent elongation of the Moon from the Sun is 7°, known as the Danjon limit (Danjon, 1932, 1936, Ilyas, 1983b, Fatoohi et al, 1998). This limit is obtained by extrapolating observations made at larger elongations. Allowing 1° for horizontal parallax of the Moon, and ignoring the effect of refraction, an apparent elongation of 7° is equivalent to $ARCL$ = 8°. With $ARCL$ = 8°and DAZ = 0° the corresponding lower limit on q is –0·293. However, in Yallop's table 4 there are 21 entries with only 3 positive sighting with a binocular for $q < -0.232$ and no sighting claim with un-aided eye. In table in appendix-III (second last column) there are 38 cases in the range $-0{\cdot}232 \geq q > -0{\cdot}299$ out of which there are 7 sighting with binocular or telescopes and only 2 claims of un-aided sightings (observation nos. 389 and 455). Both these observations deviate from all the criteria considered up to this point.

(F) The table in appendix-III (second last column) of this work shows 66 cases with $-0{\cdot}299 \geq q$ and there is one extra ordinary claim of sighting crescent without optical aid (obs. no. 389). Apart from this there is no positive sighting with or without optical aid. In table 4 of Yallop there is no claim of crescent sighting in this range.

According to the visibility classification shown above, the surface of Earth is divided into 5 regions by four constant-q values. As the actual visibility of the crescent

depends on its width and on its altitude above horizon at the time of sunset according to Bruin (Bruin, 1977) and Yallop (Yallop, 1998) a constant q-value describes a curve on the globe indicating similar visibility conditions along all points of the curve. Such a curve is a pseudo-parabolic curve with vertex on the east-most longitude. The longitudinal position of this vertex varies month to month and the latitude of the vertex depends on the declination of the Moon and the Sun on the celestial sphere. During summers in northern hemisphere the sun's declination is extreme north and if the declination of the Moon is north of the sun this vertex moves to extreme north and the crescent visibility is easier in the north latitudes. During summers of the northern latitudes if the Moon is south of the sun then this vertex does not reach its extreme northern position and still the new crescent visibility is better in the northern latitudes. The situation is reverse for the southern hemisphere. The parabola opens westward above (northwards) and below (southwards) from the vertex. The Curve A is the collection of points on the globe for which the q-value is 0.216. All regions within the two branches of the parabola west of the vertex are the regions where the q-value is greater than 0.216 and the crescent is easily visible to the naked eye in this region. Curve B is the collection of all points where the q-value is -0.014. In all the regions between curves A and B the crescent is visible to the naked eye only under perfect visibility conditions.

The regions between curve B and C (q-value -0.16) are the regions in which an observer would require optical aid to locate the crescent and then it may be visible to naked eye. For regions with q-value less than -0.16 the crescent would not be visible to the naked eye. For a common, untrained observer it is highly unlikely that the crescent would be seen in regions east of the curve A. The scientifically recorded observations (on which all the study of the twentieth century is based) do not prohibit observation of crescent with naked eye in region between curves A and C. In such regions, in fact, the probability of observation increases with the number of keen trained and experienced observers.

4.3 SCHAEFFER'S LIMITING MAGNITUDE MODEL

Bruin (1977) used only average brightness of sky during twilight and the variation of brightness of the full Moon to obtain approximate contrast for crescents of various widths to develop his "crescent visibility curves" discussed above. The resulting model formalized in terms of a relation between crescent width and the relative altitude of crescent at "best time" deduced by Yallop (1998) proved to be highly successful. Although, the basic data extracted by Yallop from Bruin's "visibility curves" was replaced by the basic data due to Schoch (1930) (the Indian method) to arrive at his q-value conditions (discussed above) for the new crescent visibility it has been seen that results from Bruin's data are marginally better than the Indian method for the data set used in chapter 3.

Schaeffer (1988a, 1988b) on the other hand has used the physics of "visibility" extensively that resulted into a tool (that Schaeffer converted into a computer program) to determine the brightness of sky at any point of time and for different atmospheric temperatures and relative humidity. In this work Schaeffer's program is reproduced and made part of the lunar crescent visibility software developed, Hilal01, to evaluate visibility conditions. This part of the software is used to compute the sky brightness (or limiting magnitude) at points close to the crescent and the apparent magnitude of the lunar crescent to study the varying contrast during twilight for various temperatures and relative humidity. If the apparent brightness of the crescent is more than the brightness of the twilight sky the crescent should be visible otherwise not. Schaeffer (1988a) himself has applied a similar technique to analyse the visibility or invisibility data available in his time. The technique is applied to the data set of the chapter 3 and the results obtained are presented in Table No. 4.3.1. Before a discussion on these results Schaeffer's methodology for computing sky brightness under different atmospheric conditions is briefly discussed.

After the conjunction the new lunar crescent can be seen in the western sky close to the horizon and the point of sunset. Similarly the last crescent can be seen in the

eastern sky before sunset. The contrast between the brightness of crescent and that of the twilight sky depends on a number of factors. These include:

- Position of the crescent which itself is significantly affected by the atmospheric refraction.
- The scattering of the light from crescent and the sunlight due to (a) the total amount of air-mass the light travels-through, (b) the total amount of aerosol present in this air, and (c) the stratospheric ozone through which the light has to travel. These scattering sources cause the intensity of light to decrease.
- Sources of light that include the Sun, the Moon and other sources (like artificial light that is not considered in this work as they are not so affective during twilight).
- The atmospheric temperature and the relative humidity.

Taking into consideration all these affects the total brightness of the sky is computed at the point where crescent is present. If the brightness of the sky is more than or equal to the brightness of the crescent the crescent can not be seen. Even if the brightness of the crescent is marginally more than the brightness of the sky it is very difficult to locate the crescent without any optical aid. In the following the quantitative tools are discussed briefly for all these computations:

For altitude well above horizon the apparent position of the crescent is raised by an amount R, the angle of refraction (Smart, 1953, Green 1985), given by:

$$R = 58''.2\left[\frac{0.372P}{273+T}\right]\tan z \qquad (4.3.1)$$

where P is the atmospheric pressure, T is the temperature and z is the zenith distance of the crescent. For altitudes closer to the horizon the following relations (Saemundsson, 1986) are better suited:

$$R = 1'{*}\cot\left[h + \frac{7.3}{h + 4.4}\right] \tag{4.3.2}$$

$$R = 1'.02 * \cot\left[h' + \frac{10.3}{h' + 5.11}\right] \tag{4.3.3}$$

where $h = 90^0 - z$ and $h' = 90^0 - z - R$. The software Hilal01 we developed in this work uses this optionally.

For the determination of visual limiting magnitude the major steps of calculations (adopted from Schaefer's program) are listed below with brief description. References and detailed descriptions can be found in Schaefer (1993).

The program function *limmagnit()* takes as input/pre-calculated values listed below:

- *malt*, the altitude of moon above horizon, *mazm*, the azimuth of the moon
- *palt*, the altitude of the place above sea level in meters, *phum*, estimated relative humidity of the place, *plat*, latitude of the place, *ptemp*, estimated temperature of the place in ^{0}C.
- *salpha*, the right ascension of the sun at the time of observation,
- *wasch[i]* (= 0.365, 0.44, 0.55, 0.7, 0.9) the wavelengths corresponding to U, B, V, R and I bands
- *bosch[i]* (= 8×10^{-14}, 7×10^{-14}, 1×10^{-13}, 1×10^{-13}, 3×10^{-13}) parameter values in the night time brightness associated with each wavelength selected.
- *ozsch[i]* (= 0, 0, 0.031, 0.008, 0) parameter values in the extinction coefficient corresponding to the ozone factor associated with each wavelength selected
- *wtsch[i]* (= 0.074, 0.045, 0.031, 0.02, 0.015) parameter values in the extinction coefficient corresponding to the weather factors (humidity, temperature etc.) associated with each wavelength selected
- *mosch[i]* (= -10.93, -10.45, -11.05, -11.9, -12.7) the magnitude of full moon corresponding to different selected wavelength bands
- *mssch[i]* (= -25.96, -26.09, -26.74, -27.26,. -27.55) the solar magnitude corresponding to different selected wavelength bands

- *cmsch[i]* (= 1.36, 0.91, 0.00, -0.76, -1.17) correction for lunar magnitudes corresponding to different selected wavelength bands
- *year*, the year of the observation,
- *elongp*, the elongation of the moon from the sun at the time of observation

The function starts with selecting a point F with sky position given by *falt* = *malt* + 0.1, *fazm* = *mazm* + 0.1, i.e. a point close to the centre of the lunar disc. The zenith distance of this point, *zendist* is used to calculate the gas, aerosol and the ozone mass components as follows:

$$X_g = [\cos(zendist) + 0.0286 * \exp(-10.5 * \cos(zendist))]^{-1} \tag{4.3.4}$$

$$X_a = [\cos(zendist) + 0.0123 * \exp(-24.5 * \cos(zendist))]^{-1} \tag{4.3.5}$$

$$X_o = \left[1 - \left\{\frac{\sin(zendist)}{1 + 20 / 6378}\right\}^2\right]^{-0.5} \tag{4.3.6}$$

This is followed by the calculation of the extinction coefficients components corresponding to five different wavelengths selected in the array *wasch[]*:

$$K_r = 0.1066 * \exp\left(-\frac{palt}{8200}\right) * \left(\frac{wasch[i]}{0.55}\right)^{-4} \tag{4.3.7}$$

$$\begin{aligned} K_a = 0.1 * \left(\frac{wasch[i]}{0.55}\right)^{-1.3} * \exp\left(-\frac{palt}{1500}\right) * \left(1 - \frac{0.32}{\ln(phum / 100)}\right)^{1.33} \\ * \left(1 + 0.33 * \sin(salpha) * \frac{plat}{|plat|}\right) \end{aligned} \tag{4.3.8}$$

$$K_o = ozsch[i] * (3 + 0.4 * (plat * \cos(salpha) - \cos(3 * plat))) / 3 \tag{4.3.9}$$

$$K_w = wtsch[i] * 0.94 * \left(\frac{phum}{100}\right) * \exp\left(\frac{ptemp}{15}\right) * \exp\left(-\frac{palt}{8200}\right) \tag{4.3.10}$$

For each wavelength band these extinction coefficients are accumulated in *ksch[i]*:

$$ksch[i] = K_r + K_a + K_o + K_w \tag{4.3.11}$$

And their linear combination with mass components are gathered into array *dmsch[]*:

$$dmsch[i] = K_r * X_g + K_a * X_a + K_o * X_o + K_w * X_g \tag{4.3.12}$$

The equations (4.3.7) to (4.3.12) are placed in a loop that runs five times once for each i 0, 1, 2, 3 and 4. After the execution of this loop the air mass at point F, at the position of the Moon and that of the Sun are calculated using:

$$stpoef = [\cos(zendist) + 0.025 * \exp(-11 * \cos(zendist.)]^{-1} \tag{4.3.13}$$

$$\begin{aligned} &if\ (malt \le 0)\ \ mnposef = 40 \\ &else\ \ \ mnpoef = [\cos(90 - malt) + 0.025 * \exp(-11 * \cos(90 - malt)]^{-1} \end{aligned} \tag{4.3.14}$$

$$\begin{aligned} &if\ (salt \le 0)\ \ snposef = 40 \\ &else\ \ \ snpoef = [\cos(90 - salt) + 0.025 * \exp(-11 * \cos(90 - salt)]^{-1} \end{aligned} \tag{4.3.15}$$

The magnitude of the Moon is then calculated using:

$$\begin{aligned} mnmag = -12.73 + 0.026 * |180 - elongp| + 4 * (180 - elongp)^4 * 10^{-9} \\ + cmsch[i] \end{aligned} \tag{4.3.16}$$

Followed by night-time brightness, moon-light brightness, twilight brightness and the day-light brightness:

$$cthree = 10^{-0.4 * ksch[i] * mnposef} \tag{4.3.17}$$

$$\begin{aligned} fem = 10^{(6.15 - felongmn\ /\ 40)} + 6.2 * 10^7 / (felongmn)^2 + \\ 10^{5.36} * (1.06 + \cos^2(felongmn)) \end{aligned} \tag{4.3.18}$$

where *felongmn* is the elongation of F from the centre of the lunar disc.

$$\begin{aligned} moonb = 10^{-0.4 * (mnmag - mosch[i] + 43.27)} * \left(1 - 10^{-0.4 * ksch[i] * stpoef}\right) * \\ (fem * cthree + 440000 * (1 - cthree)) \end{aligned} \tag{4.3.19}$$

$$\begin{aligned} twilb = 10^{-0.4 * (msch[i] - mosch[i] + 32.5 - salt - zendist\ /(360\ /\ ksch[i]))} * \\ \left(\frac{100}{felongmn}\right) * \left(1 - 10^{-0.4 * ksch[i] * stposef}\right) \end{aligned} \tag{4.3.20}$$

$$cfour = 10^{-0.4 * ksch[i] * snposef} \tag{4.3.21}$$

$$\begin{aligned} fes = 10^{(6.15 - felongsn\ /\ 40)} + 6.2 * 10^7 / (felongsn)^2 + \\ 10^{5.36} * (1.06 + \cos^2(felongsn)) \end{aligned} \tag{4.3.22}$$

$$\begin{aligned} dayb = 10^{-0.4 * (msch[i] - mosch[i] + 43.27)} * \left(1 - 10^{-0.4 * ksch[i] * stposef}\right) * \\ (fes * cfour + 440000 * (1 - cfour)) \end{aligned} \tag{4.3.23}$$

If the twilight brightness *twilb* dominates over the day-light brightness *dayb* then the sum of *nightb* and *twilb* are stored in *bsch[i]* otherwise the sum of *nightb* and *twilb* are stored in *bsch[i]*. Moreover, if the Moon is above horizon then *moonb* is also added to *bsch[i]*. Finally, the brightness *bsch[i]* is converted into nano-lambarts. From equation (4.3.16) till this point all computation is done in a loop that executes five times again, once for each wavelength band selected. The calculation of the limiting magnitude *lem* is done as follows:

$$bel = bsch[2]/\,0.0011 \tag{4.3.24}$$

$$\begin{array}{ll} \text{If } (bel < 1500) & \{\, cone = 10^{-9.8},\ ctwo = 10^{-1.9} \,\} \\ \text{Else} & \{\, cone = 10^{-8.35},\ ctwo = 10^{-5.9} \,\} \end{array} \tag{4.3.25}$$

$$teh = cone * \left(1 + \sqrt{ctwo * bel}\right)^2 \tag{4.3.26}$$

$$lem = -16.57 - 2.5 * \left(\frac{\ln(teh)}{\ln(10)}\right) - dmsch[2] \tag{4.3.27}$$

Schaefer (Schaefer, 1988) in his threshold contrast model calculates *R* as the log of the ratio of the actual total brightness of the Moon and the total brightness of the Moon needed for visibility for the given observing conditions. In this work we consider the magnitude of the Moon and the visual limiting magnitude calculated from the algorithm given above. The difference of Moon's magnitude *mnmag* and the visual limiting magnitude *lem* is considered as *magnitude contras* denoted as *Δmag*. A plot (Fig 4.3.1 below) shows difference of Schaefer's threshold contrast *R* and the magnitude contrast *Δmag*. Series 1 shows *Δmag* for crescents that were not seen and series 2 shows *Δmag* for crescents that were seen. Series 3 and series 4 show *R* corresponding to crescents that were not seen and that were seen respectively. The data for this fig is taken from Schaefer (1988). Positive values of contrast for crescents that were not seen and the negative values for contrast that were seen show the inconsistencies of the models with the observation. These inconsistencies may result from estimated values of temperature and relative humidity adopted for the calculations.

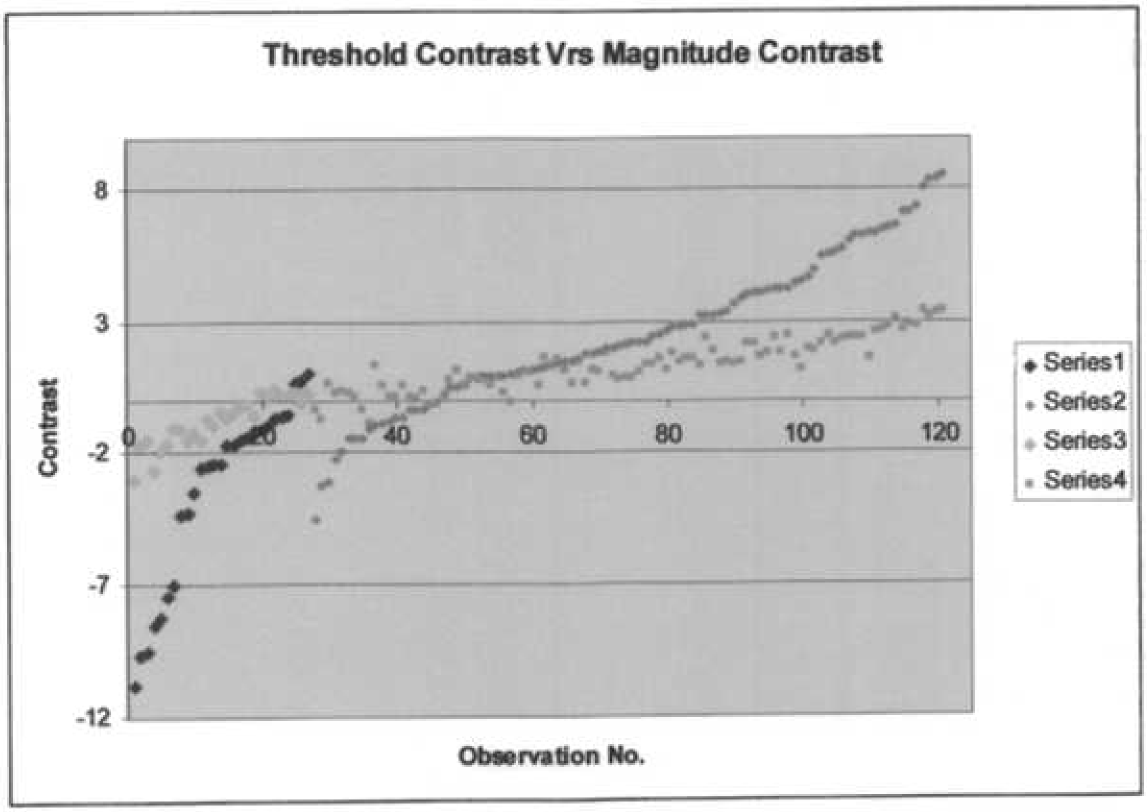


Fig. No. 4.3.1

Table 4.3.1 is developed using the same program Hilal01 in order to analyse the crescent observation records taken from literature (Schaefer, 1988a, Yallop, 1998, Odeh, 2004). In each row of the table observation number (as assigned by Odeh (Odeh, 2004)), date of observation, latitude, longitude and elevation above sea level of the place from where the crescent is observed, followed by the estimated temperature and estimated relative humidity. The next three columns contain the universal time and the corresponding *magnitude contrast* when the *magnitude contrast* becomes just favourable for sighting of crescent (column with heading "start"), when it is best for sighting (column with heading "best") and when it is favourable for sighting for the last time (column with heading "last"). This gives the time range for the possible naked eye visibility of the crescent. The *magnitude contrast* is considered only for unaided visibility of crescent. The last three columns contain the information regarding whether the crescent was claimed to be visible without any optical aid, with a binocular or with a telescope.

The observations considered in the table 4.3.1 are only those when it was claim that the crescent was seen by any means as the records when the crescent was not seen are not relevant. Moreover, by varying the estimated temperature and relative humidity it is evaluated to obtain optimum conditions for naked eye visibility of the crescent. If the *magnitude contrast* is obtained to be in favour of visibility only then the time range of naked eye crescent visibility are determined and included in the table. If the *magnitude contrast* is not found to be in favour of unaided visibility the least positive value of the *magnitude contrast* is calculated and the same value is included in all the three corresponding columns. The same positive value in all these columns is the indication that under the weather conditions considered the crescent was never visible to the naked eye.

This table shows that there are 13 cases of positive sighting claims without optical aid when magnitude contrast was never in favour of unaided visibility. All these positive cases are also not in agreement with the conditions due to Fotheringham and maunders. Lunar Ripeness law does not allow 10 of these and the two of them are only marginal cases according to it. Only one case markedly differs from Lunar Ripeness law. Only 3 of these 13 cases are not allowed by Babylonian criterion. Ten of these cases are not allowed by Indian method and the rest of the three are marginal cases in Indian method. Both Bruin's limit and Yallop's criterion do not allow nine of these cases. In case of other four claims all four cases are marginally allowed by Bruin's criterion but Yallop's criterion differs a lot in one of them.

The first page of the table 4.3.1 shows two extra ordinary claims of naked eye visibility of crescent, obs. No. 389 and 455 with q-values -0.299 and -0.236. The Yallop's criterion does not allow naked eye visibility for these q-values and the *magnitude contrast* is also not favourable even with highly exaggerated weather conditions. The Modified Ripeness Function (chapter 3) values corresponding to these observations are also not favourable (-0.95 and -0.62 in table 3.5.1) for crescent visibility. Therefore these observations, as they fail to satisfy every model, are highly unreliable and are outliers.

There is only one more claim of naked eye visibility of crescent with q-value less than -0.16, observation no. 274, with q-value -0.221 (ΔR_{avr} = -1.19). This observation is not allowed by both the Yallop's criterion and the Lunar Ripeness Law but with higher elevation (1524 meters above sea level) and low humidity (estimated to be 30%) the *magnitude contrast* is favourable for unaided visibility and the observation is not unreliable. In all the rest of the crescent observations with q-values less than -0.16 the claim of visibility of crescent is with binocular or with telescope. These claims are also not unreliable as out of 25 such claims 8 have favourable *magnitude contrast* for naked eye visibility with optimum estimates of temperatures and relative humidity.

The magnitude contrast for some other reportedly positive crescent sightings without optical aid is not favourable. These are observation numbers 341, 319, 416, 316, 315, 286, 633, 314, 272, 2. The q-values (and ΔR_{avr}) for these crescents are -0.153 (-0.87), -0.11 (0.07), -0.101 (-1.06), -0.047 (-1.5), -0.027(0.367), 0.007 (-3.5), 0.01 (-0.72), 0.032 (-1.03), 0.038 (-2.88), 0.109 (-3.47). For 341, 416, 316 the three criteria (Yallop's, Lunar Ripeness Law and the magnitude contrast) are consistent. For 286, 633, 314, 272 and 2 Yallop's criterion allows naked eye visibility under "perfect visibility conditions" but both the Lunar Ripeness Law and the magnitude contrast are unfavourable for naked eye visibility. Thus it appears that Lunar Ripeness Law is more consistent with the magnitude contrast results.

In this work limiting telescopic magnitudes are not considered as the reported crescent observations with binoculars and telescopes do not provide appropriate details. Therefore the appropriate limiting telescopes can not be computed. Moreover, our work is more concerned with visibility of new crescent without any optical aid.

Table 4.3.1

O.	Date	Latit.	Long.	Elev.	Tem	Hum	Q	Instant (Magnitude Contrast)			Visibility		
No.		Deg	Deg	Meter	C	%	val	Start	Best	End	N	B	T
737	2/11/2005	32.2	-111.1	963	5	20	-0.46	0:34(2.76)	0:34(2.76)	0:34(2.76)			V
720	13/11/2004	36.1	50.3	1270	10	50	-0.32	13:37(2.01)	13:37(2.01)	13:37(2.01)		V	
389	7/1/2000	-34	18.4	200	25	30	-0.3	18:7(2.13)	18:7(2.13)	18:7(2.13)	V		
682	16/8/2004	32	35.9	940	30	50	-0.28	16:27(2.50)	16:27(2.50)	16:27(2.50)			V
477	19/8/2001	29.5	56.8	2700	30	50	-0.28	15:4(0.48)	15:4(0.48)	15:4(0.48)		V	
549	7/9/2002	31.1	56.5	1700	25	50	-0.27	14:43(0.98)	14:43(0.98)	14:43(0.98)		V	
707	14/10/2004	32.2	-111.1	963	15	50	-0.27	8:59(1.76)	0:59(1.76)	0:59(1.76)			V
552	6/10/2002	32.4	-111	843	15	50	-0.26	1:12(1.56)	1:12(1.56)	1:12(1.56)			V
557	5/11/2002	29.9	54.4	2300	10	40	-0.24	13:32(-0.07)	13:44(-0.77)	13:52(-0.06)		V	
650	21/3/2004	30.4	35.5	1646	20	50	-0.24	16:6(0.27)	16:6(0.27)	16:6(0.27)			V
455	25/3/2001	-34	18.4	200	10	30	-0.24	17:2(1.28)	17:2(1.28)	17:2(1.28)	V		
638	22/1/2004	30	51.7	2500	10	40	-0.24	14:6(-0.04)	14:21(-0.97)	14:30(-0.01)		V	
391	7/1/2000	32.7	52.3	1500	10	30	-0.24	13:51(-0.05)	13:59(-0.40)	14:6(-0.11)		V	
318	7/5/1997	36	50.8	1500	30	50	-0.23	15:38(1.39)	15:38(1.39)	15:38(1.39)		V	
558	5/11/2002	30.1	52.1	2200	10	30	-0.23	13:47(-0.10)	14:1(-0.95)	14:9(-0.12)		V	
559	5/11/2002	29.6	52.5	1500	10	30	-0.23	13:50(-0.06)	13:57(-0.31)	14:2(-0.07)		V	
274	25/2/1990	35.6	-83.5	1524	10	30	-0.22	23:35(-0.04)	23:44(-0.44)	23:51(-0.02)	V		V
310	20/1/1996	32.4	-111	853	2	30	-0.22	1:2(0.30)	1:2(0.30)	1:2(0.30)			V
375	10/9/1999	30.4	35.5	1646	20	40	-0.21	16:0(0.02)	16:6(-0.20)	16:10(-0.07)			V
311	20/1/1996	32.8	-113.2	259	4	30	-0.21	1:7(1.25)	1:7(1.25)	1:7(1.25)			V
478	19/8/2001	30.2	35.5	1724	20	40	-0.2	16:25(-0.00)	16:33(-0.32)	16:38(-0.09)		V	V
560	5/11/2002	31.9	35.8	939	10	30	-0.2	14:58(0.23)	14:58(0.23)	14:58(0.23)		V	
321	7/5/1997	32.7	52.3	1500	25	30	-0.2	15:27(-0.03)	15:32(-0.16)	15:36(-0.05)		V	
573	3/1/2003	32.5	3.7	550	10	30	-0.19	17:5(0.87)	17:5(0.87)	17:5(0.87)		V	
328	30/12/1997	-34	18.4	350	22	30	-0.18	18:7(1.18)	18:7(1.18)	18:7(1.18)		V	
432	26/12/2000	30.2	35.5	1680	20	60	-0.18	15:3(0.17)	15:3(0.17)	15:3(0.17)		V	V
312	20/1/1996	34.1	-118.3	530	10	30	-0.18	1:27(0.62)	1:27(0.62)	1:27(0.62)			V
629	24/11/2003	41.5	-112	1350	15	30	-0.16	0:14(0.41)	0:14(0.41)	0:14(0.41)		V	
433	26/12/2000	-34	18.4	200	20	30	-0.16	18:13(0.63)	18:13(0.63)	18:13(0.63)	V		
281	24/5/1990	35.6	-83.5	1524	25	50	-0.16	0:57(0.17)	0:57(0.17)	0:57(0.17)			V
137	1/7/1973	-44	170.5	1189	26	50	-0.16	5:29(0.61)	5:29(0.61)	5:29(0.61)		V	
714	13/11/2004	32	35.9	953	15	30	-0.16	14:50(0.52)	14:50(0.52)	14:50(0.52)			V
341	18/1/1999	-34	18.4	200	20	20	-0.15	18:10(0.55)	18:10(0.55)	18:10(0.55)	V		
301	1/1/1995	33	-106	1219	10	50	-0.15	0:22(-0.04)	0:28(-0.22)	0:33(-0.06)			V
434	26/12/2000	-32	20.8	1800	30	40	-0.15	17:53(-0.07)	18:4(-0.51)	18:12(-0.04)	V		
435	26/12/2000	-32	20.8	1800	30	40	-0.15	17:52(-0.00)	18:4(-0.52)	18:12(-0.05)		V	
688	15/9/2004	36.6	59	2100	25	50	-0.15	14:18(-0.03)	14:27(-0.36)	14:32(-0.11)		V	
586	2/4/2003	30.2	35.5	1680	30	60	-0.14	16:11(0.62)	16:11(0.62)	16:11(0.62)		V	V
263	5/5/1989	30.3	-97	183	20	30	-0.14	1:23(0.97)	1:23(0.97)	1:23(0.97)			V
212	23/11/1984	34	-81	61	15	30	-0.13	22:25(1.71)	22:25(1.71)	22:25(1.71)			V
264	5/5/1989	42.7	-84.8	259	15	30	-0.13	1:3(0.58)	1:3(0.58)	1:3(0.58)			V
266	5/5/1989	43	-85.7	244	15	30	-0.12	1:7(0.57)	1:7(0.57)	1:7(0.57)			V
615	26/9/2003	41.8	-111.8	1460	20	50	-0.11	1:32(0.00)	1:32(0.00)	1:32(0.00)		V	
319	7/5/1997	31.8	34.9	79	20	34	-0.11	16:35(1.10)	16:35(1.10)	16:35(1.10)	V		
135	15/3/1972	35.5	-117.6	1128	-7	70	-0.11	2:14(0.14)	2:14(0.14)	2:14(0.14)		V	

Table 4.3.1 (continued)													
O.	Date	Latit.	Long.	Elev.	Tem	Hum	Q	Instant (Magnitude Contrast)			Visibility		
No.		Deg	Deg	Meter	C	%	val	Start	Best	End	N	B	T
416	31/7/2000	6.5	3.4	35	25	30	-0.1	18:17(1.06)	18:17(1.06)	18:17(1.06)	V		
412	2/7/2000	2.3	102.4	37	35	40	-0.09	11:31(2.34)	11:31(2.34)	11:31(2.34)		V	V
324	4/8/1997	31.3	34.2	600	20	30	-0.09	16:42(0.23)	16:42(0.23)	16:42(0.23)			
248	26/6/1987	42.7	-84.5	244	20	50	-0.08	1:38(1.06)	1:38(1.06)	1:38(1.06)		V	
304	31/1/1995	35.6	51.3	110	10	40	-0.07	14:16(0.73)	14:16(0.73)	14:16(0.73)		V	
691	15/9/2004	33.3	50.1	2600	30	60	-0.07	14:51(-0.01)	15:3(-0.64)	15:11(-0.03)		V	
415	2/7/2000	32.6	51.7	1500	35	40	-0.06	16:1(0.08)	16:1(0.08)	16:1(0.08)		V	
727	12/12/2004	32.4	-110.7	842	5	30	-0.06	0:26(-0.02)	0:35(-0.34)	0:41(-0.06)	V	V	V
337	26/5/1998	31.8	34.2	760	30	40	-0.06	16:48(0.56)	16:48(0.56)	16:48(0.56)			V
249	26/6/1987	37.2	-84.1	305	20	60	-0.05	1:16(1.21)	1:16(1.21)	1:16(1.21)			V
639	22/1/2004	32.5	3.7	550	10	30	-0.05	17:18(-0.00)	17:26(-0.33)	17:33(-0.02)	V	V	
282	24/5/1990	31.6	-110.5	1372	30	60	-0.05	2:37(0.38)	2:37(0.38)	2:37(0.38)			V
316	8/2/1997	-34	18.4	350	25	35	-0.05	17:52(0.87)	17:52(0.87)	17:52(0.87)	V		
283	24/5/1990	32.4	-111	842	25	50	-0.04	2:41(0.11)	2:41(0.11)	2:41(0.11)		V	V
484	17/10/2001	2.3	102.4	37	30	50	-0.04	11:8(2.07)	11:8(2.07)	11:8(2.07)		V	V
185	5/11/1983	37.2	-84.1	305	15	30	-0.04	22:51(0.14)	22:51(0.14)	22:51(0.14)			V
267	5/5/1989	39.7	-105.5	3353	30	70	-0.04	2:7(-0.06)	2:27(-1.24)	2:41(-0.09)			V
593	2/5/2003	5	114.9	63	35	40	-0.03	10:38(1.58)	10:38(1.58)	10:38(1.58)			V
661	19/5/2004	32.4	-111	842	25	60	-0.03	2:37(0.35)	2:37(0.35)	2:37(0.35)			V
291	16/3/1991	32.4	-111	842	25	60	-0.03	1:48(0.69)	1:48(0.69)	1:48(0.69)		V	V
668	18/6/2004	32	35.9	940	35	40	-0.03	17:5(0.15)	17:5(0.15)	17:5(0.15)			V
315	13/10/1996	31.8	34.9	79	20	40	-0.03	15:23(0.88)	15:23(0.88)	15:23(0.88)	V		
290	15/2/1991	33.4	73.1	500	10	40	-0.02	13:1(-0.05)	13:11(-0.54)	13:19(-0.08)	V		
485	17/10/2001	32.6	51.7	1500	25	70	-0.02	14:15(0.12)	14:15(0.12)	14:15(0.12)		V	
334	27/2/1998	-34	18.4	350	20	30	-0.01	17:31(-0.01)	17:39(-0.33)	17:45(-0.06)	V		
284	24/5/1990	34.2	-118.1	530	25	50	-0.01	3:13(0.35)	3:13(0.35)	3:13(0.35)			V
88	28/11/1913	-34	18.5	91	20	35	-0	17:53(-0.05)	18:2(-0.31)	18:8(-0.06)	V		
251	26/6/1987	40.7	-111.9	1311	30	60	0	3:21(-0.01)	3:26(-0.09)	3:30(-0.03)		V	
162	9/3/1978	44.1	-84.2	9	-3	30	0	22:30(-0.01)	22:36(-0.12)	22:40(-0.01)	V		
286	20/9/1990	31.8	34.7	60	10	30	0.01	15:45(1.43)	15:45(1.43)	15:45(1.43)	V		
715	13/11/2004	4.9	114.8	30	25	40	0.01	10:15(0.88)	10:15(0.88)	10:15(0.88)		V	
594	2/5/2003	3.2	101.7	60	35	50	0.01	11:28(1.93)	11:28(1.93)	11:28(1.93)			V
633	24/12/2003	49.6	8.7	175	-3	20	0.01	15:43(1.20)	15:43(1.20)	15:43(1.20)	V	V	
486	17/10/2001	29.6	52.5	1500	30	70	0.01	14:13(0.43)	14:13(0.43)	14:13(0.43)		V	
256	19/1/1988	32.2	-111	780	15	60	0.02	1:2(-0.00)	1:3(-0.01)	1:4(-0.00)			V
376	10/9/1999	38.8	-77	36	25	50	0.02	23:36(1.42)	23:36(1.42)	23:36(1.42)		V	
332	28/1/1998	29.8	-94.4	14	10	50	0.02	0:13(0.53)	0:13(0.53)	0:13(0.53)			
404	5/4/2000	4.3	102.9	20	30	40	0.02	11:30(0.72)	11:30(0.72)	11:30(0.72)		V	
437	25/1/2001	29.6	52.5	1500	25	70	0.02	14:13(-0.03)	14:22(-0.36)	14:28(-0.06)		V	
174	28/1/1979	42	-91.7	244	21	40	0.03	23:37(0.13)	23:37(0.13)	23:37(0.13)		V	
320	7/5/1997	-34	18.4	350	10	30	0.03	16:4(-0.04)	16:22(-1.21)	16:33(-0.12)	V		
173	28/1/1979	29.9	-81.3	0	15	25	0.03	23:11(-0.01)	23:18(-0.26)	23:24(-0.03)			
543	9/8/2002	2.3	102.4	37	30	50	0.03	11:34(1.42)	11:34(1.42)	11:34(1.42)			V
525	13/4/2002	30.5	-9.7	75	20	50	0.03	19:20(0.41)	19:20(0.41)	19:20(0.41)		V	
314	21/1/1996	-34	18.4	350	25	30	0.03	18:6(0.36)	18:6(0.36)	18:6(0.36)	V		

Table 4.3.1 (continued)													
O.	Date	Latit.	Long.	Elev.	Tem	Hum	Q	Instant (Magnitude Contrast)			Visibility		
No.		Deg	Deg	Meter	C	%	val	Start	Best	End	N	B	T
175	28/1/1979	29.7	-82.4	0	21	70	0.03	23:15(2.58)	23:15(2.58)	23:15(2.58)		V	
176	28/1/1979	42	-93.6	274	21	35	0.03	23:39(-0.02)	23:47(-0.22)	23:53(-0.00)	V		
694	15/9/2004	24.4	50.5	13	30	40	0.04	14:54(1.04)	14:54(1.04)	14:54(1.04)		V	
272	1/10/1989	31.3	34.6	175	0	25	0.04	15:34(0.50)	15:34(0.50)	15:34(0.50)	V		
177	28/1/1979	38.7	-90.3	183	10	50	0.04	23:39(0.10)	23:39(0.10)	23:39(0.10)		V	
487	17/10/2001	31.9	35.8	939	27	60	0.04	15:18(0.14)	15:18(0.14)	15:18(0.14)			V
616	26/9/2003	32.4	-111	842	15	40	0.04	1:19(-0.11)	1:34(-1.17)	1:45(-0.06)	V	V	V
253	26/6/1987	33.5	-112.1	329	-7	55	0.04	2:50(-0.04)	3:5(-0.76)	3:15(-0.13)	V		
621	26/10/2003	32.9	59.2	1468	25	70	0.04	13:32(0.22)	13:32(0.22)	13:32(0.22)		V	
413	2/7/2000	30.4	35.5	1646	35	70	0.05	16:59(0.84)	16:59(0.84)	16:59(0.84)			V
218	12/12/1985	-32	20.8	1800	30	40	0.05	17:39(-0.01)	18:7(-2.08)	18:22(-0.12)	V		
254	26/6/1987	37	-122	1494	15	40	0.05	3:30(-0.09)	4:2(-2.52)	4:20(-0.29)	V		
163	9/3/1978	42.7	-73.8	183	17	40	0.06	23:8(-0.04)	23:16(-0.25)	23:22(-0.02)	V		
405	5/4/2000	32.6	51.7	1500	25	80	0.06	15:12(0.17)	15:12(0.17)	15:12(0.17)		V	
622	26/10/2003	33.3	50.1	2600	25	60	0.07	13:53(-0.03)	14:15(-1.86)	14:28(-0.25)	V	V	
417	28/9/2000	-34	18.4	200	20	40	0.07	16:54(-0.03)	17:6(-0.69)	17:15(-0.06)	V		
142	18/2/1977	43.8	-87.7	30	5	30	0.07	23:34(-0.02)	23:49(-0.75)	23:59(-0.19)	V	V	
669	18/6/2004	32.5	3.7	550	30	60	0.08	19:15(0.52)	19:15(0.52)	19:15(0.52)		V	
306	28/6/1995	-30	-71	2774	25	60	0.08	21:54(-0.25)	22:24(-2.95)	22:40(-0.11)	V		
347	18/3/1999	36	50.8	1500	15	40	0.08	14:46(-0.10)	15:11(-2.40)	15:26(-0.33)	V		
695	15/9/2004	32.5	3.7	550	20	60	0.08	18:5(0.15)	18:5(0.15)	18:5(0.15)		V	
623	26/10/2003	32.6	51.7	1500	25	75	0.08	14:3(0.29)	14:3(0.29)	14:3(0.29)		V	
607	28/8/2003	4.3	102.9	20	30	50	0.08	11:30(0.98)	11:30(0.98)	11:30(0.98)		V	V
96	8/2/1921	42.3	-71.1	30	15	35	0.08	22:19(-0.02)	22:32(-0.53)	22:41(-0.01)	V		
489	17/10/2001	24.6	46.5	620	28	60	0.08	14:41(0.37)	14:41(0.37)	14:41(0.37)			
302	1/1/1995	19.8	-155.5	4180	10	70	0.09	3:53(-0.57)	4:25(-4.16)	4:40(-0.75)	V		
396	6/2/2000	-34	18.4	350	25	40	0.09	17:52(-0.02)	18:7(-0.83)	18:17(-0.00)	V	V	
471	21/7/2001	4.1	73.3	1	25	30	0.09	13:31(-0.02)	13:40(-0.52)	13:47(-0.09)			
89	16/3/1915	49.4	8.7	213	18	45	0.09	17:44(-0.04)	17:56(-0.42)	18:5(-0.02)	V		
348	18/3/1999	-34	18.4	350	20	40	0.1	17:4(-0.07)	17:20(-1.16)	17:31(-0.10)	V	V	
512	14/1/2002	24.6	46.5	620	20	70	0.1	14:43(0.25)	14:43(0.25)	14:43(0.25)		V	
438	25/1/2001	24.6	46.5	620	20	70	0.1	14:53(0.16)	14:53(0.16)	14:53(0.16)		V	V
2	27/10/1859	38	23.7	122	10	50	0.11	15:34(2.23)	15:34(2.23)	15:34(2.23)	V		
419	28/9/2000	24.4	32.7	105	20	40	0.11	15:47(-0.03)	15:57(-0.55)	16:4(-0.10)	V		
323	5/7/1997	-34	18.5	10	10	50	0.11	15:58(-0.03)	16:14(-0.80)	16:23(-0.11)	V		
608	28/8/2003	32.6	51.7	1500	35	70	0.11	15:21(0.42)	15:21(0.42)	15:21(0.42)		V	
178	28/1/1979	47.6	-122.3	30	15	40	0.12	1:20(-0.05)	1:31(-0.36)	1:40(-0.00)	V	V	
574	3/1/2003	-34	18.4	200	30	40	0.12	18:10(-0.03)	18:24(-0.59)	18:33(-0.02)	V		
544	9/8/2002	-33	18.4	750	20	40	0.12	16:15(-0.04)	16:38(-1.84)	16:51(-0.18)	V		
333	28/1/1998	34.2	-118.1	1740	10	70	0.12	1:19(-0.05)	1:47(-2.34)	2:1(-0.19)	V		
503	16/11/2001	49.6	8.7	175	0	30	0.12	15:53(-0.01)	16:3(-0.22)	16:9(-0.05)	V	V	
93	19/4/1920	43.5	7	15	10	40	0.13	18:27(-0.01)	18:44(-0.87)	18:56(-0.10)	V		
473	21/7/2001	31.9	35.8	939	25	50	0.13	16:44(-0.07)	17:3(-1.21)	17:14(-0.18)	V		V
456	24/4/2001	32.6	51.7	1500	15	60	0.13	15:7(-0.01)	15:34(-2.39)	15:49(-0.05)	V	V	
350	18/3/1999	29.6	52.5	1500	20	40	0.13	14:39(-0.12)	15:5(-2.69)	15:19(-0.37)	V	V	

Table 4.3.1 (continued)														
O.	Date	Latit.	Long.	Elev.	Tem	Hum	Q	Instant (Magnitude Contrast)			Visibility		
No.		Deg	Deg	Meter	C	%	val	Start	Best	End	N	B	T
79	7/12/1885	50.6	5.7	213	5	50	0.14	15:44(-0.02)	16:6(-0.83)	16:20(-0.04)	V		
167	9/3/1978	40.5	-89	244	18	40	0.14	0:3(-0.05)	0:20(-0.99)	0:31(-0.07)	V		
414	2/7/2000	-32	20.8	420	10	59	0.14	15:49(-0.10)	16:7(-1.19)	16:19(-0.05)	V	V	
420	28/9/2000	32.5	3.7	550	20	40	0.14	17:36(-0.04)	17:55(-1.50)	18:7(-0.13)	V	V	
397	6/2/2000	32.6	51.7	1500	20	60	0.14	14:10(-0.01)	14:36(-2.22)	14:50(-0.31)	V	V	
545	9/8/2002	32.6	51.7	1500	25	60	0.14	15:26(-0.10)	15:48(-1.74)	16:2(-0.03)	V	V	V
553	7/101/2002	49.6	8.7	175	5	25	0.15	16:58(-0.03)	17:8(-0.32)	17:15(-0.06)	V	V	
406	5/4/2000	32	35.9	850	30	70	0.15	16:14(0.37)	16:14(0.37)	16:14(0.37)			V
342	18/1/1999	28.6	43.7	500	10	50	0.15	14:35(-0.03)	14:56(-1.61)	15:9(-0.02)	V	V	
102	30/12/1921	-34	18.5	30	16	75	0.15	18:6(1.57)	18:6(1.57)	18:6(1.57)			
169	9/3/1978	41.6	-93.6	244	16	45	0.16	0:22(-0.07)	0:38(-0.98)	0:50(-0.08)	V		
85	31/1/1911	51	-0.9	61	0	40	0.16	17:4(-0.03)	17:17(-0.35)	17:25(-0.06)	v		
223	28/4/1987	26.7	-81	0	18	50	0.16	0:4(-0.04)	0:16(-0.65)	0:24(-0.10)	V	V	
398	6/2/2000	34.4	37.2	420	10	40	0.16	15:4(-0.06)	15:27(-1.76)	15:41(-0.11)	V	V	
398	6/2/2000	34.4	37.2	420	10	50	0.16	15:6(-0.08)	15:26(-1.40)	15:39(-0.04)	V	V	
222	28/4/1987	38.9	-77	30	20	50	0.16	0:8(-0.06)	0:22(-0.57)	0:30(-0.08)	V	V	
624	26/10/2003	29.4	48	30	25	50	0.16	14:21(0.71)	14:21(0.71)	14:21(0.71)		V	
609	28/8/2003	32.4	34.4	700	30	70	0.16	16:20(0.66)	16:20(0.66)	16:20(0.66)			V
595	2/5/2003	38.2	46	1400	25	40	0.17	15:47(-0.37)	16:17(-2.78)	16:35(-0.25)	V		
529	13/5/2002	5	114.9	63	35	50	0.17	10:42(0.64)	10:42(0.64)	10:42(0.64)		V	v
343	18/1/1999	26.1	44	640	25	75	0.17	14:54(0.57)	14:54(0.57)	14:54(0.57)		V	
561	5/11/2002	-34	18.4	200	30	50	0.17	17:30(-0.01)	17:38(-0.28)	17:45(-0.00)	V		
229	28/4/1987	28	-82.5	0	18	75	0.17	0:17(1.30)	0:17(1.30)	0:17(1.30)			
122	8/12/1942	40.7	-74	0	18	35	0.17	21:38(-0.05)	21:56(-0.84)	22:7(-0.04)	V	V	
122	8/12/1942	40.7	-74	0	18	35	0.17	21:38(-0.05)	21:56(-0.84)	22:7(-0.04)	V	V	
439	25/1/2001	32.5	3.7	550	10	40	0.17	17:12(-0.00)	17:38(-2.14)	17:52(-0.34)	V	V	
439	25/1/2001	32.5	3.7	550	10	60	0.17	17:16(-0.03)	17:36(-1.36)	17:48(-0.09)	V	V	
509	15/12/2001	32.4	-111	843	10	60	0.18	0:22(-0.05)	0:45(-1.65)	0:58(-0.21)	V		
407	5/4/2000	-34	18.4	200	15	40	0.18	16:38(-0.07)	16:58(-1.56)	17:10(-0.20)	V		
108	27/5/1922	-34	18.5	30	21	50	0.18	15:56(-0.00)	16:11(-0.73)	16:20(-0.12)	V		
490	17/10/2001	-34	18.4	200	25	50	0.18	17:8(-0.04)	17:22(-0.77)	17:31(-0.10)	V		
231	28/4/1987	34.4	-81.7	1402	25	70	0.19	0:15(-0.07)	0:38(-1.66)	0:53(-0.03)	V	V	
76	30/3/1881	51.5	-2.6	0	27	45	0.19	18:56(-0.01)	19:10(-0.43)	19:20(-0.00)	V		
640	22/1/2004	-34	18.4	200	30	40	0.2	18:3(-0.08)	18:22(-1.14)	18:34(-0.03)	V		
179	28/1/1979	37.8	-122.4	61	21	50	0.2	1:41(-0.02)	1:52(-0.45)	1:59(-0.17)	V	V	

4.4 A NEW CRITERION FOR NEW CRESCENT VISIBILITY

While developing the "visibility curves" (h against s) and the "limiting visibility curves" ($h + s$ against s) for constant brightness Bruin considered the average brightness of western horizon during twilight and the variation of the brightness of the full Moon with the altitude above horizon as mentioned earlier. Instead of considering average brightness of sky we have considered actual brightness of sky and the crescent calculated using the techniques developed by Schaefer and others (Schaefer, 1988b, 1993) in the software Hilal01. We selected crescent visibility circumstances of various new Moons, cases when the crescent was reported to have been seen. For crescents of a particular width we found the altitudes h of sky points with brightness equivalent to that of the particular crescent at different solar depressions s. The averages of the altitudes of sky points for different solar depressions for particular width are tabulated in Table no. 4.4.1. The left most column of the table contains the solar depressions s and the top row gives the widths w of the crescents selected and the next one gives its magnitude. The entries of the rest of the table are the altitudes h where the sky has the same brightness as the brightness of the crescent of the width at the top of the column.

It should be noted that during the twilight the width of the crescent varies up to 4 arc seconds for very wide crescents. Season to season and for different latitudes the sky brightness for the same altitude close to the point where the sun sets also varies. For each column of the table 4.4.1 a number of cases of almost same crescent width were considered and each altitude is average of these cases. The data of the table 4.4.1 is then plotted on a graph shown in figure 4.4.1. In this figure h as a function of s ($h = f(s)$) and $s + h = H$ as a function of s ($H = g(s)$) are both plotted. $f(s)$ represent the "visibility curves" and the $g(s)$ represents the "limiting visibility curves" similar to what Bruin (Bruin, 1977) developed.

Table No. 4.4.1

Width	15"	28"	62"	122"	170"
Magnitude	-5.1	-5.4	-6.1	-6.9	-7.25
s	*h*	*h*	*h*	*h*	*h*
0	18	15.5	13	8	6.5
1	13.5	11.5	9.2	5.5	4.5
2	9.5	8	6.4	3.8	3
3	7.1	5.9	4.2	2.6	2
4	5.6	4.3	2.9	1.8	1.3
5	4.3	3.2	2.1	1.3	1
6	3.55	2.5	1.7	1.1	0.8
7	3.05	2.1	1.5	0.9	0.65
8	2.87	1.9	1.3	0.8	0.55
9	2.7	1.75	1.1	0.7	0.45
10	2.55	1.6	0.95	0.6	0.35
11	2.4	1.45	0.8	0.5	0.3
12	2.3	1.3	0.7	0.4	0.25
13	2.2	1.2	0.6	0.3	0.2
14	2.1	1.1	0.5	0.2	0.15
15	2	1	0.4	0.15	0.1

Fig. No. 4.4.1

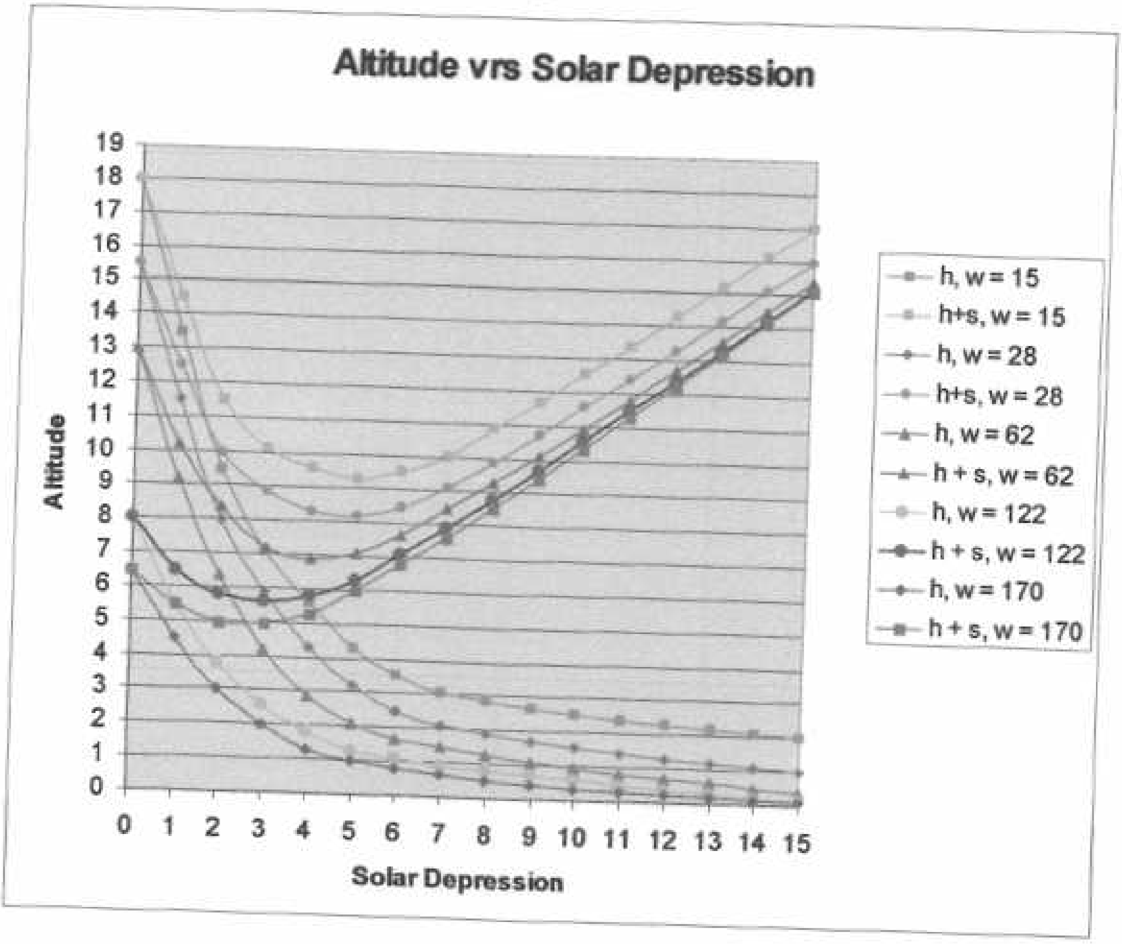
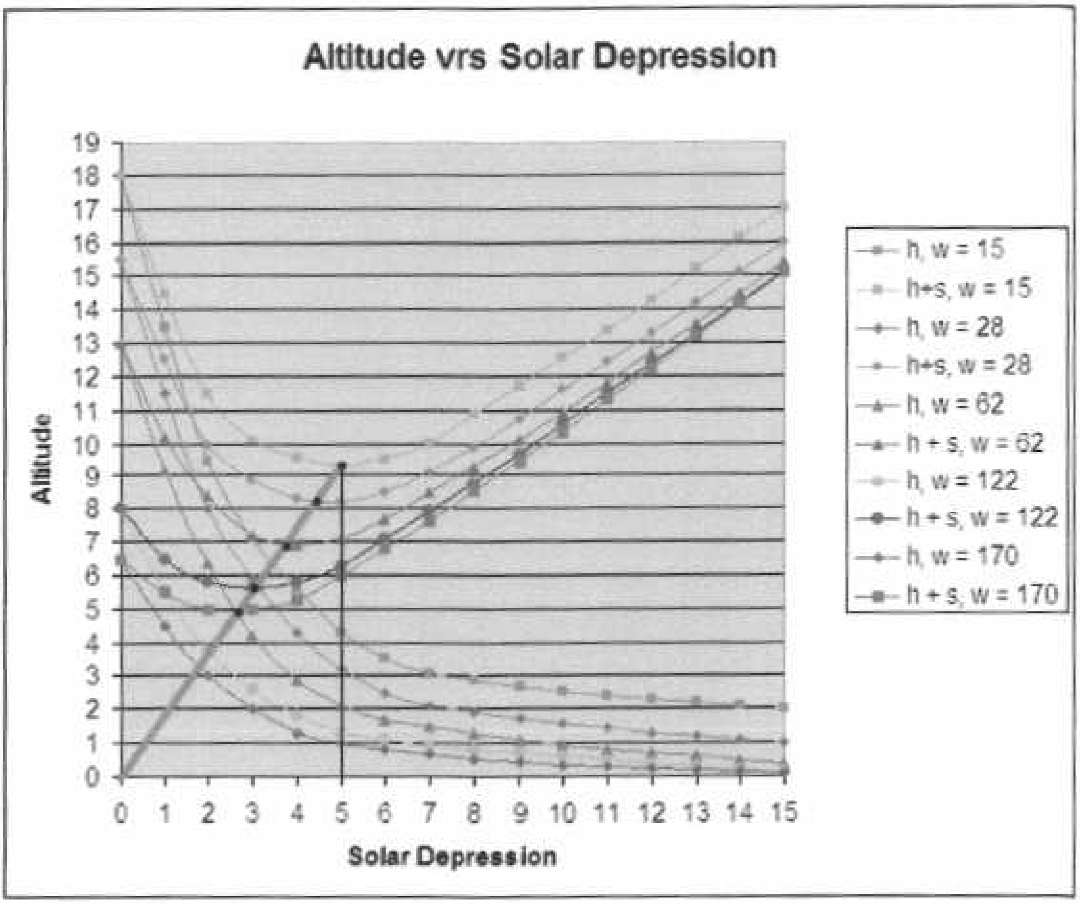


Fig. No. 4.4.2



The coordinates of the minima of *g(s)* (shown in figure 4.4.1) are then basic data for the model we have developed shown in the table 4.4.2:

Table No. 4.4.2

W	15	28	62	122	170
ARCV	9.3	8.2	6.9	5.6	4.9

Using cubic least square approximation we obtained the following relation between relative altitude of crescent ARCV and its width *W*:

$$ARCV = -0.3519637W^3 + 2.222075057W^2 - 5.42264313W + 10.4341759 \quad (4.4.1)$$

On the basis of this relation we define the visibility parameter v_p as follows:

$$v_p = (ARCV - (-0.3519637W^3 + 2.222075057W^2 - 5.42264313W + 10.4341759))/10 \tag{4.4.2}$$

Our model for earliest visibility of new lunar crescent is that if $v_p > 0$ (we call the visibility parameter v_p in (4.4.2) as the *s*-value) the crescent may be visible without optical aid otherwise not. Applying this condition on the data set used in chapter 3 and in this chapter we present the results obtained for whole data set of 463 cases in the last column of table in appendix-III. First few cases when the visibility is claimed without optical aid in order of increasing *s*-values are shown in table 4.4.3. Out of these cases only 11 cases deviate from our model. Out of these 11 cases 8 are consistent with the magnitude contrast 9 are consistent with Yallop's criterion and 8 are consistent with the Lunar Ripeness Law. The observation numbers 389, 455, 274, 341 and 316 that deviate from the Lunar Ripeness Law, the Yallop's criterion and the magnitude contrast are also negative in our model. However the observation number 416 that is negative in other models is allowed by our model. The reason is that in this case ARCV is reasonably high (9.41 degrees). The width is small (around 14 arc seconds) but the Moon is very close to perigee so closest to the Earth.

In figure 4.4.2 the minima of each visibility curve is joined resulting into a straight line which when extended intersect the origin of the (s, h) coordinate system. The slope of this line is found to be $((h + s)/s =)$ 9.3/5 or h/s = 4.3/5. This leads to a modified "best time" of crescent visibility as:

$$T_B = \frac{5T_S + 4.3T_M}{9.3}$$

or

$$T_B = \frac{5T_S + 4.3(T_S + LAG)}{9.3} = T_S + \frac{4.3}{9.3}LAG \tag{4.4.3}$$

Thus in our model the best time of crescent visibility is given by (4.4.3) that gives 1/(2.163)th part of LAG in comparison to 1/(2.25)th part of LAG in Yallop's criterion.

Table 4.4.3														
S. No.	Date	Lat	Longit	VISIBILITY			Age	Lag	ARCL	ARCV	DAZ	Width	Bruin	S-Value
		Deg	Deg	N	B	T	Hrs	Min	Deg	Deg	Deg	Min	V_P	V_F
389	7/1/2000	-34	18.4	V			24.1	35.1	11	7.19	-8.28	0.27	-0.292	-0.1933
455	25/3/2001	-34	18.4	V			15.8	36.9	9.06	8.3	-3.63	0.19	-0.245	-0.1187
274	25/2/1990	35.6	-83.5	V		V	14.8	39.3	8.53	8.51	-0.55	0.18	-0.232	-0.1022
341	18/1/1999	-34	18.4	V			26.5	37.4	13.3	7.79	-10.8	0.42	-0.13	-0.0742
433	26/12/2000	-34	18.4	V			24.9	43	11.3	8.49	-7.52	0.29	-0.149	-0.0559
434	26/12/2000	-32	20.8	V			24.7	42.9	11.2	8.62	-7.21	0.28	-0.14	-0.0451
286	20/9/1990	31.8	34.7	V			39.1	29.5	19.6	6.85	18.42	0.88	0.011	-0.0298
316	8/2/1997	-34	18.4	V			26.9	34.3	16.1	7.51	-14.3	0.66	-0.021	-0.0227
319	7/5/1997	31.8	34.9	V			19.9	40.9	11.6	8.74	7.68	0.33	-0.097	-0.0149
633	24/12/2003	49.6	8.7	V	V		30.2	53.8	18.1	7.21	16.62	0.82	0.022	-0.0088
727	12/12/2004	32.4	-110.7	V	V	V	23.2	44.2	14.4	8.13	11.93	0.53	-0.029	-0.0014
2	27/10/1859	38	23.7	V			39.2	33.6	21.4	6.8	20.34	1.08	0.076	0.00813
272	1/10/1989	31.3	34.6	V			41.9	32.1	19.5	7.34	18.1	0.84	0.047	0.01124
416	31/7/2000	6.5	3.4	V			15.9	37.7	9.58	9.41	1.8	0.23	-0.1	0.01145
314	21/1/1996	-34	18.4	V			29.4	35.2	17.9	7.51	-16.3	0.8	0.047	0.01771
639	22/1/2004	32.5	3.7	V	V		20.4	44.6	12.5	9.03	8.63	0.38	-0.029	0.03659
334	27/2/1998	-34	18.4	V			24.3	39.4	14.2	8.69	-11.3	0.51	0.018	0.04915
315	13/10/1996	31.8	34.9	V			25.2	41.6	12.7	9.27	8.73	0.38	-0.007	0.0593
290	15/2/1991	33.4	73.1	V			19.7	46.9	10.2	10.12	-0.94	0.24	-0.022	0.08687
285	24/5/1990	34.2	-118.1	V		V	15.6	52	10.2	10.14	0.81	0.26	-0.004	0.09806
88	28/11/1913	-34	18.5	V			16.4	53.6	10.3	10.25	-0.04	0.26	0.002	0.10625
162	9/3/1978	45.1	-64.2	V			20	54.6	10.7	10.16	3.46	0.28	0.01	0.10668
622	26/10/2003	33.3	50.1	V	V		25.4	43.3	14.9	9.19	11.75	0.56	0.096	0.11589
616	26/9/2003	32.4	-111	V	V	V	22.5	42	13.1	9.68	8.89	0.43	0.067	0.11939
503	16/11/2001	49.6	8.7	V	V		33.5	58.4	17.8	8.7	15.5	0.75	0.143	0.12299
553	7/10/2002	49.6	8.7	V	V		29.9	49.5	18.1	8.53	15.95	0.82	0.157	0.12492
320	7/5/1997	-34	18.4	V			19.6	49.1	11.5	10.18	-5.27	0.32	0.04	0.12507
173	28/1/1979	29.9	-81.3	V			17	48.1	10.4	10.42	0.09	0.28	0.035	0.13215
176	28/1/1979	42	-93.6	V			17.5	58.5	10.7	10.39	2.54	0.29	0.043	0.13543
253	26/6/1987	33.5	-112.1	V			21.6	56	11	10.6	2.97	0.27	0.049	0.14784
218	12/12/1985	-32	20.8	V			17.1	55.2	10.8	10.53	-2.29	0.29	0.057	0.14975
716	13/11/2004	36.8	-81.8	V	V		32.2	46.1	18.7	8.7	16.53	0.87	0.192	0.15247
417	28/9/2000	-34	18.4	V			21.3	47.2	12.5	10.26	-7.13	0.37	0.089	0.15638
418	28/9/2000	-34	18.4	V	V		21.3	47.2	12.5	10.26	-7.13	0.37	0.089	0.15638
396	6/2/2000	-34	18.4	V	V		29.1	47.5	14.1	9.99	-9.91	0.45	0.114	0.15937
163	9/3/1978	42.7	-73.8	V	V		20.7	55	11.1	10.62	3.18	0.3	0.069	0.16038
347	18/3/1999	36	50.8	V			20.3	49	12.1	10.38	6.26	0.37	0.094	0.1647
348	18/3/1999	-34	18.4	V	V		22.6	46.2	13.4	10.16	-8.67	0.44	0.124	0.17234
349	18/3/1999	-34	18.4	V			22.6	46.3	13.4	10.18	-8.66	0.44	0.126	0.17434
142	18/2/1977	43.8	-87.7	V	V		20.2	58	10.9	10.85	0.66	0.28	0.079	0.17567
306	28/6/1995	-30	-71	V			21.5	52.5	10.9	10.94	-0.24	0.27	0.081	0.18087
544	9/8/2002	-34	18.4	V			21.3	50.4	13.1	10.31	-8.08	0.43	0.128	0.18117
85	31/1/1911	51	-0.9	V			31.6	65.5	16.4	9.68	13.26	0.63	0.182	0.1861
96	8/2/1921	42.3	-71.1	V			21.9	57.9	11	10.98	0.13	0.27	0.088	0.18627

In general looking at the complete data set in table in appendix-III it can be noted that:

- There is no claim of visibility by any means when s-value < -0.193. In fact as observation number 389 is not consistent with any model we reject it and therefore we claim that there is no authentic observation (with or without optical aid) of new lunar crescent for s-value < -0.162. Thus crescent can not be seen whenever s-value < -0.16 even with a telescope.

- For -0.16 < s-value < -0.06 there are 26 (45%) claims of crescent visibility with optical aid out of 58 reported and considered observations. As unaided visibility claims 455, 274 and 341 are not consistent with any model so we conclude that for this range of s-values the crescent can be seen with optical aid only.

- In 49 cases with -0.06 < s-value < 0.05 there are 13 unaided (25.5%) and 21 sighting with optical aid (41%). We conclude that there are strong chances of sighting crescent with a binocular or a telescope and very slim chances for unaided sighting. Unaided sighting is not impossible.

- For 0.05 < s-value < 0.15, there are 35 sightings with optical aid (70%) and 14 without optical aid (28%). Thus the crescent may be easily seen with optical aid for this range of s-value and can be seen without optical aid under very good condition (weather conditions and height above sea level).

- For s-value > 0.15, out of next 213 observations the crescent was seen without optical aid 165 times (77.5%). Therefore we conclude that for s-value > 0.15 the crescent can be easily seen.

Thus our model that we call "Qureshi & Khan Criterion" can be summarized as follows:

1. Calculate s-value (or v_P according to 4.4.2) for the crescent at the best time as given by (4.4.3).
2. The visibility condition as given by our model are then given in Table 4.4.4.

Table 4.4.4	
Easily Visible (EV)	s-value > 0.15
Visible under perfect conditions (VUPC)	0.05 < s-value < 0.15
May require optical aid to find crescent (MROA)	-0.06 < s-value < 0.05
Require optical aid (ROA)	-0.16 < s-value < -0.06
Not visible with optical aid (I)	s-value < -0.16

The success of our model in terms of number of positive observations in agreement with the suggested criterion is achieved due to the fact that we have used Schaefer's brightness model, i.e. actual brightness of sky instead of the average brightness and actual brightness of crescent instead of the brightness of full Moon. The number of positive observations in agreement with a criterion is mostly interpreted as the success of the criterion.

Table No. 4.4.5

Criterion	Visibility claimed		Visibility not claimed		All Observations	
	Consistent	Not consistent	Consistent	Not consistent	Consistent	Not consistent
Babylonian	189(96.4%)	7(3.6%)	160(59.9%)	107(40.1%)	349(75.4%)	114(24.6%)
s-value	185(94.39%)	11(5.61%)	153(57.31%)	114(42.69%)	338(73%)	125(27%)
Bruin's	182(92.9%)	14(7.1%)	188(70.4%)	79(29.6%)	370(79.9%)	93(20.1%)
Ripeness	182(92.9%)	14(7.1%)	152(56.9%)	115(43.1%)	334(72.1%)	129(27.9%)
q-value	180(91.8%)	16(8.2%)	188(70.4%)	79(29.6%)	368(79.5%)	95(20.5%)
Indian	177(90.3%)	19(9.7%)	192(71.9%)	75(28.1%)	369(79.7%)	94(20.3%)
Maunder	149(76%)	47(24%)	231(86.5%)	36(13.5%)	380(82%)	83(18%)
Fotheringham	106(54.1%)	90(45.9%)	251(94.7%)	16(5.3%)	366(79%)	97(21%)

When a criterion allows optically unaided visibility of the new crescent and the crescent is not seen then it is a negative error. There can be a number of reasons for negative errors. Despite the fact that an observer may be experienced and trained astronomer and knows the location of the crescent the atmospheric conditions and the physiology of the observer's eyes may still lead to non-visibility of the crescent. These factors are still not well explored thus the high frequency of the negative errors shows that the problem is still not solved completely. A positive error occurs when a model does allow visibility of crescent and the visibility is not claimed. Smaller the number of positive errors and better is a visibility criterion.

The table 4.4.5 summarises the positive and negative observations in agreement or disagreement with different criterion from the data set we have chosen for this work. The table is arranged with decreasing success percentage in terms of visibility claims consistent with the criterion. Surprisingly the Babylonian criterion has the best success percentage followed by our *s*-value criterion. The table further shows that Bruin's criterion (4.1.2) and the Lunar Ripeness law are equally successful. These are followed by the q-value criterion of Yallop and the ARCV-DAZ-based Indian method (3.6.13). The criterion due to Maunder (3.6.10) and Fotheringham's criterion are the least successful of the methods shown in the table. It should be noted that less successful a model is in describing positive sighting stricter it is. Moreover, stricter a model it should be more consistent with negative observations (when the crescent is not seen).

The number of negative observations in agreement with the criterion is also considered as a test of a criterion by some authors (Fatoohi et al, 1999). However, it should be noted that the exploration of Schaefer shows that the brightness or magnitude contrast is highly dependent on the weather conditions. All the single parameter criteria considered in the table 4.4.5 do not consider weather conditions of individual observation. If any of these criterion allows visibility in some case it is still possible that the weather conditions are not favourable for visibility and the crescent is not actually seen.

Therefore, if Fotheringham's criterion is most successful in being consistent for negative observation this does not at all means that it is better or more dependable than Yallop's model. Both Maunders and Fotheringham's models are most consistent in cases of negative observations (when crescent is not seen) but are least successful for positive observation. This is true simply because these criteria are stricter as compared to other criteria. On the other hand Babylonian condition, *s*-value criterion and the Lunar Ripeness law are highly consistent with the positive observations but least consistent with the negative observations. These and other criteria (Bruin's limit and Yallop's *q*-value criterion) are more concerned with conditions under the visibility of new lunar crescent is possible and not with the conditions under which it is impossible.

In terms of overall consistency Maunders method appears to be best. However, Bruin's limit, Yallop's *q*-value criterion, Lunar Ripeness law and our *s*-value criterion are based on some theoretical considerations and the other methods are only empirical. The methods based on any theoretical consideration may be improved with better understanding of actual physical and physiological aspects of the problem.

4.5 DISCUSSION

In this chapter the problem of the first visibility of new lunar crescent is explored on the basis of physical models describing the brightness of crescent and that of the twilight sky. These models have accuracy of 20% which is exhibited in the success percentage of Bruin's limit, Yallop's *q*-value criterion and our *s*-value criterion with overall consistency of 79.9%, 79.5% and 75.4% (respectively) with the observations. All these models have high success percentage (90% and plus) for positive observations (when the crescent was reportedly seen). With more accurate models of brightness of crescent and twilight sky these methods can be improved further.

The physical models considered in this chapter can be divided into two classes. One that is based directly on the brightness models and includes only the algorithm due to Schaefer. The other class of models deduce visibility conditions on the basis of

visibility curves initially conceived by Bruin. This class includes Bruin's limit, Yallop's q-value criterion and the s-value criterion that is developed in this work.

We have used Schaefer's algorithm to explore the reported observations that do not come under the "Easily Visible" condition due to Yallop and when the crescent was reportedly seen. Amongst these observations cases that have unfavourable magnitude contrast are critically examined and some are rejected (especially observation numbers 389 and 455) as they are not consistent with any of the visibility criterion considered in this work. Both these reported cases are not considered reliable as even under highly exaggerated atmospheric temperature and relative humidity the magnitude contrast is found to be unfavourable for crescent visibility. We found at least one observation when the crescent was reportedly seen without any optical aid (obs. no. 274) and that is not consistent with any criterion but have a favourable magnitude contrast for relative humidity less than 50% and atmospheric temperature around 10 degree centigrade.

Apart from observation numbers 389 and 455 there are 11 other cases when the crescent was reportedly seen without optical aid but the magnitude contrast was not favourable. However, these cases of positive observations are consistent with at least one other criterion considered in this work. As the brightness model are still not perfect therefore these 11 observations can not be ruled out as unreliable.

The models that are deduced from the Bruin's visibility curves and limiting visibility curves, the Bruin's limit and Yallop's single parameter criterion, are found to be consistent with each other. Both have almost equivalent success percentage but Bruin's limit has a slight edge.

We have developed new visibility curves and limiting visibility curves using the brightness models due to Schaefer and others. On the basis of these new curves a new data set and a new single parameter criterion is deduced. The new limiting visibility curves have lead to a slightly modified "best time" of crescent visibility. Our new visibility criterion, the s-value criterion, is found to be more successful for positive

observations but less successful in comparison to the Bruin's limit and Yallop's criterion for negative observations.

In view of the fact that all the visibility criteria are aimed at exploring conditions under which the new lunar crescent may be seen the success of a model for positive observation is more important an issue as compared to its success for negative observations. None of the model is aimed at deducing conditions under which the visibility of new lunar crescent is impossible. Therefore, a models success for being consistent with the positive observation is the success of the model.

In view of the magnitude contrast model based on brightness of crescent and of twilight sky it has been seen that the visibility of new lunar crescent is greatly affected by (i) the elevation above sea level of the observation site, (ii) the atmospheric temperature and (iii) the relative humidity. Higher is the elevation more is the magnitude contrast in favour of visibility. Lower is the temperature or humidity the magnitude contrast is more favourable for crescent sighting. Even if semi-empirical criteria, like Yallop's q-value criterion or our s-value criterion, does not allows crescent visibility without any optical aid the magnitude contrast may be in favour of naked eye visibility of the new crescent. It may be due higher elevation of the site or very low temperature or humidity.

Thus in making decisions about the authenticity of any claim of visibility of new lunar crescent any semi-empirical or a simple empirical criterion alone may not prove to be sufficient. Such a criterion must be supplemented by an analysis of magnitude contrast before any authentication is done.

All the empirical and the semi-empirical models are deduced from some basic set of data that is deduced without taking the atmospheric conditions and elevation above sea level. This basic set of data may be based on an ARCV-DAZ relation (like Fotheringham's, Maunders or Schoch models) or ARCV-WIDTH relation (like Yallop's model or our model developed in this work) only. Although, atmospheric conditions can not be predicted accurately, the seasonal averages for temperature and relative humidity

may be considered for an advance prediction. In case of verifying a visibility claim the actual atmospheric conditions may be recorded.

Therefore, a possible strategy for verification may be to use estimated elevation, E_0, estimated temperature, T_0 and estimated relative humidity, H_0 and get the results for q-value or s-value. If it allows crescent visibility for the evening in question and the claim is made, the claim is accepted. If the semi-empirical criterion does not allows crescent visibility and crescent sighting is claimed, evaluate magnitude contrast M_0. If $M_0 > 0$ the program Hilal01 allows for variation in elevation, temperature and relative humidity so one can adjust for these quantities and verify whether a favourable magnitude contrast ($M < 0$) is obtained or not. If the new magnitude contrast is favourable accept the claim otherwise reject it.

On basis of the data generated by program Hilal01 (shown in figures on the next page) relations between magnitude contrast and the quantities on which it depends (E, T, and H) are obtained as follows:

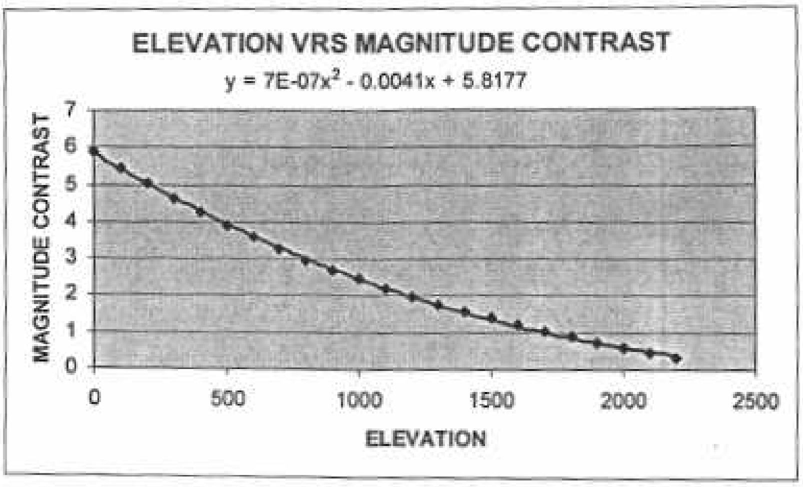
$$M = 5.8177 - 0.0041E + 0.0000007E^2$$

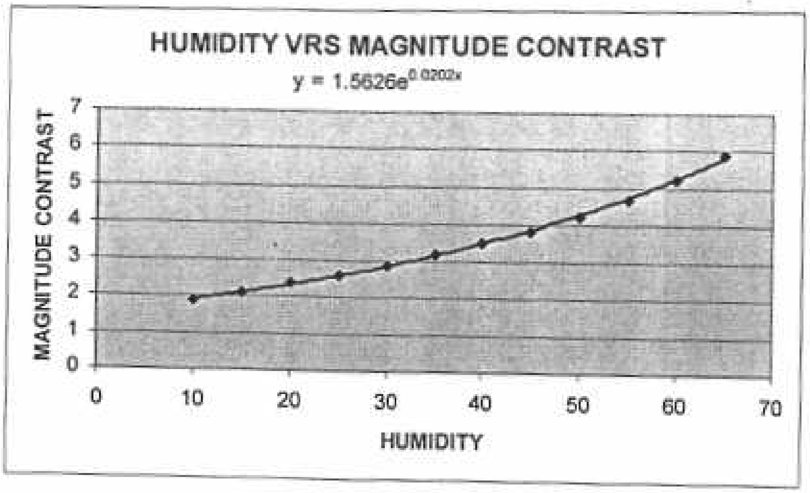
$$M = 1.5626\, e^{(0.0202H)}$$

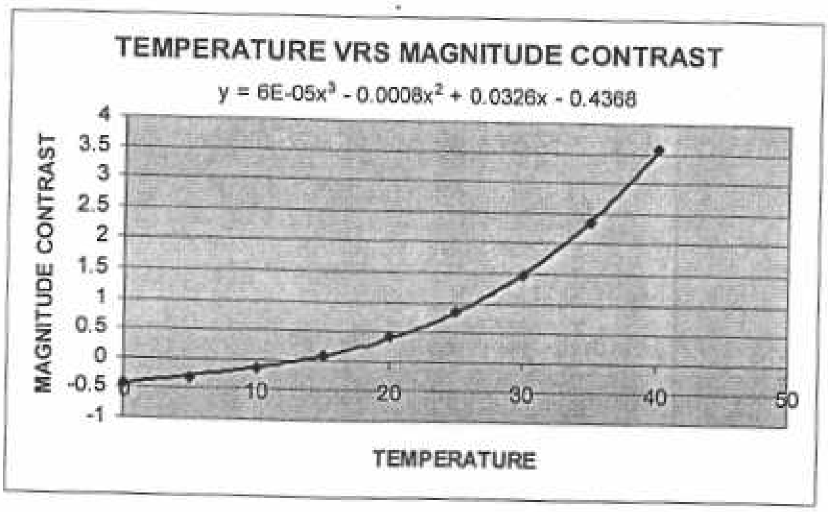
$$M = -0.4368 + 0.0326T - 0.0008T^2 + 0.00006T^3$$

Thus for meter increase in elevation M decreases by $0.00000014E - 0.0041$, for each percentage increase in relative humidity M varies by increases by $0.03156e^{(0.0202H)}$ and for each degree centigrade increase in temperature M increases by $0.0326 - 0.0008T + 0.000006T^2$. This may lead to the appropriate elevation, relative humidity and atmospheric temperature required for favourable magnitude contrast. An appropriate elevation may be the elevation of a hill top or building roof from where an observation may be made or may have been made. The appropriate temperature and relative humidity

may those that may be the average for the season or the values that were actually recorded at the time of observation.







Chapter No. 5

APPLICATIONS

During this work it is observed that since the Babylonian era till recently a number of prediction criteria, mathematical as well as observational, were developed to determine when the new lunar crescent would be first seen for a given location. As the first appearance of new lunar crescent marks the beginning of a new month in observational lunar calendars these criteria and models are significant for calendarical purposes. Whether an actual observational lunar calendar, like the Islamic Lunar calendar, utilizes these criteria for arranging its calendar or not these criteria provides a guidance for both testing an evidence of crescent sighting by common people and tracing down the dates of a calendar in history where appropriate dates are not well recorded. Thus the main utility of the prediction criteria for the earliest visibility of new crescent is to regulate the observational lunar calendar.

Although first order approximations, like Arithmetic Lunar Calendar that are based on the concept of Leap Years and the average motion of the Moon have been in use, Muslims have been following actual sighting of crescents at least for the months of fasting (Ramazan) and pilgrimage (Zil hajjah). The actual motion of the Moon varies greatly due to various factors which cause the observational calendar to be different from the arithmetic calendar. The Calendars if based on a prediction criterion like that of Yallop or the one developed in this work are the closest to the observational calendar. In this chapter, we compare these calendars with the actual observational calendar in practice in Pakistan for the years 2000 to 2007. It is found that on average 93.7% observations are according to the Yallop's *q*-value

criterion or our s-value (or Q&K) criterion. The disagreement is the result of either the bad weather due to which the new lunar crescent could not be sighted and the Lunar month began one day late, or too optimistic claims of observation and the Lunar month began one day earlier than predicted.

Further, in this work another application of these models is considered. This is the use of these models to develop a computational tool to determine the length of crescent from cusp to cusp. The new lunar crescent as well as crescent on next few evenings is observed to be shorter than its theoretical length i.e. 180 degree from one cusp to the other. A number of authors have described the reasons for the shortening of the observed crescent (Danjon, 1932, Schaefer, 1991, McNally, 1983). However, few have attempted devising a mathematical technique to determine the extent of this shortening of length of crescent. On the basis of one parameter criteria we have used crescent of minimum visible width as limit on the length of crescent and devised a simple technique to calculate it. The chapter begins with a description of the same.

5.1 LENGTH OF LUNAR CRESCENT

The fact, that the new lunar crescent appears shorter than 180^0 in length, is known for centuries. It was Danjon who first gave an explanation for the phenomenon (Danjon, 1932 & 1936) and attributed it to the lunar terrain close to the cusps. McNally attributed a different reason with this phenomenon discarding Danjon's hypothesis (McNally, 1983). McNally proved that the length of the shadows close to the lunar cusp and the departure of lunar surface from being perfect sphere couldn't diminish the brightness of the regions of crescent close to cusps to be the cause of the phenomenon. He attributed the length shortening of crescent to the "seeing affect" due to the turbulence of the atmosphere. McNally has also developed a formula for calculating the length of the crescent. Lately Sultan (Sultan, 2005) has attributed the shortening of length of crescent due to the Blackwell Contrast Threshold (Blackwell, 1946) and has developed a formula to calculate crescent's length. We have also developed a simple technique for calculating

length of new lunar crescent (Qureshi and Khan, 2007). In the following we reproduce this effort with slight modification and a correction.

Schaeffer rejected McNally's explanation on the basis of his view that the shortening of the crescent length is simply because of the sharp decline of the brightness of the crescent close to the cusps (Schaeffer, 1991). Using the accurate model of Hapke (Hapke, 1984) for calculating the surface brightness of the crescent Schaeffer claims that Danjon's collected observations and his own new data fits the model. However, neither Danjon nor Schaefer have suggested a method for calculating crescent length. Hapke's model may be accurate for theoretical setting related to the elongation of the Moon but as far as the observed crescents are concerned there ought to be a departure from smooth relation between elongation and the crescent length. The reason we consider is based on observations of some morning and evening crescents.

Most of the early description of the phenomena concentrated on relating it to the phase (or elongation) of the Moon that is generally the reason behind the phenomenon. As the elongation increases the length of the crescent from cusp to cusp keeps increasing which a common observation. The mathematical description of the phenomenon in terms of deficiency arc, by Danjon was shown to be incorrect by McNally. However, an over estimated limit on the minimum width of visible crescent (2 to 6 arc seconds) by McNally lead to very small values for Danjon Limit. The description due to McNally is logically sound so is that of Sultan and both resulted into techniques for calculating length of crescent. However both McNally and Sultan have not reported the application of their description on the recorded historical data found in literature (Yallop, 1998, Schaeffer 1991 etc.). According to Danjon, as described by Fatoohi et. al.

$$Sin(\alpha) = Sin(a).Cos(\omega) \qquad (5.1.1)$$

Where the arc PQ = α is the deficiency arc in Fig. 1 (Fatoohi et. Al.. 1998), a is the elongation and ω is half the crescent length. It appears that Danjon used the Sine

formulae by assuming spherical angle at Q to be right angle. McNally rejected Danjon's argument and using four-part formula arrives at:

$$Tan(\alpha) = Tan(a).Cos(\omega) \tag{5.1.2}$$

Numerically, for small angles α and a there is only a marginal difference between the two results. Generally the elongation (a) can be calculated and the crescent length (2ω) is observed, so the two formulas can be used to find the deficiency arc. However, to calculate the crescent length none of these can be used. McNally develops a formula for angular separation φ from a cusp in terms of crescent width R as:

$$Cos\varphi = \frac{1}{SinE}\sqrt{1 - \frac{Cos^2 E}{\left\{1 - \frac{\Delta R}{R}\right\}^2}} \tag{5.1.3}$$

where ΔR is the minimum visible width measured in radial direction away from the centre of the lunar disc. Therefore the length of the crescent he obtains is 180^0 - 2φ.

Using the Blackwell threshold contrast Sultan (Sultan, 2005) arrives at the minimum visible width (in terms of diameter of the smallest equivalent Blackwell (Blackwell, 1946) disc) of crescent at perigee and apogee at his location of observation. The formula that he developed for calculating the crescent length is:

$$l = \left(\frac{L}{2r}\right).180^0 \tag{5.1.4}$$

where r is the semi-diameter of the Moon, and

$$L = 2r - w\left(\frac{2r + W}{W}\right) \qquad (5.1.5)$$

with w is the diameter of the smallest visible equivalent Blackwell disc and W is central width of the crescent. (5.1.5) is the corrected form of (1.5) in Qureshi & Khan (Qureshi & Khan, 2007). Sultan considers minimum diameter of Blackwell disc to be 0.14 arc minutes when the Moon is near perigee and 0.16 arc minutes when the Moon is near apogee.

The present work is based on the observation of the last (old) crescent on February 26, 2006. During this observation that started from the beginning of morning twilight till well past sunrise, it was noticed that for this –48 hours Moon, more than 27 degrees away from the sun the crescent length started to decrease with the rising sun. A similar observation on March 28th when the age of Moon was around –33 hours and around 18.5 degrees away from the sun, the crescent length decreased more rapidly with decreasing contrast. The last time this crescent was seen without optical aid well after sunrise was less than 90 degrees in length. Two days later new crescent with age 27.6 hours at 15 degrees away from the sun was observed till setting. Close to the horizon through thick humid atmosphere the crescent length was again observed to be decreasing. These observations clearly demonstrate that there is much more to be explored about the phenomenon of shortening of crescent length apart from Hapke's accurate model that shows dependence of crescent length on elongation alone.

Therefore, in this work, to begin with we consider a simple geometrical model for the crescent. The model describes the phenomenon of shortening of length that depends on the actual semi-diameter of the Moon as well as the relative altitude (ARCV) of the crescent over local sky. This is derived from the single parameter (q-value or s-value) criterion of earliest visibility of crescent (chapter 4) and is based on the fact that whenever the width (or brightness) of crescent close to cusp is below the minimum visible central width (or brightness) of the crescent the whole length of crescent with

smaller width would not be visible. Applying our model on the recorded visibility and invisibility data available the length of crescent in each case is calculated. The calculated lengths of crescent are also compared with the observed lengths and with the lengths calculated using formulas (5.1.3) due to McNally and (5.1.4 & 5.1.5) due to Sultan. In case of using McNally's formula ΔR is considered from the criterion used in this work and for Sultan's formula the minimum width of Blackwell disc (w) is considered to be in the range 0.14 arc minute to 0.16 arc minute and depends on the distance between Earth and the Moon obtained using simple linear interpolation.

In view of Qureshi & Khan (Qureshi & Khan, 2007) the brightness of crescent at displacement ψ from a cusp is given by

$$B_{\psi} = rF_m Cos\psi(1-CosE)\frac{a}{R^2} \tag{5.1.6}$$

where r is the radius of Moon, F_m is the maximum flux of sunlight received at the surface of Moon, E is the separation between the Sun and the Moon in our sky, a is the albedo of the lunar surface and R is the distance of the Moon from the Earth. Similarly the width of the crescent at angular separation ψ from the cusp is given by

$$W_{\psi} = rCos\psi(1-CosE) = W_c Cos\psi \tag{5.1.7}$$

where W_c is the width of the crescent in the middle. In both these equations ψ varies from 0^0 to 90^0 along the length of the crescent from centre to a cusp respectively.

For the development of any model that describes the minimum possible width that can be visible through naked eye one requires to seek guidance from the actual observations. In the history of scientifically reported observation of the very young crescent Moon, the record is that due to Pierce on February 25, 1990 (reported by Schaefer and Yallop). The crescent he claims to have seen with naked eye was just 14.8 hours and its width was 0.18 arc minute. Amongst all the recorded observations the

sighting of such a young and thin crescent was never reported. In the model that is developed in this work the lower limit of the width of visible crescent is considered to be 0.18 arc minute. However this minimum is not the absolute minimum for all crescents for all possible relative altitudes (ARCV). In this work we consider this minimum of 0.18 arc-minutes of crescent width when the relative azimuth, DAZ of the Moon is zero and the relative altitude ARCV gives the q-value of -0.22. Yallop's criterion (4.2.3) for this *q*-value then yields:

$$ARCV = 9{\cdot}6371 - 6{\cdot}3226\, W + 0{\cdot}7319\, W^2 - 0{\cdot}1018\, W^3 \qquad (5.1.8)$$

The values of ARCV according to this criterion giving the q-value equal to -0.22 would yield a different lower limit on the visible width of the crescent. This is caused by different relative azimuths DAZ. For the least possible ARCV (zero) the width of the invisible crescent would be around 108 arc-seconds that occur at a large value of DAZ. In view of this criterion the minimum width of visible crescent for any ARCV is termed W_m and as the crescent is just invisible for this width:

$$W_c Cos\psi - W_m = 0 \qquad (5.1.9)$$

Where W_c is the theoretical central width of the crescent and $W_c Cos\psi$ is the reduced width at angle ψ from the center of the crescent. W_m is the width reduced by the relative altitude ARCV. In order that some part of crescent is visible $W_c Cos\psi > W_m$ at some value $\psi = \psi_m$. Thus the effective visible width of the crescent for ψ ranging from 0^0 to ψ_m is given by:

$$W_{eff} = W_c Cos\psi - W_m \qquad (5.1.10)$$

The brightness of crescent falls sharply as θ approaches $90^0 - E$ the actual visible width of crescent at any value of ψ must be less than the geometric value of width given by equation (5.1.8). Therefore the "effective visible width" given by (5.1.10) is justified.

Thus it is not only the length of crescent that shortens but the visible width of crescent has to diminish also. Whenever the crescent is invisible in view of (5.1.9) W_{eff} has to vanish so that:

$$Cos\psi = \frac{W_m}{W_c} = 1 \tag{5.1.11}$$

In all other cases, i.e. whenever the crescent is visible its width at some angle ψ_m must vanish as the crescent is never seen a complete 180^0 in length. Thus at $\psi = \psi_m$

$$W_c Cos\psi_m - W_m = 0 \tag{5.1.12}$$

Therefore in (5.1.12) ψ_m is a measure of half the length of the crescent so that the total length of the crescent is given by:

$$l = 2Cos^{-1}\left(\frac{W_m}{W_c}\right) \tag{5.1.13}$$

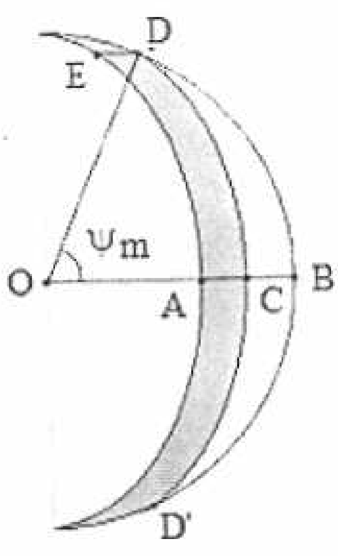


Fig. 5.1.1

Thus the crescent length can be evaluated whenever the theoretical width W_c exceeds the minimum width W_m visible according to Yallop's q-value criterion or our s-value criterion, for the particular values of ARCV. In the Fig. 5.1.1 the segment ED or AC is the minimum width W_m at any ARCV invisible according to Yallop's criterion. The segment AB is the theoretical width W_c at the centre of the crescent. At angular separation ψ_m from the centre of crescent ED equals $W_c Cos\psi_m$. Therefore, the points on the outer limb of the crescent that has angular separation from centre greater than $\psi_m = \angle D\hat{O}B$ should not be visible. The visible crescent then extends from D to D' and has length $2\psi_m$. One should note that whenever W_m (minimum visible width according to Yallop's criterion) is greater than W_c (theoretical width) (5.1.13) can not be used and the crescent is not visible, i.e. it has no length.

The model developed in this work to compute the length of the crescent has been applied to a number of observations reported in literature (Schaeffer, 1984, Yallop, 1998). The results for the crescent length against the elongation, also known as "arc of light" or ARCL, are presented in table 5.1.1 and in Fig. 5.1.2. The lengths are calculated by selecting the minimum visible width ψ_m of crescent for the value of ARCV according to (5.1.8) in the every case and then using (5.1.13).

The columns of the table 5.1.1 show the date of observation, the coordinates of the location of observer (Latitude and longitude respectively), the relative azimuth (DAZ), the relative altitude (ARCV) and the elongation (ARCL) of the Moon at the best time of observation, followed by semi-diameter and the central width of crescent in arc minutes, the q-value, the minimum crescent width visible at the relative altitude calculated from (5.1.8) and finally the crescent length calculated using (5.1.13). The table is arranged in chronological order of observations and contains only part of the complete data set that has been used for comparing models for earliest crescent sighting in previous chapters.

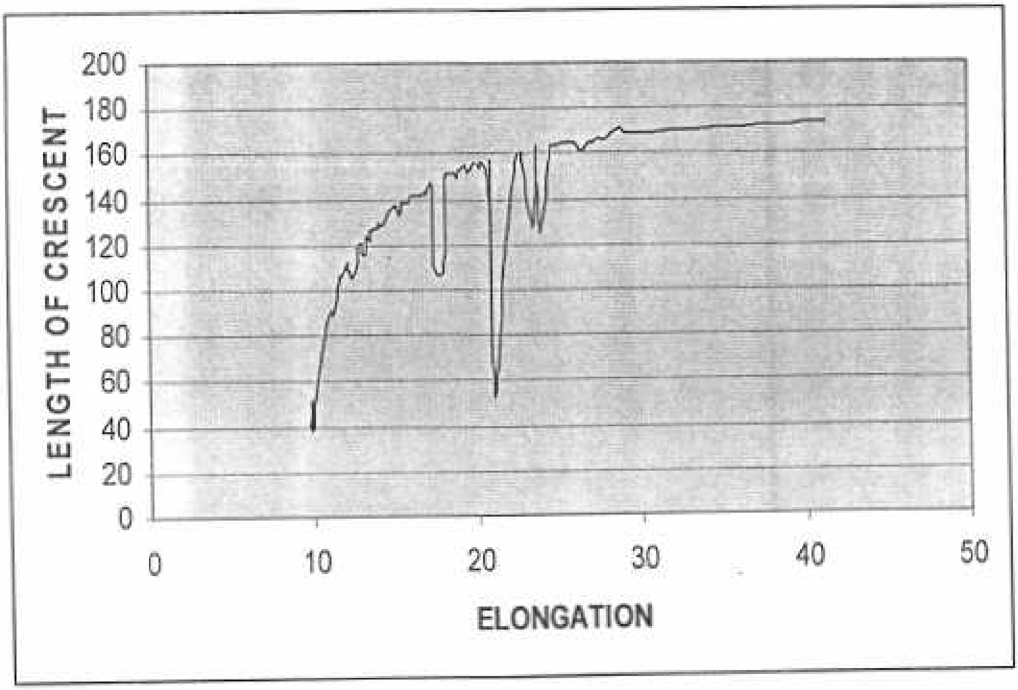


Fig. 5.1.2: Crescent Lengths vrs Variations due ARCV

The Fig 5.1.2 shows that the functional relation between crescent length and the elongation is not smooth as reported by Schaeffer on the basis of Hapke's model. The main reasons for this are (i) the crescent length has to be affected by the Earth-Moon distance as claimed above (ii) the atmospheric turbulence close to horizon must affect the crescent length as claimed by McNally and Sultan. Moreover, very wide crescents that are invisible due to their close vicinity with the horizon must not vanish suddenly. For an invisible crescent, the elongation may be large and must not have zero length (i.e. must not be invisible) according to claims of Schaefer. The invisibility of a wide crescent is due to its closeness to horizon (small ARCV). However, with suitable altitude (ARCV) the same crescent shall be visible. Our observation and claim is that for a crescent of same (sufficient) width the varying altitude (ARCV) must cause to decrease the length of crescent from its maximum (at DAZ = 0) to its minimum (zero length when its q-value is below limits of vision) smoothly and not suddenly. In the associated fig. 5.1.2 only those observations are considered when the crescent was claimed to have been seen or the crescent length is computable.

Table 5.1.1

Observ Date			Location		DAZ	ARCV	ARCL	VIS	SD	Wc	Q-VAL	Wm	Crescent
Y	M	D	LAT	LONG	Deg	Deg	Deg	REP	arc-min			arc-min	LENGTH
1859	7	1	38	23.7	10.2	11.6	15.4	V	33.9	0.61	0.33	0.21	139
1859	10	27	38	23.7	20.4	5.37	21	V	31.5	1.05	-0.1	0.94	52.6
1860	1	23	38	23.7	3.28	5.25	6.19	I	32.5	0.09	-0.6	0.96	0
1860	2	23	38	23.7	2.48	19.4	19.5	V	29.9	0.86	1.25	0.21	152
1860	6	20	38	23.7	12.7	14.3	19	V	32.8	0.9	0.76	0.21	153
1861	3	12	38	23.7	1.63	12.2	12.3	V	31.8	0.36	0.26	0.21	109
1861	8	7	38	23.7	15.1	4.08	15.7	I	32.7	0.61	-0.4	1.18	0
1861	9	7	38	23.7	36.6	12.1	38.2	V	33.3	3.57	1.81	0.21	173
1861	10	5	38	23.7	19.4	4.28	19.8	I	33.7	1	-0.2	1.15	0
1861	11	4	38	23.7	25.1	11.9	27.6	V	33.3	1.89	1.01	0.21	167
1861	12	3	38	23.7	16.4	12	20.2	V	33	1.02	0.59	0.21	156
1862	1	1	38	23.7	7.02	11.5	13.4	V	32.5	0.44	0.23	0.21	123
1862	3	31	38	23.7	0.3	15.3	15.3	V	30.2	0.53	0.66	0.21	133
1862	4	29	38	23.7	0.1	7.82	7.82	I	29.9	0.14	-0.3	0.51	0
1862	7	28	38	23.7	20.5	7.48	21.8	I	31.1	1.11	0.19	0.57	119
1864	1	10	38	23.7	6.18	17.4	18.4	V	33.7	0.86	1.05	0.21	152
1864	2	8	53.5	2.3	4.48	12.8	13.6	V	33.7	0.47	0.38	0.21	126
1864	3	9	38	23.7	2.32	20.3	20.4	V	33.2	1.04	1.43	0.21	156
1864	5	6	39.6	26.2	4.22	6.87	8.05	I	31.8	0.16	-0.4	0.67	0
1864	6	6	38	23.7	19.8	17.4	26.1	V	31.8	1.62	1.43	0.21	165
1864	8	4	38	23.7	22.1	7.67	23.4	I	29.8	1.22	0.27	0.53	128
1864	9	3	38	23.7	24.9	9.33	26.5	V	29.9	1.57	0.6	0.26	161
1864	11	1	38	23.7	18.8	13.1	22.8	V	31.1	1.22	0.81	0.21	160
1865	1	28	38	23.7	3.12	16.9	17.2	V	33.5	0.75	0.95	0.21	147
1865	3	28	38	23.7	5.42	19.1	19.9	V	33.7	1	1.3	0.21	156
1865	4	26	38	23.7	7.4	12.9	14.8	V	33.3	0.55	0.43	0.21	135
1865	6	24	38	23.7	16.1	8.17	18	I	31.5	0.77	0.08	0.45	108
1865	7	24	38	23.7	22.1	8.83	23.7	V	30.5	1.29	0.41	0.34	149
1865	10	21	38	23.7	17.3	12	20.9	V	29.9	0.99	0.58	0.21	155
1866	1	17	38	23.7	0.73	9.83	9.86	I	31.8	0.23	-0.1	0.21	51.4
1866	4	16	38	23.7	9.3	17.1	19.4	V	33.8	0.96	1.08	0.21	155
1867	2	5	38	23.7	1.78	9.7	9.86	I	31.1	0.23	-0.1	0.21	45.3
1867	11	27	38	23.7	8.85	12.8	15.5	V	29.9	0.54	0.42	0.21	134
1868	6	22	38	23.7	22.9	18.6	29.1	V	33.6	2.12	1.78	0.21	169
1869	5	12	38	23.7	9.68	8.27	12.7	I	31.3	0.38	-0.1	0.43	0
1871	2	20	38	23.7	8.68	10.5	13.6	V	31.2	0.43	0.12	0.21	122
1871	4	20	38	23.7	7.1	7.47	10.3	I	30.1	0.24	-0.3	0.57	0
1871	5	20	38	23.7	8.22	10.5	13.3	I	29.9	0.4	0.11	0.21	116
1871	6	18	38	23.7	4.38	4.52	6.29	I	30	0.09	-0.7	1.10	0
1871	8	17	38	23.7	12.2	12	17	V	31.4	0.69	0.42	0.21	144
1872	6	7	38	23.7	9.13	14.4	17	V	30	0.66	0.64	0.21	142
1872	7	6	38	23.7	5.63	8.78	10.4	I	29.9	0.25	-0.2	0.35	0
1872	9	4	38	23.7	14.4	12.2	18.8	V	30.7	0.82	0.51	0.21	150
1872	10	3	38	23.7	9.02	7.9	12	I	31.3	0.34	-0.2	0.49	0
1872	12	31	38	23.7	14.9	11.4	18.7	I	33.5	0.88	0.47	0.21	152

Table 5.1.1 (Continued)													
Observ Date			Location		DAZ	ARCV	ARCL	VIS	SD	Wc	Q-VAL	Wm	Crescent
Y	M	D	LAT	LONG	Deg	Deg	Deg	REP	arc-min			arc-min	LENGTH
1873	4	27	38	23.7	4.12	8.18	9.15	I	32.3	0.21	-0.2	0.45	0
1873	5	27	38	23.7	6.62	14.3	15.7	I	31.2	0.58	0.59	0.21	138
1873	12	20	38	23.7	10	4.12	10.8	I	32.1	0.29	-0.6	1.18	0
1874	4	17	38	23.7	4.3	14.4	15	I	33.5	0.57	0.59	0.21	136
1875	6	4	51.5	-2.6	7.08	10.9	13	I	33.6	0.43	0.16	0.21	121
1875	7	4	38	23.7	12.4	16.4	20.4	I	32.7	1.03	1.04	0.21	156
1876	2	26	38	23.7	5.37	15.1	16	V	31.6	0.61	0.69	0.21	139
1876	6	22	38	23.7	4.6	10.8	11.7	I	34	0.35	0.11	0.21	106
1877	3	16	38	23.7	1.7	17.3	17.3	V	30.9	0.7	0.95	0.21	145
1877	6	12	38	23.7	6.53	13.7	15.2	V	33.4	0.58	0.53	0.21	137
1877	11	7	38	23.7	27.2	10.4	28.9	V	43.6	2.72	1.24	0.21	171
1878	1	5	38	23.7	14.8	17.2	22.5	V	30	1.15	1.18	0.21	159
1878	6	2	38	23.7	8.9	17.8	19.8	V	31.9	0.94	1.13	0.21	154
1878	7	31	38	23.7	21.9	11.2	24.5	V	33.3	1.5	0.75	0.21	164
1878	10	27	38	23.7	22.9	7	23.9	V	33	1.41	0.29	0.65	125
1879	5	23	38	23.7	12.6	24.1	27	V	30.6	1.67	2.13	0.21	165
1879	7	22	38	23.7	37.3	19.2	41.3	V	32.2	4.01	2.75	0.21	174
1881	3	30	51.5	-2.6	1.3	10.5	10.6	V	31.8	0.27	0.04	0.21	77.8
1900	5	29	38.7	-0.7	9.3	10.9	14.3	V	31.8	0.49	0.2	0.21	129
1901	4	19	50.7	-2.8	5.77	10.2	11.7	V	33.9	0.35	0.05	0.21	107
1908	2	3	56	-3.2	15.8	9.92	18.6	V	33.3	0.87	0.31	0.21	152
1909	2	21	51.1	0	14	11.8	18.2	V	33.7	0.84	0.48	0.21	151
1911	6	27	49.9	2.3	13.2	12.5	18.1	V	33.6	0.83	0.55	0.21	151
1911	8	25	49.9	2.3	19.2	8.48	20.9	V	32	1.05	0.26	0.40	135
1913	11	28	-34	18.5	0.48	9.08	9.1	V	32.5	0.2	-0.1	0.30	0
1915	3	16	49.4	8.7	1.62	9.92	10	V	30.4	0.23	-0	0.21	49.5
1916	4	3	49.4	8.77	1.43	12.9	13	V	31.1	0.4	0.35	0.21	116
1918	3	13	50.2	5.1	1.47	12.9	12.9	V	33.8	0.43	0.36	0.21	121
1919	4	1	53.9	-1.6	3.7	11.6	12.2	V	33.8	0.38	0.21	0.21	112
1921	2	8	42.3	-71.1	0.32	9.9	9.91	V	30.1	0.22	-0.1	0.21	38.7
1921	8	4	-34	18.5	0.1	11.6	11.6	V	33.8	0.34	0.18	0.21	104
1921	10	31	-34	18.5	5.13	7.27	8.89	I	32.1	0.19	-0.3	0.60	0
1921	12	30	-34	18.5	15.2	8.53	17.4	I	30.2	0.69	0.07	0.39	111
1922	2	28	-34	18.5	18.1	9.9	20.6	V	29.9	0.95	0.35	0.21	154
1922	3	29	-34	18.5	10.1	6.87	12.2	I	30.3	0.34	-0.3	0.67	0
1922	4	27	-34	18.5	2.63	4.53	5.24	I	30.7	0.06	-0.7	1.10	0
1922	4	28	-34	18.5	11.6	12.1	16.7	V	31	0.66	0.41	0.21	142
1922	5	27	-34	18.5	4.82	10.2	11.2	V	31.7	0.3	0.02	0.21	91.7
1953	4	14	51.1	5.3	2.25	12.6	12.8	FV	33.1	0.41	0.33	0.21	118
1954	3	5	44.5	-88	0.2	11.9	11.9	V	33.7	0.36	0.22	0.21	108
1961	1	17	34	-118	5.18	14.7	15.6	V	33.4	0.61	0.65	0.21	140
1962	4	5	-25	-28.2	7.2	12.1	14.1	FV	33.4	0.5	0.33	0.21	130
1970	6	4	26.3	-98.2	0.13	11.3	11.3	V	30.5	0.3	0.13	0.21	88.9
1971	3	27	51	0	2.52	13.1	13.3	V	33.7	0.45	0.39	0.21	124
1972	3	15	35.5	-118	2.6	8.12	8.52	BI	33.7	0.19	-0.3	0.45	0

Table 5.1.1 (Continued)													
Observ Date			Location		DAZ	ARCV	ARCL	VIS	SD	Wc	Q-VAL	Wm	Crescent
Y	M	D	LAT	LONG	Deg	Deg	Deg	REP	arc-min			arc-min	LENGTH
1973	3	5	40	-85	0.97	12.2	12.2	FV	32.3	0.36	0.25	0.21	109
1973	7	1	-44	170.5	15.7	18	23.7	BV	33.1	1.39	1.38	0.21	163
1976	12	21	42	-91.6	5.17	10.6	11.7	FV	32.8	0.34	0.08	0.21	104
1977	2	18	43.8	-87.7	0.77	9.73	9.76	FV	31.1	0.23	-0.1	0.21	40.1
1978	3	9	45.1	-64.2	3.43	9	9.63	V	32	0.23	-0.1	0.31	0
1979	1	28	29.9	-81.3	0.3	9.22	9.22	V	33.9	0.22	-0.1	0.28	0
1980	7	13	41.4	-70.7	17.5	10.2	20.2	V	30.5	0.94	0.37	0.21	154
1984	1	3	15.6	35.6	3.6	3.2	4.82	I	30.3	0.05	-0.8	1.36	0
1984	2	2	15.6	35.6	4.6	6.32	7.81	I	29.8	0.14	-0.5	0.77	0
1984	3	3	15.6	35.6	3.73	9.23	9.95	I	29.9	0.23	-0.1	0.21	40
1984	4	2	15.6	35.6	1.85	12.2	12.4	I	30.5	0.35	0.25	0.21	106
1984	5	2	15.6	35.6	0.22	16.6	16.6	V	31.2	0.65	0.86	0.21	142
1984	5	31	15.6	35.6	0.43	11	11	I	32	0.3	0.1	0.21	88.6
1984	6	30	15.6	35.6	3.48	19.4	19.7	V	32.9	0.96	1.3	0.21	155
1984	7	29	15.6	35.6	4.23	15.5	16.1	V	33.6	0.66	0.75	0.21	142
1984	8	27	15.6	35.6	3.52	11.6	12.1	I	33.9	0.38	0.2	0.21	112
1984	9	25	15.6	35.6	1.65	7.12	7.3	I	33.9	0.14	-0.4	0.63	0
1984	10	25	15.6	35.6	8.63	12.2	14.9	I	33.3	0.56	0.37	0.21	136
1984	11	23	15.6	35.6	5.32	6.38	8.3	I	32.7	0.17	-0.4	0.76	0
1984	12	23	15.6	35.6	7.85	11.8	14.1	V	31.6	0.48	0.28	0.21	127
1985	1	21	19	-155	6.55	11.1	12.8	V	30.5	0.38	0.15	0.21	112
1986	10	5	40.8	-73.2	26.4	9.52	27.9	V	32.8	1.91	0.78	0.23	166
1986	12	31	39	-77	10.9	4.58	11.8	IB	33.5	0.36	-0.5	1.10	0
1987	4	28	39	-77	1.12	10.5	10.5	V	30.8	0.26	0.02	0.21	70.5
1987	6	26	-30	-71	8.08	2.93	8.6	IB	29.7	0.17	-0.8	1.41	0
1990	2	25	35.6	-83.5	0.53	7.47	7.49	V	32.9	0.14	-0.3	0.57	0
1990	5	24	35.6	-83.5	0.02	7.85	7.85	I(V)	32.9	0.15	-0.3	0.50	0
1990	11	19	39	-76.8	24.6	11.9	27.2	V	29.7	1.64	0.89	0.21	165
1991	3	17	39	-76.8	0.87	20.3	20.3	V	32.4	1.01	1.42	0.21	156
1991	5	15	39	-76.8	10.4	22.5	24.7	V	33.4	1.52	1.9	0.21	164
1995	1	1	33	-106	0.17	8.33	8.33	I(V)	32.6	0.17	-0.2	0.42	0

During this work the 70 observations of Danjon (mentioned by Schaeffer and Fatoohi et. al.) could not be accessed, however the pictorial data of crescent length is being generated at the Astronomical Observatory at University of Karachi. The observed crescent length from photographic records is given in Table 5.1.2. This includes some observations made by others during past few years and their pictures are available from www.icproj.org maintained by Odeh.

The model developed in this work for both the brightness and the length of crescent is mainly geometric, supplemented by the single parameter criterion for the earliest visibility of new lunar crescent. The model provides a simple method of calculating length of lunar crescent and takes into account the atmospheric affects indirectly through single parameter criteria for the visibility of new lunar crescent only. Whenever $W_c < W_m$ the crescent length is not calculated and the crescent was not seen according to recorded observation (table 5.1.1). The model has been tested in two ways. First, for some of the recent observations whose photographic records are available the calculated and observed crescent lengths are compared and shown in Table 5.1.2 and in Fig 5.1.7. The crescent lengths calculated using our formula (5.1.13) are greater than the observed values and those due to Sultan's technique are generally closer to the observed values. In calculations using the formula of McNally $\Delta R = W_m$ and R = semi-diameter of lunar disc.

The columns in table 5.1.2 show the date of observation the coordinates of the location of the observer (latitude and longitudes in degree respectively), the elongation of the Moon from the sun in degrees, the semi-diameter of the Moon, the central width of the crescent, minimum visible width of crescent (all in arc minutes), followed by the crescent length calculated by our model and the observed crescent length, the name of the observer. The last two columns contain crescent lengths as calculated using the models due to Sultan and McNally respectively. The data in this table has been arranged in order of increasing elongation or ARCL.

To determine the length of a crescent from pictorial record the digital photograph is opened in any graphic software tool. Selecting coordinates of three of the points (two of them close to each of the cusp and one close to supposed centre of the crescent) on the visible outer limb of the crescent an equation of circle is developed that leads to the coordinates of the centre of the lunar disc. Joining the centre of the lunar disc with the two visible cusps of the crescent the angle made at the centre by the two cusps is measured. The picture of one of the crescent measured in this way is shown in Fig 5.1.7.

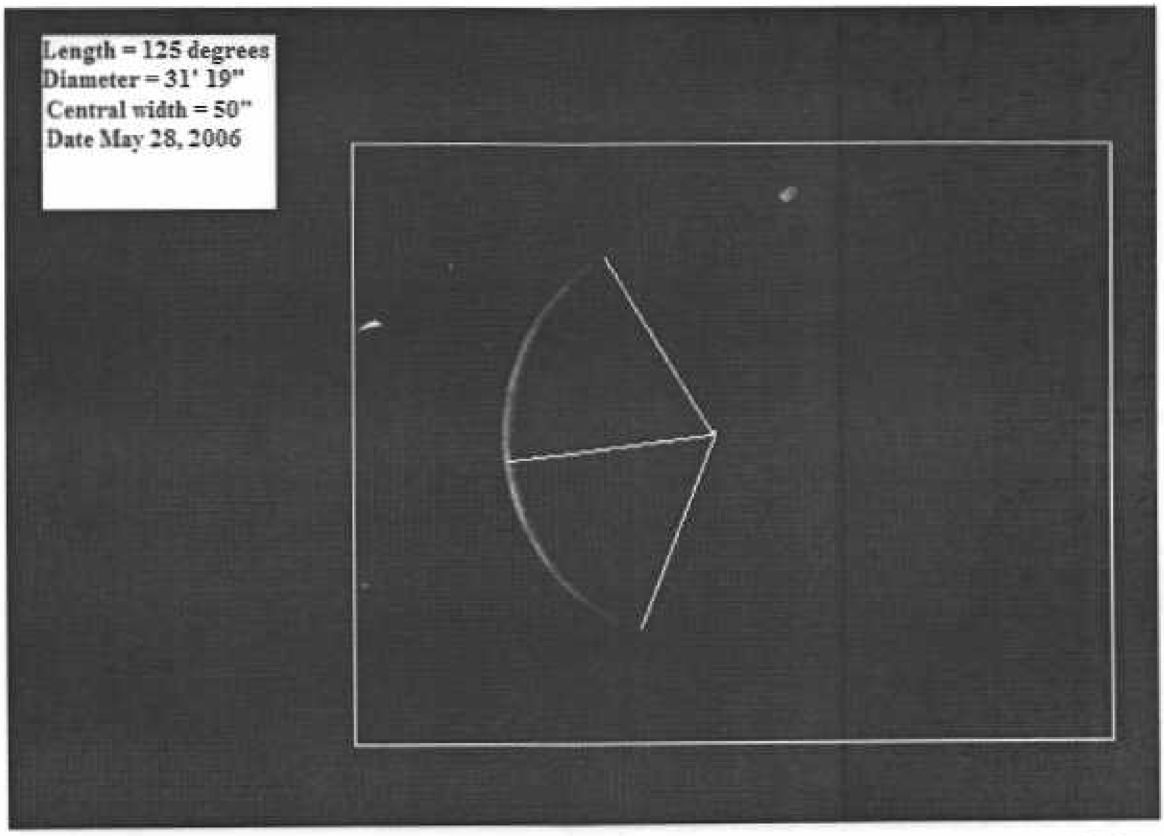


Fig. No. 5.1.7: Measurement of Length for Crescent of May 28, 2006 as photographed at Karachi University observatory.

The data for the observed crescent length shown in table 5.1.2 and the chart in fig 5.1.7 shows interesting pattern. Claimed by Schaefer (Schaefer, 1991) the crescent length is a smooth function of elongation, but this data set shows a trend that clearly exhibits deviation from any such smooth relation. The data sample is small but there are two subsets each having nearly smooth relations, separately, between the crescent length and the elongation. However, when considered as a combined data set the observations with elongation 11.93 degrees (by Anwar), 14.41 degrees (by Omer), 16.67 degrees (by Rahimi) and 20.32 degrees (by Qureshi) deviate markedly from the apparent smooth relation exhibited by the rest of the data set. Crescent lengths in these four cases are much smaller than the trend shown by the rest of observations as well as the results of each the models considered above for calculating the length of the crescent.

All these four deviating cases are photographically recorded and have the least possibility of "observational" errors. Out of the rest of the eight cases another six are photographic. The records due to Schaefer are the only cases that "observational" length of new lunar crescent is concerned its relation with elongation must not be smooth as shown in figure 5.1.2 as well as table 5.1.2 (or figure 5.1.7).

It is further noted that the root mean square error calculated for the three computational methods (Our's, Sultan's and McNally's) it is found that Sultan's method has the least error (4.76 degrees) followed by McNally's that has an error of 6.16 degrees and our model has an error of 7.22 degrees. The major difference between our model and those Sultan's and McNally's is that our model gives consistently higher values of the length of crescent, whereas the other two models have both positive and negative errors. The functional relation between elongation and the crescent length is similar in our model and that due to Sultan's, but the one exhibited by McNally's model is markedly different. McNally's model is giving better results for larger elongation but as the elongation becomes smaller the error given by McNally's model becomes larger and larger.

Table No 5.1.2 *Observed & Calculated Lengths of Crescents*

Obs. Date			Location		Elong	SD	Wc	Wm	LENGTH		Report	Length	
Y	M	D	LAT	LONG	Deg		Arcmin		Cal	Obs	By	Sultan	McNally
1989	5	5	39.7	-106	9.88	32.92	0.244	0.18	84.9	82	Schaefer	64.78	105.85
2006	4	28	21.5	39.2	11.93	32.21	0.348	0.18	118	75	Anwar	100.6	119.69
1999	3	18	32	35.9	12.68	32.78	0.4	0.18	127	110	Odeh	109.7	124.24
1999	10	10	32	35.9	14.27	30	0.463	0.18	134	124	Odeh	124.1	128.74
1999	4	16	33.2	-112	14.41	33.3	0.524	0.18	140	92	Omer	125.3	132.06
1989	4	6	34	-107	14.67	33.29	0.544	0.18	141	123	Schaefer	127.3	132.99
2001	2	24	32.7	51.7	14.79	29.74	0.493	0.18	137	122	Rahimi	127.9	130.52
2006	3	30	24.9	67.1	16.29	32.84	0.659	0.18	148	138	Qureshi	136.9	137.83
2001	11	16	32.7	51.7	16.67	31.48	0.661	0.18	148	117	Rahimi	138.7	137.96
2006	5	28	24.9	67.1	18.21	31.15	0.78	0.18	153	138	Qureshi	145.3	141.68
2006	1	1	24.9	67.1	20.32	33.02	1.027	0.18	160	127	Qureshi	151.9	147.1
2006	6	24	24.9	67.1	20.78	31.25	1.016	0.18	160	154	Qureshi	153	146.99

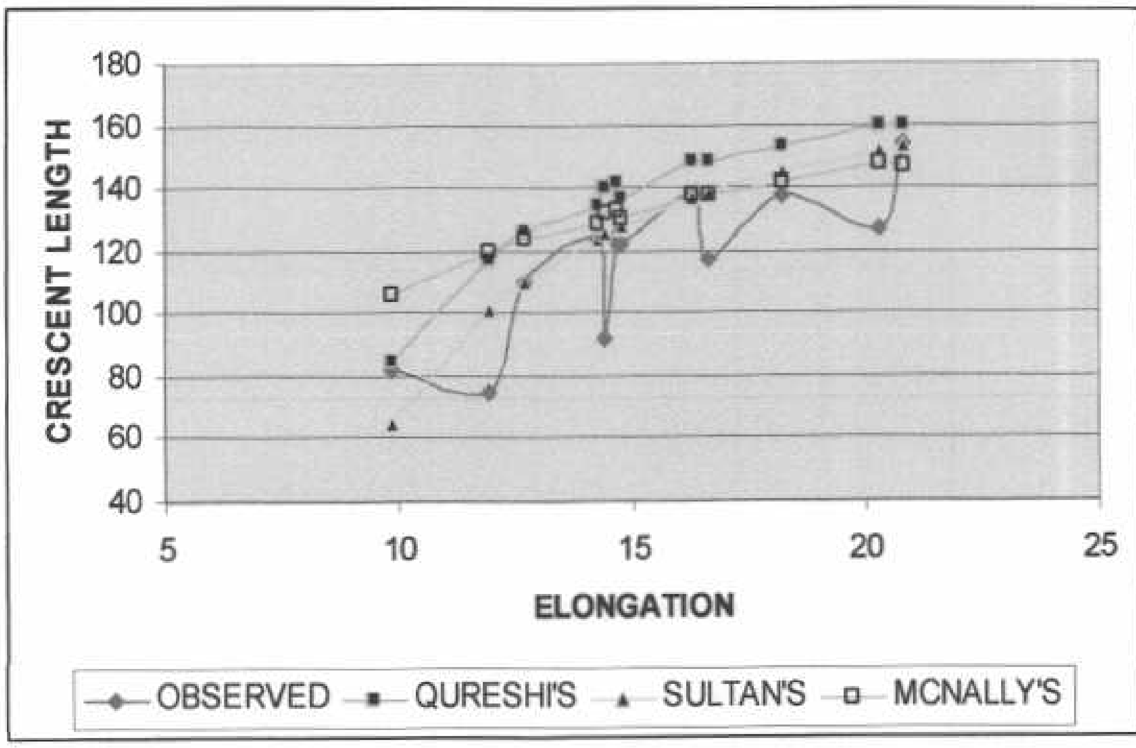


Fig. No. 5.1.7

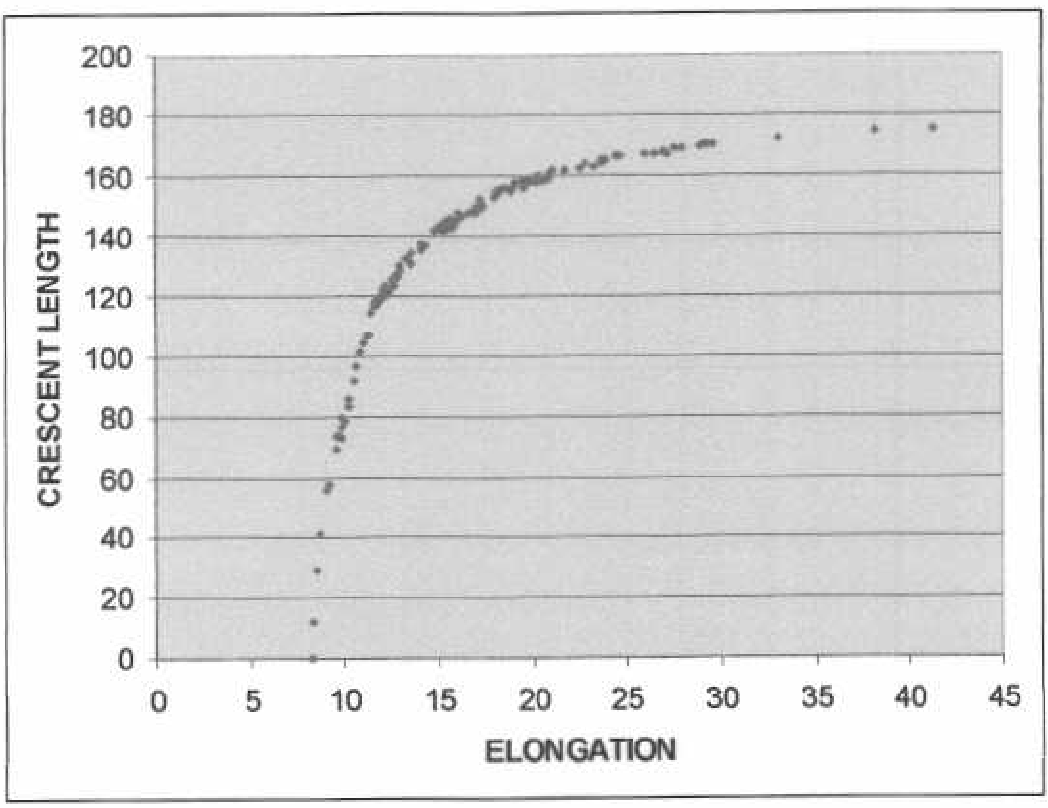


Fig. 5.1.8: Crescent Lengths against Elongation without variations due to ARCV

The model for the calculation of crescent length may be used as the earliest visibility criterion much in the same way as Yallop's criterion can be used. However, our emphasis was not to develop an alternate criterion for the same. This work was intended for a better understanding of the geometry of the lunar crescent and to develop a method for calculating its length.

The second and indirect test of the model is its comparison with the results of Danjon mentioned in Fatoohi et. al. If the minimum visible width W_m for various ARCV according to Yallop's criterion is replaced by the minimum ever visible central width of 0.18 arc minutes that is equivalent to ignoring the atmospheric affect for lower ARCV then the relation between crescent length and elongation becomes smooth as presented by Schaefer (Schaefer, 1991). The same is shown in Fig. 5.1.8. The chart in figure 5.1.8 also shows that limiting value of elongation for possible visibility of the crescent is around 8 degrees. As our computations do consider the affect of parallax this limit is equivalent to the 7 degree limit very popularly known as Danjon's limit.

On the basis of the calculated lengths of crescent and its elongation using our model the Danjon deficiency arcs are calculated ignoring the affects of ARCV by formula given by Danjon and that given by McNally (shown in Fig. 5.1.9 and 5.1.10 respectively). These results are in close agreement with the Fig 1 in Fatoohi et al (that is a reproduction of Danjon's fig 2).

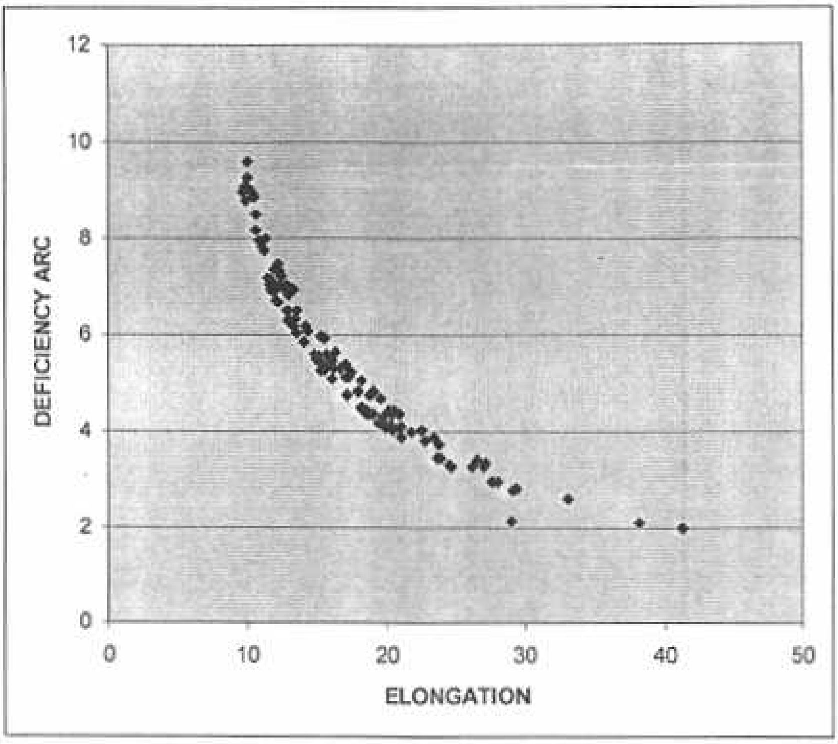


Fig. 5.1.9: Deficiency Arc against Elongation according to Danjon

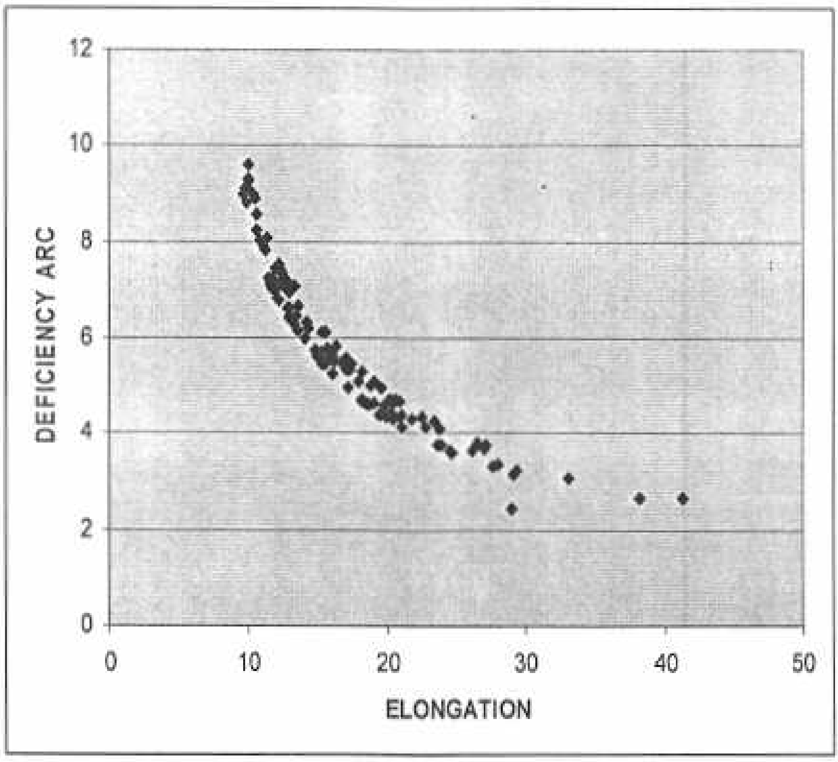


Fig. 5.1.10: Deficiency Arc against Elongation according to McNally

6.2 LUNAR CALENDAR FOR PAKISTAN

Most of the Islamic countries follow an observational lunar calendar, at least for their religious purposes. Although substantial work is done to evolve a prediction criterion to develop a universal calendar by Ilyas (1984b, 1988, 1991, 1994a, 1994b, 1997) and others, a truly universal calendar could not be developed. As according to common Islamic belief actual sighting of the new Lunar crescent is necessary to begin a Lunar month, such a universal calendar seems to be impossible. This is true especially because just after conjunction the new lunar crescent is not visible on the same day throughout the world, even if two places of observation are on same longitude. According to recent developments (Scahefer, 1993, Yallop, 1998, Qureshi &Khan, 2007) observational lunar calendars for each Islamic country can be developed but are not generally accepted by various Islamic communities.

In Pakistan an Official Committee (Ruet-e-Hilal Committee) decides about when to begin a Lunar month on the basis of public evidence and observations. This committee gathers information about the claims of sighting the crescent. The claims are judged on the basis of the directives of the Islamic Laws. Some representatives of various scientific organizations are also consulted. Once the claim/s is justified religiously and/or scientifically, the announcement is made about beginning of the next Lunar month. In a way, the beginning of new lunar month is based on "public observations" verified in view of the principals laid down by Islamic shariaat laws (listed in article 1.3).

The advantage is that a large number of "observers" take part in the exercise and with it the probability of sighting new crescent is increased. Moreover, most of these observers are from rural areas where there is least industrial/traffic and light pollution, and it is highly probable that the observing conditions are near perfect. Therefore in this study these public observations are considered to have a high degree of authenticity. In this work all these public observation-based dates of start of each Lunar month from year 2000 to 2007 are reproduced and are studied in comparison with the Yallop's q-value and

our *s*-value criterion-based criteria. A similar work for the period 2000-2005 has already been reported (Qureshi and Khan, 2005)

The results of the 95 lunation during the January 2000 to September 2007 (Shawwal, 1420 AH to Ramazan, 1427 AH) along with observational data are presented in yearly tables in appendix-IV. All the computations for these tables are based for the city of Karachi (Latitude 24° 56', Longitude 67° 3'). The first column indicates last conjunction with month, day, hour, minute and second for the local time of conjunction in the sub-columns. The calculations are done for the day of last conjunction, for the next day and where required two days later, indicated under column headed "Last Conj" followed by the Date in Gregorian calendar. Next two columns give the relative altitude (ARCV) and relative azimuth (DAZ) in degrees followed by moonset-sunset lag in minutes. The age of Moon (in hours), the arc of light or elongation (ARCL in degrees) and the crescent width (in arc minutes) appear the next three columns, followed by columns indicating the *q*-value, visibility condition on *q*-value, the *s*-value and the visibility conditions on *s*-value. The visibility conditions are those described in table 4.2.4 (for Yallop's criterion) and table 4.4.4 (for Qureshi & Khan criterion). Under the heading of "Month" the column gives the name of Islamic month that begins after a sighting reported on the previous evening followed by the Gregorian date of start of this month. In this last column under Gregorian date of starting lunar month, number of days in the month, are also given. The last column contains a comment. If the decision of starting new lunar month is in agreement with the prediction criterion (*s*-value criterion) the column contains "PROPER". The "observational decision" and the model are considered to be in agreement only when the *s*-value criterion show EV, i.e. crescent is easily visible. The comment in the last column contains "LATE" if the decision of starting new lunar month is one day late as indicated by the day of easy visibility on *s*-value and "EARLY" if the lunar month has been started one day earlier than that indicated by the prediction criterion. As stated earlier all these calculations are done for Karachi and for the best local time for crescent visibility. However, it should be noted that observation claims are collected from all over the country.

Out of the 95 public observations reported in this work, there were only six occasions (June, 2000, June, 2001, July 2004, April, 2006, July, 2006 and July, 2007) when the sighting was reported one day late in comparison to the prediction criteria according to which the New Crescent was visible on the previous day but was not reported. There was only one occasion when the early sighting is reported (Nov. 13, 2004). Thus, there were only 6.3% errors in the decisions of the moon sighting committee of Pakistan during last seven years. These results are based on verified claims. The data regarding number of crescent sighting claims that were not verified is not available. The only case of "EARLY" sighting accepted by the authorities was for the month of Shawwal.

The main reason for late sightings on the five indicated occasions is the overcast sky all over the country in all these cases. In case of the only early sighting for the last seven years attempts for crescent sighting were made at the Observatory of University of Karachi using 6" Codé refractor telescope. The prediction criterion based on *s*-value allowed crescent visibility with optical aid and that based on *q*-value did not allowed visibility even with telescope. We failed in sighting of the crescent but the officials accepted claims of naked eye sightings from area close to Karachi. Although, the two criteria considered did not allow naked eye visibility on this occasion the misconception that if the phase of the crescent is more than 1% it should be visible (Astronomical Almanac, 2007) may have lead the decision makers to accept the claims.

The low percentage of errors in this observational effort is much more promising as compared to the results of "moon watch" programs conducted in United States in the late 1980s and reported by Dogget & Schaeffer (1994). According to this report 15% positive errors (wrong claims of observing the crescent) and 2% negative errors (crescent was visible but the observers missed them) were found amongst the experienced moon watchers. These experienced moon watchers were generally considered to know where to find the crescent and what the orientation of the "horns" was.

Frequently, claims of very early sighting are made in some regions of Pakistan (particularly for starting the months of Ramazan and Shawwal as mentioned before) that are not accepted by the authorities. One of the major causes of these early claims of sighting of new crescent in Pakistan is the keenness of the claimer and the misconception amongst masses that when the "sighting" has been reported in the Kingdom of Saudi Arabia the crescent must be sighted the next day in Pakistan. However, the official decision about starting new lunar month in the Kingdom of Saudi Arabia is not based on actual sighting of the new crescent. These decisions are based on criteria that have changed three times over the past two decades (Odeh 2000).

i) Up to 1999 the criteria was as follows: If the Moon's age at the sunset is 12 hours or more after the New Moon, the previous day is the first day of the Islamic month. This actually means that the lunar month begins on the day of conjunction of Moon which occurs one or two days earlier than the time when the new crescent becomes visible to the naked eye.

ii) From 1999 the criterion was changed to the following: The lunar month begins on the evening when the sunset is before the moon set according to Mecca. This was more disastrous as it is possible that the sunset before moonset will occur even before the conjunction. Therefore, the lunar month may begin three days before the crescent becomes visible to naked eye.

iii) From 2003 onwards the criterion is:

   a) The geocentric conjunction occurs before Sunset.

   b) The Moon sets after the Sun.

   This is more realistic but it is scarcely possible that the crescent becomes visible on the day of conjunction.

The main parameters significant in the earliest visibility criterion for the lunar crescent are LAG, AGE, ARCV, DAZ, ARCL, Phase and the width of crescent. As mentioned above none of these parameters alone decide the visibility or non-visibility of

the lunar crescent. Generally the critical value of the Phase is considered to be 1% (Astronomical Almanac, 2007). During the period from January, 2000 to Sept., 2007 that were considered in this study there was not a single incidence when the sighting of crescent with phase less than 1% was reported and accepted. There were two occasions when the phase was greater than 1% but there was no claim of sightings and sighting was not possible according to both the criteria considered (Oct. 23, 2006 and Dec. 21, 2006). In the case of Nov. 13, 2004 the phase was 1.19% that may have been the reason that the sighting claim was accepted (as mentioned earlier).

The youngest crescent of age 19.1 hours (at best time of visibility) seen during the period of study was that of May 17, 2007. Apart from being the youngest crescent seen during the period of study the observation has another interesting feature. This was the only case when the crescent was seen on the same Gregorian date as it was born according to Pakistan standard time. Apart from this record observation there are four other claims of young Moons' sighting, Nov. 13, 2004 (22.3 hours), Jan. 30, 2006 (23.45 hours), June 26, 2006 (22.76 hours) and Feb. 17, 2007 (21.6 hours). In all there were 22 (23% of the studied) cases of sighting claims when the age of Moon was less than 30 hours.

The observation of Feb. 17, 2007 was made at the astronomical observatory at the Institute of Space & Planetary Astrophysics, University of Karachi. We spotted the crescent using the 6" Codé Refractor Telescope. Both the prediction criteria allowed crescent sighting without optical aid but we could see the crescent without telescope. No other claim of crescent sighting was received by the authorities on this day. The authorities accepted our claim. This was the youngest crescent seen at our observatory during the period of study. It is a record for our observatory and the world record for this particular crescent's earliest observation (www.moonsighting.com and www.icoproject.org). Our photographic record is also posted on relevant websites.

There were 6 cases when the age of Moon was between 24 hours and 30 hours and the crescent was not seen. Thus, this study also supports the idea that age is not a dependable factor for any prediction criterion.

Another important parameter concerned is the moonset-sunset LAG. There are 22 occasions when the crescent of lag 60 minutes or less was seen. Out of these only two were with lag less than 50 minutes, Nov. 13, 2004 with lag 35 minutes and April 10, 2005 with lag 44 minutes. In the second of these cases the prediction criteria allowed naked eye visibility. There were 15 cases when the lag was between 40 and 50 minutes and the crescent was not seen and both the prediction criteria did not allow crescent visibility without optical aid. Thus the moonset-sunset lag alone is also not a dependable parameter for any prediction criterion. Out of 95 new Moons the prediction criteria differ from the Babylonian criterion (Fatoonhi et al, 1999) only 9 times when Babylonian criterion (ARCL + LAG(in degrees) < 22^o) allows naked eye visibility but q-value and the s-value criteria do not. However, on these nine occasions the crescent was never reported to be seen.

In view of these facts we conclude that both q-value and s-value criteria are well suited prediction criteria with s-value criterion developed in this work has marginally better success percentage for positive observations (chapter 4). Particularly in view of the fact that these criteria do not match actual crescent visibility only in one out 95 cases (1.05%) and that occasion was controversial in view of the above discussion. The five cases when the crescent was seen later than predicted are not considered as an error as the problem occurred due to overcast skies. Therefore, for the formulation of future observational lunar calendar these prediction criteria are highly dependable.

There has been only one occasion when there were three consecutive months of 29 days each (during June, July and August 2000). This is the maximum for repetition of 29 days lunar months anticipated as early as 10^{th} century AD by Muslims (Ilyas, 1994a). The first (Rabi ul Awwal, 1421) of this triplet of months began a day later than predicted

otherwise this maximum repetition could not have been there. There was no occasion of four (maximum) consecutive months of 30 days each in this period of study.

Late sightings can cause naturally occurring repetition of four 30 days' month to five or six. Although, the Islamic Sharia Law (chapter 1) allows for correction as and when observation indicate an error, but for advance planning and development calendar an observation lunar calendar based on prediction criteria may greatly help. The Appendix-V shows such a lunar calendar in which computations are based in Karachi on the *s*-value criterion.

Chapter No. 6

DISCUSSION

This work was intended to explore and compare the mathematical models for the criteria under which the new lunar crescent could be visible at given location on the Earth. Moreover, it was intended that a comparison of these model is conducted and the models are modified if possible. The task of comparison and modification of the model has been successfully achieved and a new model the *s*-value criterion has been developed. A summary of this work is presented below with a discussion on the major achievements of the whole effort.

First of all a better understanding of the issues, computational, astronomical and observational, associated with the problem of the first sighting of the new lunar crescent is developed. We have explored the computational techniques and the astronomical algorithm and their application to the extent that is necessary for the calculations involved in solving the problem described in the previous paragraph. Initially the techniques were implemented on Microsoft Excel work sheets but due to lengthy calculations and the use of long formulas we were forced to develop a computer program Hilal01. The program has been used to do all calculations for determining coordinates of the sun and the Moon as well as the parameters involved in our problem. The results of the application of some of the models are obtained within the program and those for other models are done on the basis of the data generated by program that is saved as an output file. This file is then transformed into an MS-Excel work sheet and the results of other models are obtained there. The tables comprising these results within the main text and that appear in the appendices are all developed from these work sheets.

The model due to Babylonians as described by Fatoohi et el (Fatoohi et al, 1999) that was modified by the Muslims/Arabs is briefly described in the first chapter (article

1..2) and is its application is studied in comparison with other models in chapter 3 (article 3.1) and chapter 4. This rule is based on the sum of the elongation and the arc of vision. The Lunar Ripeness rule deduced by Muslims of the medieval era in explored more extensively. It is found that though, the Lunar Ripeness function is far more sophisticated as compared to the Babylonian rule, the two methods produce almost equivalent results when applied to the recent observational records.

The methods based on relations between arc of vision (ARCV) and the relative azimuths (DAZ) that were extensively developed during the early part of the 20^{th} century are found more successful during this study in comparison to the ancient and medieval methods. The success in measured in how many observations are in agreement with the models. How many times the crescent is seen when the models suggests its visibility and how many times the crescent is not seen when the model also suggests invisibility.

The reason that the models based on ARCV-DAZ relations are more successful is that with increasing relative azimuth the brightness contrast of the crescent against the sky brightness improves and the crescents of lower relative altitude are visible. Smaller is the DAZ this brightness contrast deteriorates and the crescent is only visible at higher ARCV. In comparison to these models the Lunar Ripeness Function is strongly based on arc of separation. The problem with arc of separation is that with large DAZ it can be much smaller then suggested by the Lunar Ripeness law (10 to 12 degrees) for a visible crescent. In these cases with large DAZ the Ripeness function R_{day} can be large so that the larger values of R_{vis} are required. Consequently, for larger latitudes large arc of separation is required. But larger DAZ for larger latitudes allows smaller ARCV and consequently smaller arc of separation. Therefore, especially for locations with larger latitudes Lunar Ripeness law becomes more inconsistent as compared to the observations and the ARCV-DAZ models.

It is seen during the discussion at the end of the 4^{th} chapter that three ARCV-DAZ based models, the Fotheringham's model, the Maunder's model and the Indian model are successively better. These improvements are due to deduction of better and better basic

data of minimum arc of vision for different relative azimuths. However, all these models ignore the width of the new lunar crescent that varies greatly with the Earth-Moon distance for the same elongation. This causes variations in the actually brightness of the crescent of same elongation. Consequently, the same pair of ARCV and DAZ the brightness of crescent varies for different Earth-Moon distances. The lack of this consideration is the main cause of lesser success percentage of these methods as compared to the later models for positive observations.

The realization of varying brightness of crescent with the width of crescent for same elongation (and same pair of ARCV-DAZ values) the physical description of the problem by Bruin lead Yallop to deduce basic data relating widths of crescent to the arc of vision. Yallop deduced this data from the minima of the limiting visibility curves of Bruin. Consequently, Yallop was successful in deducing his single parameter test, the q-value criterion. This criterion produced better results in comparison to all the previous methods. The deduction of various visibility conditions on the basis of different ranges of q-values and that of the 'best time of visibility' of crescent from the limiting visibility curves of Bruin are the most remarkable of Yallop's contribution. The visibility conditions provide guidelines about under what conditions the crescent would be easily visible, when it will be visible under perfect weather conditions, when the optical would be required and when the crescent would be simply not visible with or without any optical aid. These conditions have proved to be more and more reliable with increasing number of observations.

The major contribution of our work is the comparison of all the models theoretically, mathematically, physically and in view of their success percentages for a set of observations collected from the late 19[th] century till recently. Another significant contribution of this work is to convert all the models into single parameter criteria. These are listed below in the increasing order of success percentage:

Fotheringham's: $V_P = (\text{ARCV} - (12.^o0 - 0^o.008(DAZ)^2))/10$

Maunder's: $$V_P = \left(ARCV - \left\{ -\frac{DAZ^2}{100} - \frac{|DAZ|}{20} + 11 \right\} \right) \Big/ 10$$

Indian: $$V_P = \left\{ ARCV - \left(10.3743 - 0.0137|DAZ| - 0.0097 DAZ^2 \right) \right\} / 10$$

Yallop's: $$q = (ARCV - (11.8371 - 6.3226W + 0.7319W^2 - 0.1018W^3))/10$$

Muslims': $$R_{avr} = R_{vis} - R_{day} = \frac{a_s \cos\varphi}{\cos(\Delta + \varphi)} - \{\lambda_M - \lambda_S + \beta_M \tan(\Delta + \varphi)\}$$

Bruin's: $$V_P = (ARCV - (12.4023 - 9.4878W + 3.9512W^2 - 0.5632W^3))/10$$

Babylonian: $$V_p = \text{ARCL} + \text{arc of separation} - 21^\circ$$

In this work Schaefer's model of relative brightness is also explored in terms of magnitude contrast and the results are in agreement with his work (Schaefer, 1988a). This has been achieved by implementing the techniques developed by Schaefer and others to evaluate sky brightness (in terms of limiting magnitude) and the brightness of the crescent. Brightness of sky and that of crescent depend heavily on various atmospheric affects. Amongst these temperature and relative humidity are kept variable in the program Hilal01 in order to explore possible conditions under which claims of naked eye visibility may be tested for different conditions. This leads to what we have termed as Magnitude Contrast (Δmag = magnitude of Moon – sky's limiting magnitude). If magnitude contrast is negative it is in favour of crescent visibility otherwise not. For extremely critical naked eye observation claims of new lunar crescent the magnitude contrast has been evaluated minutely. It is found that some of these cases appear doubtful as the magnitude contrast was never in favour of crescent visibility even with highly exaggerated weather conditions (very low relative humidity and temperature) for the whole duration of the moonset-sunset lag time. Using the program Hilal01 the times have been evaluated of (i) when the magnitude contrast is maximum, (ii) when the magnitude contrast just becomes favourable for crescent visibility and (iii) when the magnitude contrast was last in favour of crescent visibility. These results are similar to what is described by a vertical line over a limiting visibility curve of Bruin.

The major achievement of this work is the formulation of a new single parameter criterion on the basis of the techniques developed by Bruin and Yallop using the brightness models developed by Schaefer and others. The visibility curves and the limiting visibility curves are developed for crescents of different widths that were actually observed and have been reported in literature. Bruin developed these curves on the basis of (i) the average brightness of the full Moon and the way it decreases with the decreasing altitude above horizon and (ii) the average brightness of the sky during twilight and the way it depends on the dollar depression below horizon. On the other hand we have used the actual brightness for the observed crescents of a fixed width. Then for the site of observation we have calculated the altitude of sky points having the same brightness as that of the crescent for different solar depressions. Such computations are repeated for a number of observed crescents of the same width at different locations and times. The altitudes of the sky points thus obtained are then averaged out. The whole process is then repeated for crescent of different widths.

These computations resulted into our visibility curves and the limiting visibility curves. The limiting visibility curves we have obtained are slightly different from those of Bruin with minima slightly displaced. The straight line joining the minima of these curves has a slope of 9.3/5 as compared to 9/5 for Bruin's curves. This leads to slightly different "best time of visibility" which is 4.3/9.3 time the moonset-sunset lag after the sunset (as compared to Yallop's best time which 4/9 times the moonset-sunset lag after the sunset). The basic data obtained from the minima of our limiting visibility curve fitted to a third degree polynomial results into the following visibility parameter:

$$s = (ARCV - (-0.3519637W^3 + 2.222075057W^2 - 5.42264313W + 10.4341759))/10$$

We have also deduced visibility conditions in a manner similar to that of Yallop. Our visibility conditions are slightly different from those of Yallop. However, applying our model on the observational data the success percentage is found to be the best amongst all the recent models that exist and that are tested and compared in this work. The reason is that our model is based on the actual sky brightness and the brightness of

crescent with varying weather conditions whereas Bruin's model is based on average brightness of sky and the Full Moon.

As claimed by Schaefer the brightness models may be in error by as much as 20% still our model yielded better results as compared to those due to Bruin and Yallop as far as the positive observations are concerned. It is hoped that with better models of sky brightness still better criterion for first visibility of new lunar crescent can be developed.

Some of the applications of the criteria of earliest visibility of new lunar crescent have been considered in this work. The first and the most important application are to determine when the new lunar crescent would be visible at some location on the globe on the evening after the conjunction. Another area of application is to deduce an "observational lunar calendar" for a region. Yet another area of application we have explored is to determine the "length of new lunar crescent".

It may be recalled that the first appearance of new lunar crescent marks the beginning of a new month in observational lunar calendars these criteria and models are significant for calendarical purposes. Whether an actual observational lunar calendar, like the Islamic Lunar calendar, utilizes these criteria for arranging its calendar or not these criteria provides a guidance for both testing an evidence of crescent sighting by common people and tracing down the dates of a calendar in history where appropriate dates are not well recorded. Thus the main utility of the prediction criteria for the earliest visibility of new crescent is to regulate the observational lunar calendar and testify the claims of visibility of new lunar crescent.

Although first order approximations, like Arithmetic Lunar Calendar that are based on the concept of Leap Years and the average motion of the Moon have been in use Muslims have been following actual sighting of crescents at least for the months of fasting (Ramazan) and pilgrimage (Zil hajjah). The Calendars if based on a prediction criterion like that of Yallop or the one developed in this work are the closest to the observational calendar. Comparison of these criteria with the actual observational

calendar in practice in Pakistan for the years 2000 to 2005 have been done (Qureshi and Khan, 2005). In this work the study is extended to the year 2007. It is found that on average 95% observations are according to the Yallop's s-value criterion or our s-value (or Q&K) criterion. The disagreement is the result of either the bad weather due to which the new lunar crescent could not be sighted and the Lunar month began one day late, or too optimistic claims of observation and the Lunar month began one day earlier than predicted.

This remarkable success for the visibility criteria for calendarical purpose has motivated us to deduce an "observational lunar calendar" for Pakistan that has been reported in appendix-V.

APPEDNIX-I

COMPUTER PROGRAM

HILAL01

```
void main(void)
{       clrscr();       mainmenu();  clrscr();       mainroutine();          fclose(fptr4);}
void mainmenu(void)
{       gotoxy(10,4);cout<<"Welcome to the New Moon Calculator";
        gotoxy(25,20);cout<<"Press Enter to start";
        sel=getche();}
void mainroutine(void)
{       fptr4=fopen("Schrnge.txt","a");
        do
        {       inputdatetime();        clrscr();       month_change();
        do
        {       first_calculation();
                jd=juliandate(lyear,lmonth,ldate,frac);
                jde=jd+deltat[int(lyear-1620.0)]/(3600.0*24.0);
                nutation();             stjd=juliandate(lyear,lmonth,ldate,0.0);
                st=(stjd-2451545.0)/36525.0;        sid_time(st);
                outinfo();              t=(jde-2451545.0)/365250.0;
                sun_coord(t);           display_scoord();
                t*=10.0;
                moon_coord(t);                  display_mcoord();
                elongp=sin(mdeltap*PI/180.0)*sin(sdeltap*PI/180.0);
                elongp=elongp+cos(mdeltap*PI/180.0)*cos(sdeltap*PI/180.0)
                        *cos((salphap-malphap)*15.0*PI/180.0);
                elongp=acos(elongp)*180.0/PI;
                gotoxy(52,7); printf("T. Elon  = ");gotoxy(65,7);printf("%7.3lf
                        d",elongp);
                phasep=(1.0-cos(elongp*PI/180.0))/2.0;
                gotoxy(52,8);printf("T. Phase  = ");gotoxy(65,8);printf("%7.3lf
                        %",phasep*100.0);
                elong=sin(mlat*PI/180.0)*sin(slat*PI/180.0);
                elong=elong+cos(mlat*PI/180.0)*cos(slat*PI/180.0)*cos((slong-
                        mlong)*PI/180.0);
                elong=acos(elong)*180.0/PI;
                phase=(1.0-cos(elong*PI/180.0))/2.0;
                msemidia=(mrad/mdst)*180.0/PI;
                msemidia*=60.0;                 wid=msemidia*phasep;
                gotoxy(52,9);printf("Width     = ");gotoxy(65,9);printf("%7.3lf am",wid);
                gotoxy(52,10);printf("DAZ      = ");gotoxy(65,10);printf("%7.3lf
                        d",sazm-mazm);
                gotoxy(52,11);printf("ARCV     = ");gotoxy(65,11);printf("%7.3lf
                        d",malt-salt);
                qval=((malt-salt)-(11.8371-6.3226*wid+0.7319*wid*wid-
                        0.1018*pow(wid,3)))/10.0;
                oval=(malt-salt)-(7.1651-6.3226*wid+0.7319*wid*wid-
                        0.1018*pow(wid,3));
```

```
gotoxy(52,12);printf("Age       = ");gotoxy(65,12);printf("%7.3lf h",(jde-
        nmjd)*24.0);
gotoxy(52,13);printf("Qvalue    = ");gotoxy(65,13);printf("%7.3lf",qval);
gotoxy(52,14);printf("Visib.    = ");gotoxy(65,14);cout<<"      ";
gotoxy(68,14);
if(qval>0.216)
        cout<<"EV";
else
        if(qval>-0.014)
                cout<<"VUPC";
        else
                if(qval>-0.232)
                        cout<<"MNOATFC";
                else
                        if(qval>-0.293)
                                cout<<"VWOAO";
                        else
                                cout<<"CI";
gotoxy(52,15);printf("Lag       = ");
if(ltms-ltss<0.0)cout<<"-";convert_hms(fabs(ltms-ltss));
gotoxy(52,16);printf("Rday      = ");gotoxy(65,16);printf("%.5lf",rday);
gotoxy(52,17);printf("Rday1     = ");gotoxy(65,17);printf("%.5lf",rday1);
gotoxy(52,18);printf("Rsky      = ");gotoxy(65,18);printf("%.5lf",rsky);
gotoxy(52,19);printf("S-value   = ");gotoxy(65,19);printf("%.5lf",rsky-
        rday);
gotoxy(52,20);printf("philem    =
                ");gotoxy(65,20);printf("%.5lf",philemda);
gotoxy(52,21);printf("angle hor =
        ");gotoxy(65,21);printf("%.5lf",angle_hor);
gotoxy(52,23);printf("Lim Mag   =
        ");gotoxy(65,23);printf("%.2lf",limmagnit());
gotoxy(52,22);printf("Moons Mag =
        ");gotoxy(65,22);printf("%.2lf",mnmag);
gotoxy(52,24);printf("Rel Hum   =
        ");gotoxy(65,24);printf("%.2lf",phum);
gotoxy(52,25);printf("Temperature=
        ");gotoxy(65,25);printf("%.2lf",ptemp);
gotoxy(5,28);printf("sec(s/r)  min(m/n) hour(h/l) humid(a/b) temp(c/d)
                day(e/f) print(p) range(q)");
nexts=getch();
switch(nexts)
{
        case 's':       {       inc_sec();      break; }
        case 'r':       {       dec_sec();      break; }
        case 'm':       {       inc_min();      break; }
        case 'n':       {       dec_min();      break; }
```

```
                case 'h':       {       inc_hour();     break; }
                case 'l':       {       dec_hour();     break; }
                case 'a':       {       phum++;         break; }
                case 'b':       {       phum--;         break; }
                case 'c':       {       ptemp++;        break; }
                case 'd':       {       ptemp--;        break; }
                case 'e':       {       inc_day();      break; }
                case 'f':       {       dec_day();      break; }
                case 'p':       {       fileout1();     break; }
                case 'q':       {       time_range();   break; }
        }}
        while(nexts!='\r');
        gotoxy(50,29);cout<<"New Calculation?";
        gotoxy(77,29);nexts=getch(); clrscr();
        }
        while(nexts!='\r');
        gotoxy(10,10);cout<<"Allah Hafiz"; getch();
}
void inputdatetime(void)
{       gotoxy(20,2);cout<< "Enter Local Time & Date";
        gotoxy(10,4);cout<< "Observation No.            : ";cin>>ocnt;
        gotoxy(10,6);cout << "Day                   : ";cin >>date;
        gotoxy(10,8);cout << "Month                 : ";cin >>month;
        gotoxy(10,10);cout << "Year                 : ";cin >>year;
        gotoxy(10,12);cout << "Long.(+ for East)    : ";cin >>plong;
        gotoxy(10,14);cout << "Lat. (+ for North)   : ";cin >>plat;
        gotoxy(10,16);cout<< "Altitude above Sea    : ";cin >>palt;
        gotoxy(10,18);cout<< "Estim Temperature     : ";cin >>ptemp;
        gotoxy(10,20);cout<< "Estim Humidity              : ";cin >>phum;
        press=1010.0; swr=0.0;
}
void first_calculation()
{
        if(leapcheck(int(year)))
                monthdays[1]=29.0;
        else    monthdays[1]=28.0;
        ldate=date;lmonth=month;lyear=year;
        zontim=plong/15.0;
        if((zontim-long(zontim))>0.5)
                zontim=double(long(zontim+1.0));
        else
                zontim=double(long(zontim));
        lhour=(hour+(min+sec/60.0)/60.0)-zontim;
        if(lhour>=24.0)
        {       lhour=lhour-24.0;
                ldate=ldate+1.0;
```

```
			if(ldate>monthdays[int(lmonth-1.0)])
			{	ldate=1.0;
				lmonth=lmonth+1.0;
				if(lmonth>12.0)
				{	lmonth=1.0;
					lyear=lyear+1.0;
				}}}
		if(lhour<0.0)
		{	lhour+=24.0;
			ldate=ldate-1.0;
			if(ldate<1.0)
			{
				lmonth=lmonth-1.0;
				if(lmonth<1.0)
				{	lmonth=12.0;
					lyear=lyear-1.0;
				}
				ldate=monthdays[int(lmonth-1.0)];
			}	}
		ttdate=ldate;ttmonth=lmonth;ttyear=lyear;
		tthour=lhour+deltat[int(lyear-1620.0)]/3600.0;
		if(tthour>=24.0)
			{	tthour=tthour-24.0;
				ttdate=ttdate+1.0;
				if(ttdate>monthdays[int(ttmonth)])
				{	ttdate=1.0;
					ttmonth=ttmonth+1.0;
					if(ttmonth>12.0)
					{	ttmonth=1.0;
						ttyear=ttyear+1.0;
					}}}
		frac=lhour/24.0;
}
void month_change()
{
		dt_new_moon(date,month,year);
		gotoxy(5,1);cout<<"New Moon ";printf("JD = %lf",nmjd);
		printf(" or %d/%d/%d",int(nmdate),int(nmmonth),int(nmyear));
		nmdate=double(long(nmdate));
		nmmonth=double(long(nmmonth));
		nmyear=double(long(nmyear));
		printf(" at %d:%d:%d (TD)",int(nmhour),int(nmmin),int(nmsec));
		printf(" or %d:%d:%d (UT)",int(uhour),int(umin),int(usec));
		frac=(uhour+(umin+usec/60.0)/60.0)/24.0+deltat[int(nmyear-1620.0)]/86400.0;
		first_calculation();	setting_rt();
		hour=btim;
```

```
        if(hour>=24.0)          hour-=24.0;
        if(hour<0.0)            hour+=24.0;
        min=(hour-double(long(hour)))*60.0;
        sec=(min-double(long(min)))*60.0;
        min=double(long(min));
        sec=double(long(sec));
        hour=double(long(hour));
}
void setting_rt(void)
{
        sun_set();      moon_set();
        btim=ltss+4.0*(ltms-ltss)/9.0;
        gotoxy(2,5);cout<<"(LT) of Sun Set : ";convert_hms(ltss);
        gotoxy(52,5);cout<<", Best Time : ";convert_hms(btim);
        gotoxy(30,5);cout<<", Moon Set : ";convert_hms(ltms);
        gotoxy(2,6);cout<<"(LT) of Sun Rise : ";convert_hms(ltsr);
        gotoxy(30,6);cout<<", Moon Rise : ";convert_hms(ltmr);
}
void outinfo(void)
{
        gotoxy(2,3);cout<<"Lat. ";
        gotoxy(8,3);convert_dms(fabs(plat));
        if(plat<0.0)    printf("S");    else            printf("N");
        gotoxy(20,3);cout<<"Long. ";gotoxy(27,3);convert_dms(fabs(plong));
        if(plong>0.0)           printf("E");    else            printf("W");
        gotoxy(40,3);cout<<"TZone ";
        gotoxy(48,3);
        if(zontim<0.0)          cout<<"-";      else            cout<<"+";
        gotoxy(49,3);printf("%2.0lf",fabs(zontim));
        gotoxy(2,4);printf("LMT %2d:%2d:%2d",int(hour),int(min),int(sec));
        gotoxy(15,4);printf("(%2d/%2d/%4d) UT",int(date),int(month),int(year));
        gotoxy(31,4);convert_hms(lhour);
        gotoxy(42,4);printf(" (%2d/%2d/%4d), JD = ",int(ldate),int(lmonth),int(lyear));
        gotoxy(60,4);printf("%.6lf",jde);
        gotoxy(65,3);cout<<"TD: ";convert_hms(tthour);
        gotoxy(22,8);cout<<"S u n";gotoxy(40,8);cout<<"M o o n";
        gotoxy(2,10);cout<<"Geoc. Long.";
        gotoxy(2,11);cout<<"Geoc. Latit.";
        gotoxy(2,12);cout<<"Geoc. Dist.";
        gotoxy(2,14);cout<<"Geoc. RA:";
        gotoxy(2,15);cout<<"Geoc. DELT";
        gotoxy(2,16);cout<<"Hour Angle:";
        gotoxy(2,18);cout<<"Top. RA:";
        gotoxy(2,19);cout<<"Top. DELt:";
        gotoxy(2,20);cout<<"Top. HA:";
        gotoxy(2,22);cout<<"Altitude:";
```

```
        gotoxy(2,23);cout<<"Azimut:";
        gotoxy(2,24);cout<<"Semi-Dia:";
}
int leapcheck(int n)
{       int k=0;
        if((n%400)!=0)
        {       if((n%100)==0)          k=0;
                else if((n%4)==0) k=1;
        }
        else k=1;
        return(k);
}
double juliandate(double yr,double mn,double de,double fr)
{
        double a,b,jd;
        if(mn<3){       yr=yr-1;mn=mn+12;  }
        a=(double (long (yr/100.0)));
        b=2-a+(double (long (a/4.0)));
        jd=(double (long (365.25*(yr+4716.0))))+(double (long
                (30.6001*(mn+1.0))))+de+b-1524.5+fr;
        return jd;
}
void nutation(void)
{
        t=(jde-2451545.0)/36525.0;
        epsilon=23.0+(26.0+21.448/60.0)/60.0-(46.815/3600.0)*t-
                (0.00059/3600.0)*t*t*+(0.001813/3600.0)*t*t*t;
        elong=297.85036+445267.11148*t-0.0019142*t*t+t*t*t/189474.0;
        sanom=357.52772+35999.05034*t-0.0001603*t*t-t*t*t/300000.0;
        manom=134.96298+477198.867398*t+0.0086972*t*t+t*t*t/56250.0;
        marg=93.27191+483202.017538*t-0.0036825*t*t+t*t*t/327270.0;
        lnode=125.04452-1934.136261*t+0.0020708*t*t+t*t*t/450000.0;
        deltasi=0.0;
        deltaepsi=0.0;
        for(i=0;i<63;i++)
        {
                deltasi=deltasi+(nut_obl[i][5]+nut_obl[i][6]*t)*sin((nut_obl[i][0]*elong+
                        nut_obl[i][1]*sanom+nut_obl[i][2]*manom+nut_obl[i][3]*marg+
                        nut_obl[i][4]*lnode)*PI/180.0);
                deltaepsi+=((nut_obl[i][7]+nut_obl[i][8]*nt)*cos((nut_obl[i][0]*elong+
                        nut_obl[i][1]*sanom+nut_obl[i][2]*manom+nut_obl[i][3]*marg+
                        nut_obl[i][4]*lnode)*PI/180.0));
        }
        deltasi*=(1.0/(10000.0*3600.0));
        deltaepsi*=(1.0/(10000.0*3600.0));
        epsilon=epsilon+deltaepsi;
```

```
}
void sid_time(double)
{
	gmstzero=6.0+(41.0+50.54841/60.0)/60.0+(8640184.812866*st+0.093104*st*st-
		0.0000062*st*st*st)/3600.0;
	gastzero=gmstzero+deltasi*cos(epsilon*PI/180.0)/15.0;
	gmstzero=((gmstzero/24.0)-double(long(gmstzero/24.0)))*24.0;
	gastzero=((gastzero/24.0)-double(long(gastzero/24.0)))*24.0;
	if(gmstzero<0.0)		gmstzero+=24.0;
	if(gmstzero>24.0)		gmstzero-=24.0;
	if(gastzero<0.0)		gastzero+=24.0;
	if(gastzero>0.0)		gastzero-=24.0;
	gmstcurr=gmstzero+frac*24.0*1.00273790935;
	gastcurr=gastzero+frac*24.0*1.00273790935;
	if(gmstcurr>24.0)		gmstcurr-=24.0;
	if(gmstcurr<0.0)		gmstcurr+=24.0;
	if(gastcurr>24.0)		gastcurr-=24.0;
	if(gastcurr<0.0)		gastcurr+=24.0;
	lmstcurr=gmstcurr+plong/15.0;
	lastcurr=gastcurr+plong/15.0;
	if(lastcurr>24.0)		lastcurr-=24.0;
	if(lmstcurr>24.0)		lmstcurr-=24.0;
	if(lastcurr<0.0)		lastcurr+=24.0;
	if(lmstcurr<0.0)		lmstcurr+=24.0;
}
void convert_dms(double coord)
{
	cdeg=double(long(coord));
	cmin=(coord-cdeg)*60.0;
	csec=(cmin-double(long(cmin)))*60.0;
	printf("%3do%2dm%2ds",int(cdeg),int(cmin),int(csec));
}
void convert_hms(double coord)
{
	cdeg=double(long(coord));
	cmin=(coord-cdeg)*60.0;
	csec=(cmin-double(long(cmin)))*60.0;
	printf("%2dh%2dm%2ds",int(cdeg),int(cmin),int(csec));
}
void sun_coord(double)
{
	fptr=fopen("vsopeart.txt","r");
	//	LONGITUDE OF EARTH
	for(i=0;i<6;i++)
	{	temp=0.0;
		for(j=0;j<lngf[i];j++)
```

```
		{	fscanf(fptr,"%lf %lf %lf",&ax,&bx,&c);
			  temp=temp+ax*cos(bx+c*t);
		}
		slong+=(temp*pow(t,i));
}
slong=slong/100000000.0;
slong=slong*180.0/PI;
slong=(slong/360.0-double(long(slong/360.0)));
slong*=360.0;slong+=180.0;
if(slong<0.0)
		slong+=360.0;
if(slong>360.0)
		slong=slong-360.0;
//		Latitude OF EARTH
for(i=0;i<2;i++)
{		temp=0.0;
		for(j=0;j<latf[i];j++)
		{	fscanf(fptr,"%lf %lf %lf",&ax,&bx,&c);
			  temp=temp+ax*cos(bx+c*t);
		}
		slat+=(temp*pow(t,i));
}
slat=-slat/100000000.0;
slat*=(180.0/PI);
for(i=0;i<5;i++)
{		temp=0.0;
		for(j=0;j<dstf[i];j++)
		{	fscanf(fptr,"%lf %lf %lf",&ax,&bx,&c);
			  temp=temp+ax*cos(bx+c*t);
		}
		sdst=sdst+temp*pow(t,i);
}
dst=sdst;
fclose(fptr);
sdst=sdst/100000000.0;
dst=sdst*149597870.0;
slong=slong+deltasi-(20.4898/sdst)/3600.0;
salpha=sin(slong*PI/180.0)*cos(epsilon*PI/180.0);
salpha=salpha-tan(slat*PI/180.0)*sin(epsilon*PI/180.0);
salpha=salpha/(cos(slong*PI/180.0));
salpha=atan(salpha)*180.0/PI;
if((slong>90.0)&&(slong<270.0))			salpha+=180.0;
if(slong>270.0)				salpha+=360.0;
salpha=salpha/15.0;
if(salpha>24.0)				salpha=salpha-24.0;
sha=lastcurr-salpha;
```

```
        if(sha<0.0)              sha+=24.0;
        sdelta=sin(slat*PI/180.0)*cos(epsilon*PI/180.0);
        sdelta=sdelta+cos(slat*PI/180.0)*sin(epsilon*PI/180.0)*sin(slong*PI/180.0);
        sdelta=asin(sdelta)*180.0/PI;
        paralx(sha,sdelta);
        salphap=salpha+deltaalpha;
        sha=sha-deltaalpha;
        sdeltap=deltap;
        salt=sin(plat*PI/180.0)*sin(sdeltap*PI/180.0)+cos(plat*PI/180.0)*cos(sdeltap*PI/
                180.0)*cos(sha*15.0*PI/180.0);
        salt=asin(salt)*180.0/PI;
        if(swr==1)
        {
        srefr=(1.02/(tan((salt+10.3/(salt+5.11))*PI/180.0)))*(283.0/(273.0+tempr))*(press
                    /1010.0);
                srefr=srefr/60.0;
                seeta=sin(plat*PI/180.0)-sin(sdeltap*PI/180.0)*sin(salt*PI/180.0);
                seeta=seeta/(cos(sdeltap*PI/180.0)*cos(salt*PI/180.0));
                seeta=acos(seeta)*180.0/PI;
                sdeltar=sdeltap+srefr*cos(seeta*PI/180.0);
                shar=sha-(srefr*sin(seeta*PI/180.0)/cos(sdeltar*PI/180.0))/15.0;
        }
        else
        {       sdeltar=sdelta;         shar=sha;       }
        salt=sin(plat*PI/180.0)*sin(sdeltar*PI/180.0)+cos(plat*PI/180.0)*cos(sdeltar*PI/
                180.0)*cos(shar*15.0*PI/180.0);
        salt=asin(salt)*180.0/PI;
        salt+=(acos(emaj/(emaj+hite/1000.0))*180.0/PI)/15.0;
        sazm=sin(sdeltar*PI/180.0)-sin(plat*PI/180.0)*sin(salt*PI/180.0);
        sazm=sazm/(cos(plat*PI/180.0)*cos(salt*PI/180.0));
        sazm=acos(sazm)*180.0/PI;
        if(shar==0.0)           sazm=180.0;
        else            if(shar==12.0)          sazm=360.0;
                else    if(shar<12.0)           sazm=360.0-sazm;
        ssemidia=(srad/dst)*180.0/PI;
        ssemidia*=60.0;
}
void moon_coord(double)
{
        fptr2=fopen("elp2000.txt","r");
        sumv=0.0;sumvp=0.0;sumvpp=0.0;sumvppp=0.0;
        for(i=0;i<218;i++)
        {       fscanf(fptr2,"%lf %lf %lf %lf %lf
                        %lf",&v,&alp0,&alp1,&alp2,&alp3,&alp4);
                sumv=sumv+v*sin((alp0+alp1*t+alp2*t*t/10000.0
                        +alp3*pow(t,3)/1000000.0
```

```
            +alp3*pow(t,4)/100000000.0)*PI/180.0);
}
for(i=0;i<244;i++)
{       fscanf(fptr2,"%lf %lf %lf",&v,&alp0,&alp1);
        sumvp=sumvp+v*sin((alp0+alp1*t)*PI/180.0);
}
for(i=0;i<154;i++)
{     fscanf(fptr2,"%lf %lf %lf",&v,&alp0,&alp1);
        sumvpp=sumvpp+v*sin((alp0+alp1*t)*PI/180.0);
}
for(i=0;i<25;i++)
{       fscanf(fptr2,"%lf %lf %lf",&v,&alp0,&alp1);
        sumvppp=sumvppp+v*sin((alp0+alp1*t)*PI/180.0);
}
mlong=218.31665+481267.88134*t-13.268*t*t/10000.0
        +1.856*pow(t,3)/1000000.0-1.534*pow(t,4)/100000000.0
        +sumv+(sumvp+sumvpp*t+sumvppp*t*t/10000.0)/1000.0;
mlong=(mlong/360.0-(long (mlong/360.0)))*360.0;
if(mlong<0)
        mlong+=360;
sumu=0.0;sumup=0.0;sumupp=0.0;sumuppp=0.0;
sumr=0.0;sumrp=0.0;sumrpp=0.0;sumrppp=0.0;
for(i=0;i<188;i++)
{     fscanf(fptr2,"%lf %lf %lf %lf %lf
            %lf",&v,&alp0,&alp1,&alp2,&alp3,&alp4);
sumu=sumu+v*sin((alp0+alp1*t+alp2*t*t/10000.0+alp3*pow(t,3)/1000000.0
            +alp3*pow(t,4)/100000000.0)*PI/180.0);
}
for(i=0;i<64;i++)
{     fscanf(fptr2,"%lf %lf %lf",&v,&alp0,&alp1);
        sumup=sumup+v*sin((alp0+alp1*t)*PI/180.0);
}
for(i=0;i<64;i++)
{     fscanf(fptr2,"%lf %lf %lf",&v,&alp0,&alp1);
        sumupp=sumupp+v*sin((alp0+alp1*t)*PI/180.0);
}
for(i=0;i<12;i++)
{     fscanf(fptr2,"%lf %lf %lf",&v,&alp0,&alp1);
        sumuppp=sumuppp+v*sin((alp0+alp1*t)*PI/180.0);
}
mlat=sumu+(sumup+sumupp*t+sumuppp*t*t/10000.0)/1000.0;
for(i=0;i<154;i++)
{     fscanf(fptr2,"%lf %lf %lf %lf %lf
        %lf",&v,&alp0,&alp1,&alp2,&alp3,&alp4);
sumr=sumr+v*cos((alp0+alp1*t+alp2*t*t/10000.0+alp3*pow(t,3)/1000000.0
            +alp3*pow(t,4)/100000000.0)*PI/180.0);
```

```
}
for(i=0;i<114;i++)
{     fscanf(fptr2,"%lf %lf %lf",&v,&alp0,&alp1);
        sumrp=sumrp+v*cos((alp0+alp1*t)*PI/180.0);
}
for(i=0;i<68;i++)
{     fscanf(fptr2,"%lf %lf %lf",&v,&alp0,&alp1);
        sumrpp=sumrpp+v*cos((alp0+alp1*t)*PI/180.0);
}
for(i=0;i<9;i++)
{     fscanf(fptr2,"%lf %lf %lf",&v,&alp0,&alp1);
        sumrppp=sumrppp+v*cos((alp0+alp1*t)*PI/180.0);
}
fclose(fptr2);
mdst=385000.57+sumr+sumrp+sumrpp*t+sumrppp*t*t/10000.0;
mlat=(mlat/360.0-(long (mlat/360.0)))*360.0;
mlong+=(-0.00019524-0.00001059*sin((225.0+477198.9*t)*PI/180.0));
mlat+=(-0.00001754*sin((183.3+483202.0*t)*PI/180.0));
mdst+=(0.0708*cos((225.0+477198.9*t)*PI/180.0));
mlong+=deltasi;
malpha=sin(mlong*PI/180.0)*cos(epsilon*PI/180.0);
malpha=malpha-tan(mlat*PI/180.0)*sin(epsilon*PI/180.0);
malpha=malpha/(cos(mlong*PI/180.0));
malpha=atan(malpha)*180.0/PI;
if((mlong>90.0)&&(mlong<270.0))            malpha+=180.0;
if(mlong>270.0)                    malpha+=360.0;
malpha=malpha/15.0;
if(malpha>24.0)                    malpha=malpha-24.0;
mha=lastcurr-malpha;
if(mha>24.0)            mha-=24.0;
if(mha<0.0)             mha+=24.0;
mdelta=sin(mlat*PI/180.0)*cos(epsilon*PI/180.0);
mdelta=mdelta+cos(mlat*PI/180.0)*sin(epsilon*PI/180.0)*sin(mlong*PI/180.0);
mdelta=asin(mdelta)*180.0/PI;
paralx(mha,mdelta);
malphap=malpha+deltaalpha;
mdeltap=deltap;
mha=mha-deltaalpha;
malt=sin(plat*PI/180.0)*sin(mdeltap*PI/180.0)+cos(plat*PI/180.0)*cos(mdeltap*
            PI/180.0)*cos(mha*15.0*PI/180.0);
malt=asin(malt)*180.0/PI;
if(swr==1)
{
mrefr=(1.02/(tan((malt+10.3/(malt+5.11))*PI/180.0)))*(283.0/(273.0+tempr))*(pr
        ess/1010.0);
        mrefr=mrefr/60.0;
```

```
        meeta=sin(plat*PI/180.0)-sin(mdeltap*PI/180.0)*sin(malt*PI/180.0);
        meeta=meeta/(cos(mdelta*PI/180.0)*cos(malt*PI/180.0));
        if(fabs(meeta)<=1.0)
        {     gotoxy(45,20);cout<<"                    ";
                meeta=acos(meeta)*180.0/PI;
                mdeltar=mdeltap+mrefr*cos(meeta*PI/180.0);
                mhar=mha-
                (mrefr*sin(meeta*PI/180.0)/cos(mdeltar*PI/180.0))/15.0;

malt=sin(plat*PI/180.0)*sin(mdeltar*PI/180.0)+cos(plat*PI/180.0)*cos(mdeltar*
                PI/180.0)*cos(mhar*15.0*PI/180.0);
                malt=asin(malt)*180.0/PI;
                malt+=(acos(emaj/(emaj+hite/1000.0))*180.0/PI);

        }
        else
                cout<<"eeta greater than 1";
}
else
{       mdeltar=mdeltap;
        mhar=mha;
}
mazm=sin(mdeltar*PI/180.0)-sin(plat*PI/180.0)*sin(malt*PI/180.0);
mazm=mazm/(cos(plat*PI/180.0)*cos(malt*PI/180.0));
mazm=acos(mazm)*180.0/PI;
if(mha==0.0)
        mazm=180.0;
else
        if(mha==12.0)
                mazm=360.0;
        else
                if(mha<12.0)
                        mazm=360.0-mazm;
msemidia=(mrad/mdst)*180.0/PI;
msemidia*=60.0;
echh=acos(cos(slong*PI/180.0)*sin(epsilon*PI/180.0)/(pow(1.0-
        pow(sin(epsilon*PI/180.0)*sin(slong*PI/180.0),2.0),0.5)));
echh=echh*180.0/PI;
iii=sin(plat*PI/180.0);
iii=iii/(pow(1.0-pow(sin(epsilon*PI/180.0)*sin(slong*PI/180.0),2.0),0.5));
iii=acos(iii)*180.0/PI;
ell=echh-iii;
philemda=echh-iii;
angle_hor=90.0-philemda;
rday=(25.5-msemidia/2.2)*cos(plat*PI/180.0)/cos(philemda*PI/180.0);
rday1=10.5*cos(plat*PI/180.0)/cos(philemda*PI/180.0);
```

```
        if((mlong<90.0)&&(slong>300.0))
                delong=mlong+360.0-slong;
        else
                delong=mlong-slong;
        rsky=mlat*tan(philemda*PI/180.0)+delong;
}
double modfunc(double xy)
{       xy=xy/360;
        xy=(xy-(double (long(xy))))*360;
        if(xy<0)                xy=360+xy;
        return xy;
}
void paralx(double hh,double dd)
{     double xx,yy,hp;
        uu=atan(emin*tan(plat*PI/180.0)/emaj)*180.0/PI;
        hh*=15.0;
        platp=plat-(692.73*sin(2.0*plat*PI/180.0)-1.16*sin(4.0*plat*PI/180.0))/3600.0;
        xx=emin*sin(uu*PI/180.0)/emaj+hite*sin(plat*PI/180.0)/cons;
        yy=cos(uu*PI/180.0)+hite*cos(plat*PI/180.0)/cons;
        rho=pow(xx*xx+yy*yy,0.5)*emaj;
        pie=rho/dst;
        deltaalpha=atan((-pie*cos(plat*PI/180.0)*sin(hh*PI/180.0))/
                    (cos(dd*PI/180.0)-pie*cos(platp*PI/180.0)*cos(hh*PI/180.0)));
        deltaalpha*=(180.0/PI);
        hp=hh-deltaalpha;
        deltaalpha/=15.0;
        deltap=atan((cos(hp*PI/180.0)*(sin(dd*PI/180.0)-pie*sin(platp*PI/180.0)))/
                    (cos(dd*PI/180.0)*cos(hh*PI/180.0)-pie*cos(platp*PI/180.0)));
        deltap*=(180.0/PI);
        hp/=15.0;
}
void dt_new_moon(double,double,double)
{
        yy=year+(month-1.0+date/30.0)/12.0;
        kay=(yy-2000.0)*12.3685;
        if(kay<0.0)
                kay=(double (long (kay)))-1.0;
        else    kay=(double (long (kay)));
        tee=kay/1236.85;
        nmjd=2451550.09766+29.530588861*kay+0.00015437*tee*tee-
                0.00000015*tee*tee*tee+0.00000000073*tee*tee*tee*tee;
        msun=modfunc(2.5534+29.1053567*kay-0.0000014*tee*tee-
                0.00000011*tee*tee*tee);
        mmoon=modfunc(201.5643+385.81693528*kay+0.0107582*tee*tee+0.00001238
                *tee*tee*tee-0.000000058*tee*tee*tee*tee);
```

```
farg=modfunc(160.7108+390.67050284*kay-0.0016118*tee*tee-
        0.00000227*pow(tee,3)+0.000000011*pow(tee,4));
omeg=modfunc(124.7746-
        1.56375588*kay+0.0020672*tee*tee+0.00000215*pow(tee,3));
eee=1.0-0.002516*tee-0.0000074*tee*tee;
msun*=(PI/180.0);mmoon*=(PI/180.0);farg*=(PI/180.0);omeg*=(PI/180.0);
per=-0.4072*sin(mmoon)+0.17241*eee*sin(msun)+0.01608*sin(2.0*mmoon);
per+=(0.01039*sin(2.0*farg)+0.00739*eee*sin(mmoon-msun)-
        0.00514*eee*sin(msun+mmoon));
per+=(0.00208*pow(eee,2.0)*sin(2.0*msun)-0.00111*sin(mmoon-2.0*farg)-
        0.00057*sin(mmoon+2.0*farg));
per+=(0.00056*eee*sin(2.0*mmoon+msun)-
        0.00042*sin(3.0*mmoon)+0.00042*eee*sin(msun+2.0*farg));
per+=(0.00038*eee*sin(msun-2.0*farg)-0.00024*eee*sin(2.0*mmoon-msun)-
        0.00017*sin(omeg));
per+=(-0.00007*sin(mmoon+2.0*msun)+0.00004*sin(2.0*mmoon-
        2.0*farg)+0.00004*sin(3.0*msun));
per+=(0.00003*sin(mmoon+msun-
        2.0*farg)+0.00003*sin(2.0*mmoon+2.0*farg));
per+=(-0.00003*sin(mmoon+msun+2.0*farg)+0.00003*sin(mmoon-
        msun+2.0*farg));
per+=(-0.00002*sin(mmoon-msun-2.0*farg)-
        0.00002*sin(3.0*mmoon+msun)+0.00002*sin(4.0*mmoon));
a1=(299.77+0.107408*kay-0.009173*tee*tee);a1*=(PI/180.0);
a2=(251.88+0.016321*kay);a2*=(PI/180.0);
a3=(251.83+26.651886*kay);a3*=(PI/180.0);
a4=(349.42+36.412478*kay);a4*=(PI/180.0);
a5=(84.66+18.206239*kay);a5*=(PI/180.0);
a6=(141.74+53.303771*kay);a6*=(PI/180.0);
a7=(207.14+2.453732*kay);a7*=(PI/180.0);
a8=(154.84+7.306860*kay);a8*=(PI/180.0);
a9=(34.52+27.261239*kay);a9*=(PI/180.0);
a10=(207.19+0.121824*kay);a10*=(PI/180.0);
a11=(291.34+1.844379*kay);a11*=(PI/180.0);
a12=(161.72+24.198154*kay);a12*=(PI/180.0);
a13=(239.56+25.513099*kay);a13*=(PI/180.0);
a14=(331.55+3.592518*kay);a14*=(PI/180.0);
addcr=0.000325*sin(a1);     addcr+=0.000165*sin(a2);
addcr+=0.000164*sin(a3);    addcr+=0.000126*sin(a4);
addcr+=0.000110*sin(a5);    addcr+=0.000062*sin(a6);
addcr+=0.000060*sin(a7);    addcr+=0.000056*sin(a8);
addcr+=0.000047*sin(a9);    addcr+=0.000042*sin(a10);
addcr+=0.000040*sin(a11);   addcr+=0.000037*sin(a12);
addcr+=0.000035*sin(a13);   addcr+=0.000023*sin(a14);
nmjd=nmjd+per+addcr;        z=(double(long(nmjd+0.5)));
ff=nmjd-z+0.5;
```

```
        if(z<2299161.0)      aa=z;  else
        {       alph=(double (long((z-1867216.25)/36524.25)));
                aa=z+1.0+alph-(double (long (alph/4.0)));
        }
        bb=aa+1524.0;
        cc=(double (long ((bb-122.1)/365.25)));
        dd=(double (long (365.25*cc)));
        ee=(double (long ((bb-dd)/30.6001)));
        nmdate=bb-dd-(double (long (30.6001*ee)))+ff;
        if(ee<14.0)   nmmonth=ee-1.0;    else   nmmonth=ee-13.0;
        if(nmmonth>2.0)     nmyear=cc-4716.0;   else   nmyear=cc-4715.0;
        nmhour=(nmdate-double(long(nmdate)))*24.0;
        uhour=nmhour-deltat[int(nmyear-1620.0)]/3600.0;
        nmmin=(nmhour-double(long(nmhour)))*60.0;
        nmsec=(nmmin-double(long(nmmin)))*60.0;
        umin=(uhour-double(long(uhour)))*60.0;
        usec=(umin-double(long(umin)))*60.0;
}
void inc_sec(void)
{
        sec++;
        if(sec>=60.0)
        {     sec=0.0;
                min++;
                if(min>=60.0)
                {       min=0.0;
                        hour++;
                        if(hour>=24.0)
                        {     hour=0.0;
                                date++;
                                if(date>monthdays[int(month-1.0)])
                                {       date=1.0;
                                        month++;
                                        if(month>=12.0)
                                        {       month=1.0;
                                                year++;
                                        }}}}}
        setting_rt();
}
void inc_min(void)
{     min++;
        if(min>=60.0)
        {       min=0.0;
                hour++;
                if(hour>=24.0)
                {     hour=0.0;
```

```
                    date++;
                    if(date>monthdays[int(month-1.0)])
                    {       date=1.0;
                            month++;
                            if(month>=12.0)
                            {       month=1.0;
                                    year++;
                            }}}}
        setting_rt();
}
void inc_hour(void)
{
        hour++;
        if(hour>=24.0)
        {     hour=0.0;
                date++;
                if(date>monthdays[int(month-1.0)])
                {       date=1.0;
                        month++;
                        if(month>=12.0)
                        {       month=1.0;
                                year++;
                        }}}
        setting_rt();
}
void inc_day(void)
{
        date++;
        if(date>monthdays[int(month-1.0)])
        {       date=1.0;
                month++;
                if(month>=12.0)
                {       month=1.0;
                        year++;
                }}
        if(sel!='d')
                setting_rt();
}
void inc_mon(void)
{
        month++;
        if(month>=12.0)
        {       month=1.0;
                year++;
        }
        month_change();
```

```
        if(sel!='d')
                setting_rt();
}

void dec_sec(void)
{       sec--;
        if(sec<1.0)
        {       sec=59.0;
                min--;
                if(min<1.0)
                {       min=59.0;
                        hour--;
                        if(hour<1.0)
                        {       hour=23.0;
                                date--;
                                if(date<1.0)
                                {
                                        month--;
                                        if(month<1.0)
                                        {       month=12.0;
                                                year--;
                                        }
                                        date=monthdays[int(month-1.0)];
                                }}}}
        setting_rt();
}
void dec_min(void)
{
        min--;
        if(min<1.0)
        {       min=59.0;
                hour--;
                if(hour<1.0)
                {       hour=23.0;
                        date--;
                        if(date<1.0)
                        {
                                month--;
                                if(month<1.0)
                                {       month=12.0;
                                        year--;
                                }
                                date=monthdays[int(month-1.0)];
                        }}}
        setting_rt();
}
```

```
void dec_hour(void)
{
        hour--;
        if(hour<1.0)
        {     hour=23.0;
              date--;
              if(date<1.0)
              {
                      month--;
                      if(month<1.0)
                      {       month=12.0;
                              year--;
                      }
                      date=monthdays[int(month-1.0)];
              }}
        setting_rt();
}
void dec_day(void)
{
        date--;

        if(date<1.0)
        {
                month--;
                if(month<1.0)
                {       month=12.0;
                        year--;
                }
                date=monthdays[int(month-1.0)];
        }
        setting_rt();
}

void sun_set(void)
{     frac=0.0;
        for(lcnt=1;lcnt<4;lcnt++)
        {
                jd=juliandate(year,month,date,frac);
                jde=jd+deltat[int(year-1620.0)]/(3600.0*24.0);
                nutation();
                stjd=juliandate(year,month,date,0.0);
                st=(stjd-2451545.0)/36525.0;
                sid_time(st);
                t=(jde-2451545.0)/365250.0;
                sun_coord(t);
```

```
        numr=sin(-(50.0/60.0)*PI/180.0)-
                sin(plat*PI/180.0)*sin(sdeltap*PI/180.0);
        denm=cos(plat*PI/180.0)*cos(sdeltap*PI/180.0);
        if(fabs(numr/denm)<=1.0)
        {       hass=acos(numr/denm)*180.0/PI;
                mnot=salphap-plong/15.0-gmstzero;
                mone=mnot-hass/15.0;
                mtwo=mnot+hass/15.0;
                if(mnot<0.0)  mnot+=24.0;
                if(mnot>24.0) mnot-=24.0;
                if(mone<0.0)  mone+=24.0;
                if(mone>24.0) mone-=24.0;
                if(mtwo<0.0)  mtwo+=24.0;
                if(mtwo>24.0  mtwo-=24.0;
                stss=gmstzero+mtwo*1.00273790935;
                mtwo=mtwo+deltat[int(lyear-1620.0)]/3600.0;
                jd=juliandate(year,month,date,mtwo/24.0);
                t=(jd-2451545.0)/365250.0;
                sun_coord(t);
                caph=stss+plong/15.0-salphap;
                altss=asin(sin(plat*PI/180.0)*sin(sdeltap*PI/180.0)
                        +cos(plat*PI/180.0)*cos(sdeltap*PI/180.0)*
                        cos(caph*15.0*PI/180.0))*180.0/PI;
                deltam=((altss+50.0/60.0)/(cos(plat*PI/180.0)
                        *cos(sdeltap*PI/180.0)*sin(caph*15.0*PI/180.0)))/15.0;
                utss=mtwo+deltam;
        }
        utss=utss-deltat[int(lyear-1620.0)]/3600.0;
        utss+=((acos(emaj/(emaj+hite/1000.0))*180.0/PI)/15.0);
        frac=utss/24.0;
}
ltss=utss+zontim;
if(ltss<0.0)            ltss+=24.0;
if(ltss>24.0)           ltss-=24.0;
frac=0.0;
for(lcnt=1;lcnt<4;lcnt++)
{
        jd=juliandate(year,month,date,frac);
        jde=jd+deltat[int(year-1620.0)]/(3600.0*24.0);
        nutation();
        stjd=juliandate(year,month,date,0.0);
        st=(stjd-2451545.0)/36525.0;
        sid_time(st);
        t=(jde-2451545.0)/365250.0;
        sun_coord(t);
```

```
			numr=sin(-(50.0/60.0)*PI/180.0)-
				sin(plat*PI/180.0)*sin(sdeltap*PI/180.0);
			denm=cos(plat*PI/180.0)*cos(sdeltap*PI/180.0);
			if(fabs(numr/denm)<=1.0)
			{	hass=acos(numr/denm)*180.0/PI;
				mnot=salphap-plong/15.0-gmstzero;
				mone=mnot-hass/15.0;
				mtwo=mnot+hass/15.0;
				if(mnot<0.0)	mnot+=24.0;
				if(mnot>24.0) mnot-=24.0;
				if(mone<0.0)	mone+=24.0;
				if(mone>24.0) mone-=24.0;
				if(mtwo<0.0)	mtwo+=24.0;
				if(mtwo>24.0) mtwo-=24.0;
				stsr=gmstzero+mone*1.00273790935;
				mone=mone+deltat[int(lyear-1620.0)]/3600.0;
				jd=juliandate(year,month,date,mone/24.0);
				t=(jd-2451545.0)/365250.0;
				sun_coord(t);
				caph=stsr+plong/15.0-salphap;
				altsr=asin(sin(plat*PI/180.0)*sin(sdeltap*PI/180.0)
					+cos(plat*PI/180.0)*cos(sdeltap*PI/180.0)
					*cos(caph*15.0*PI/180.0))*180.0/PI;
				deltam=((altsr+50.0/60.0)/(cos(plat*PI/180.0)*
					cos(sdeltap*PI/180.0)*sin(caph*15.0*PI/180.0)))/15.0;
				utsr=mone+deltam;
			}
			utsr=utsr-deltat[int(lyear-1620.0)]/3600.0;
			utsr+=((acos(emaj/(emaj+hite/1000.0))*180.0/PI)/15.0);
			frac=utsr/24.0;
		}
		ltsr=utsr+zontim;
		if(ltsr<0.0)	ltsr+=24.0;
		if(ltsr>24.0)	ltsr-=24.0;
}
void moon_set()
{	frac=0.0;
	for(lcnt=1;lcnt<4;lcnt++)
	{
		jd=juliandate(year,month,date,frac);
		jde=jd+deltat[int(year-1620.0)]/(3600.0*24.0);
		nutation();
		stjd=juliandate(year,month,date,0.0);
		st=(stjd-2451545.0)/36525.0;
		sid_time(st);
		t=(jde-2451545.0)/36525.0;
```

```
        moon_coord(t);
        numr=sin(0.125*PI/180.0)-sin(plat*PI/180.0)*sin(mdeltap*PI/180.0);
        denm=cos(plat*PI/180.0)*cos(mdeltap*PI/180.0);
        if(fabs(numr/denm)<=1.0)
        {       hass=acos(numr/denm)*180.0/PI;
                mnot=malphap-plong/15.0-gmstzero;
                mone=mnot-hass/15.0;
                mtwo=mnot+hass/15.0;
                if(mnot<0.0)  mnot+=24.0;
                if(mnot>24.0) mnot-=24.0;
                if(mone<0.0)  mone+=24.0;
                if(mone>24.0) mone-=24.0;
                if(mtwo<0.0)  mtwo+=24.0;
                if(mtwo>24.0) mtwo-=24.0;
                stss=gmstzero+mtwo*1.00273790935;
                mtwo=mtwo+deltat[int(lyear-1620.0)]/3600.0;
                jd=juliandate(year,month,date,mtwo/24.0);
                t=(jd-2451545.0)/36525.0;
                moon_coord(t);
                caph=stss+plong/15.0-malphap;
                altss=asin(sin(plat*PI/180.0)*sin(mdeltap*PI/180.0)
                        +cos(plat*PI/180.0)*cos(mdeltap*PI/180.0)
                        *cos(caph*15.0*PI/180.0))*180.0/PI;
                deltam=((altss-0.125)/(cos(plat*PI/180.0)
                        *cos(mdeltap*PI/180.0)*sin(caph*15.0*PI/180.0)))/15.0;
                utms=mtwo+deltam;
        }
        utms+=((acos(emaj/(emaj+hite/1000.0))*180.0/PI)/15.0);
        frac=utms/24.0;

}
ltms=utms+zontim;
if(ltms<0.0)   ltms+=24.0;
if(ltms>24.0)  ltms-=24.0;
frac=0.0;
for(lcnt=1;lcnt<4;lcnt++)
{
        jd=juliandate(year,month,date,frac);
        jde=jd+deltat[int(year-1620.0)]/(3600.0*24.0);
        nutation();
        stjd=juliandate(year,month,date,0.0);
        st=(stjd-2451545.0)/36525.0;
        sid_time(st);
        t=(jde-2451545.0)/36525.0;
        moon_coord(t);
        numr=sin(0.125*PI/180.0)-sin(plat*PI/180.0)*sin(mdeltap*PI/180.0);
```

```
                denm=cos(plat*PI/180.0)*cos(mdeltap*PI/180.0);
                if(fabs(numr/denm)<=1.0)
                {       hass=acos(numr/denm)*180.0/PI;
                        mnot=malphap-plong/15.0-gmstzero;
                        mone=mnot-hass/15.0;
                        mtwo=mnot+hass/15.0;
                        if(mnot<0.0)  mnot+=24.0;
                        if(mnot>24.0) mnot-=24.0;
                        if(mone<0.0) mone+=24.0;
                        if(mone>24.0) mone-=24.0;
                        if(mtwo<0.0)  mtwo+=24.0;
                        if(mtwo>24.0) mtwo-=24.0;
                        stmr=gmstzero+mone*1.00273790935;
                        mone=mone+deltat[int(lyear-1620.0)]/3600.0;
                        jd=juliandate(year,month,date,mtwo/24.0);
                        t=(jd-2451545.0)/36525.0;
                        moon_coord(t);
                        caph=stmr+plong/15.0-malphap;
                        altmr=asin(sin(plat*PI/180.0)*sin(mdeltap*PI/180.0)
                                +cos(plat*PI/180.0)*cos(mdeltap*PI/180.0)
                                *cos(caph*15.0*PI/180.0))*180.0/PI;
                        deltam=((altss-0.125)/(cos(plat*PI/180.0)
                                *cos(mdeltap*PI/180.0)*sin(caph*15.0*PI/180.0)))/15.0;
                        utmr=mone+deltam;
                }
                utmr+=((acos(emaj/(emaj+hite/1000.0))*180.0/PI)/15.0);
                frac=utmr/24.0;
        }
        ltmr=utmr+zontim;
        if(ltmr<0.0)   ltmr+=24.0;
        if(ltmr>24.0)  ltmr-=24.0;
}
void display_scoord(void)
{
        if(slat<0) sd='S';      else            sd='N';
        gotoxy(16,10);convert_dms(slong);
        gotoxy(16,11);convert_dms(fabs(slat));
        printf("%c",sd);
        gotoxy(16,12);printf(" %.2lf Km",dst);
        if(sdelta<0.0)          sd='S'; else            sd='N';
        gotoxy(16,14);convert_hms(salpha);
        gotoxy(16,15);convert_dms(fabs(sdelta));
        printf("%c",sd);
        if(sdeltap<0.0)         sd='S'; else            sd='N';
        gotoxy(16,16);convert_hms(sha);
        gotoxy(16,18);convert_hms(salphap);
```

```
    gotoxy(16,19);convert_dms(fabs(sdeltap));
    printf("%c",sd);
    gotoxy(16,20);convert_hms(sha);
    gotoxy(16,23);convert_dms(sazm);
    gotoxy(16,22);convert_dms(fabs(salt));
    if(salt<0.0)            sd='B'; else            sd='A';
    printf("%c",sd);
    gotoxy(16,24);printf("%.4lf arc min",ssemidia/2.0);
}
void display_mcoord(void)
{
    if(mlat<0.0)            sd='S'; else            sd='N';
    gotoxy(36,10);convert_dms(mlong);
    gotoxy(36,11);convert_dms(fabs(mlat));
    printf("%c",sd);
    gotoxy(36,12);printf("%.8lf",mdst/emaj);dst=mdst;
    if(mdelta<0.0)          sd='S'; else            sd='N';
    gotoxy(36,14);convert_hms(malpha);
    gotoxy(36,15);convert_dms(fabs(mdelta));
    printf("%c",sd);
    gotoxy(36,16);convert_hms(mha);
    gotoxy(36,18);convert_hms(malphap);
    if(deltap<0.0)          sd='S'; else            sd='N';
    gotoxy(36,19);convert_dms(fabs(mdeltap));
    printf("%c",sd);
    gotoxy(36,20);convert_hms(fabs(mha));
    gotoxy(36,22);convert_dms(fabs(malt));
    if(malt<0.0)            sd='B'; else            sd='A';
    printf("%c",sd);
    gotoxy(36,23);convert_dms(mazm);
    gotoxy(36,24);printf("%.4lf arc min",msemidia/2.0);
}
void fileout1(void)
{
fprintf(fptr4,"\n%d\t%d/%d/%d\t",ocnt,int(date),int(month),int(year));
fprintf(fptr4,"%5.1lf\t%5.1lf\t%d\t",plat,plong,int(palt));
fprintf(fptr4,"%5.1lf\t%5.1lf\t%d:%d\t",ptemp,phum,int(utss),int((utss-int(utss))*60.0));
fprintf(fptr4,"%15.5lf\t%6.2lf\t",nmjd,(jde-nmjd)*24.0);
fprintf(fptr4,"%6.2lf\t%6.2lf\t",(ltms-ltss)*60.0,elongp);
fprintf(fptr4,"%6.2lf\t%6.2lf\t",malt-salt,sazm-mazm);
fprintf(fptr4,"%8.3lf\t%8.3lf\t%8.3lf\t",wid*60.0,qval,oval);
fprintf(fptr4,"%9.4lf\t%9.4lf\t%9.4lf\t%9.4lf\t%9.5lf\t",philemda,mlat,mlong,slong,mse
midia);
fprintf(fptr4,"%6.2lf\t%8.3lf\t%8.3lf\t%8.3lf\t%8.3lf\t%8.3lf\t",25.5-
msemidia/2.2,rday,rday1,rsky,rsky-rday,rsky-rday1);
fprintf(fptr4,"%8.3lf\t%8.3lf\t",mnmag,lem);
```

```
}
void time_range()
{
fprintf(fptr4,"%d:%d(%8.3lf)\t",int(lhour),int((lhour-int(lhour))*60),mnmag-lem);
}
double limmagnit()
{
	falt=malt+.1;   fazm=mazm+.1;
	zendist=90.0-falt;
	felonmon=(acos(sin(malt*PI/180.0)*sin(falt*PI/180.0)+cos(malt*PI/180.0)*cos(f
		alt*PI/180.0)*cos((mazm-fazm)*PI/180.)))*180.0/PI;
	felonsun=(acos(sin(salt*PI/180.0)*sin(falt*PI/180.0)+cos(salt*PI/180.0)*cos(falt
		*PI/180.0)*cos((sazm-fazm)*PI/180.)))*180.0/PI;
	gacom=1.0/(cos(zendist*PI/180.0)+0.0286*exp(-10.5*cos(zendist*PI/180.0)));
	aacom=1.0/(cos(zendist*PI/180.0)+0.0123*exp(-24.5*cos(zendist*PI/180.0)));
	oacom=pow(1.0-pow(sin(zendist*PI/180.0)/(1.0+20.0/6378.0),2.0),-0.5);
	for(i=0;i<5;i++)
	{
	kr=0.1066*exp(-palt/8200.0)*pow(wasch[i]/0.55,-4.0);
	ka=0.1*pow(wasch[i]/0.55,-1.3)*exp(-palt/1500.0);
	ka=ka*pow(1.0-0.32/log(phum/100.0),1.33)
		*(1.0+0.33*sin(salpha*PI/180.0)*(plat/fabs(plat)));
	ko=ozsch[i]*(3.0+0.4*(plat*(PI/180.0)*cos(salpha*PI/180.0)-
		cos(3.0*plat*PI/180.0)))/3.0;
	kw=wtsch[i]*0.94*(phum/100.0)*exp(ptemp/15.0)*exp(-palt/8200.0);
	ksch[i]=kr+ka+ko+kw;
	dmsch[i]=kr*gacom+ka*aacom+ko*oacom+kw*gacom;
	}
	stposef=1.0/(cos(zendist*PI/180.0)+0.025*exp(-11.0*cos(zendist*PI/180.0)));
	if(malt<=0.0)		mnposef=40.0;
	else
		mnposef=1.0/(cos((90.0-malt)*PI/180.0)+0.025*exp(-11.0*cos((90.0-
			malt)*PI/180.0)));
	if(salt<=0.0)		snposef=40.0;
	else
		snposef=1.0/(cos((90.0-salt)*PI/180.0)+0.025*exp(-11.0*cos((90.0-
			salt)*PI/180.0)));
	for(i=0;i<5;i++)
	{
	nightb=bosch[i]*(1.0+0.3*cos(6.283*(year-1992.0)/11.0));
	nightb=nightb*(0.4+0.6/pow(1.0-0.96*pow(sin(zendist*PI/180.0),2.0),-0.5));
	nightb=nightb*pow(10.0,(-0.4*ksch[i]*stposef));
	mnmag=-12.73+0.026*fabs(180.0-elongp)+4.0*pow((180.0-
		elongp),4.0)*(pow(10.0,-9.0));
	mnmag=mnmag+cmsch[i];
	cthree=pow(10.0,(-0.4*ksch[i]*mnposef));
```

```
fem=pow(10.0,(6.15-felonmon/40.0))+6.2*pow(10.0,7.0)/pow(felonmon,2.0);
fem=fem+pow(10.0,5.36)*(1.06+pow(cos(felonmon*PI/180.0),2.0));
moonb=pow(10.0,(-0.4*(mnmag-mosch[i]+43.27)));
moonb=moonb*(1.0-pow(10.0,(-0.4*ksch[i]*stposef)));
moonb=moonb*(fem*cthree+440000.0*(1.0-cthree));
twilb=pow(10.0,(-0.4*(mssch[i]-mosch[i]+32.5-salt-zendist/(360.0*ksch[i])))));
twilb=twilb*(100.0/felonsun)*(1.0-pow(10.0,-0.4*ksch[i]*stposef));
cfour=pow(10.0,(-0.4*ksch[i]*snposef));
fes=6.2*pow(10.0,7.0)/pow(felonsun,2.0);
fes+=pow(10.0,(6.15-felonsun/40.0));
fes=fes+pow(10.0,5.36)*(1.06+pow(cos(felonsun*PI/180.0),2.0));
dayb=pow(10.0,(-0.4*(mssch[i]-mosch[i]+43.27)));
dayb=dayb*(1.0-pow(10.0,-0.4*ksch[i]*stposef));
dayb=(fes*cfour+440000.0*(1.0-cfour))*dayb;
if(dayb<twilb)          bsch[i]=nightb+dayb;
else             bsch[i]=nightb+twilb;
if(malt>0.0)            bsch[i]=bsch[i]+moonb;
bsch[i]=bsch[i]*pow(10.0,12.0);
}
bel=bsch[2]/0.0011;//*1000.0/1.11;
if(bel<1500.0) {       cone=pow(10.0,-9.8);              ctwo=pow(10.0,-1.9); }
else    {       cone=pow(10.0,-8.350001);                 ctwo=pow(10.0,-5.9); }
teh=cone*pow((1.0+pow(ctwo*bel,0.5)),2.0);
lem=-16.57-2.5*log(teh)/log(10.0)-dmsch[2];
return(lem);
}
```

APPENDIX-II

ANCIENT, MEDIEVAL AND EARLY 20TH CENTURY MODELS

S. No.	Date	Lat	Long.	VISIBILITY N	B	T	Age	Lag	ARCL	ARCV	DAZ	MODEL A	B	C	D	E
565	4/12/2002	30.9	35.8				7.07	9.77	4.04	2.58	3.12	6.48	-8.6	-0.93	-0.84	-0.77
514	12/2/2002	43.9	18.4				8.55	4.83	5.94	1.56	5.73	7.15	-6.7	-1.02	-0.94	-0.84
481	17/9/2001	4.1	73.3				2.72	12.5	5.23	3.73	-3.7	8.36	-7.6	-0.82	-0.7	-0.65
730	10/1/2005	11.2	7.6				5.41	10.6	5.94	3	5.13	8.59	-7.4	-0.88	-0.8	-0.7
345	16/2/1999	33.3	44.4				8.27	16.8	4.47	4.05	1.89	8.67	-4.9	-0.79	-0.69	-0.63
498	15/11/2001	24.3	54.3				7.07	18.4	4.62	4.61	0.37	9.22	-6.5	-0.74	-0.64	-0.58
189	3/1/1984	15.6	35.6				10.2	16.2	5.51	4.21	3.56	9.55	-6.1	-0.77	-0.67	-0.6
566	4/12/2002	10.3	9.8				9.53	18.8	5.42	4.86	2.41	10.1	-5.8	-0.71	-0.61	-0.54
500	15/11/2001	31.9	35.8				8.11	21.1	5.06	4.85	1.45	10.3	-6.2	-0.71	-0.61	-0.55
699	14/10/2004	26.6	50				11.5	17	6.21	4.41	4.37	10.5	-7.6	-0.74	-0.66	-0.57
567	4/12/2002	6.5	3.4				10.1	20.7	5.73	5.31	2.14	10.9	-5.3	-0.67	-0.56	-0.5
700	14/10/2004	30.4	35.5				12.5	17.5	6.7	4.37	5.08	11.1	-7.8	-0.74	-0.66	-0.57
482	17/9/2001	26.6	50				4.41	21.8	5.62	5.5	-1.2	11.1	-6	-0.65	-0.54	-0.48
499	15/11/2001	-34	18.4				10.9	19.9	6.36	4.46	-4.5	11.3	-4.5	-0.74	-0.61	-0.57
659	19/5/2004	29.4	48				10.9	24	5.37	5.36	-0.2	11.4	-4	-0.66	-0.56	-0.5
501	15/11/2001	10.3	9.8				10.4	23.6	6.11	6.11	0.03	12	-4.7	-0.59	-0.49	-0.43
106	27/4/1922	-34	18.5				11.3	23.7	6.13	5.54	-2.6	12.1	-5.5	-0.64	-0.53	-0.47
444	23/2/2001	-34	18.4				9.34	25.6	5.98	5.85	-1.3	12.4	-5.1	-0.61	-0.51	-0.45
521	14/3/2002	4.1	73.3				11.4	23.9	7.16	6.52	2.96	13.1	-4.2	-0.54	-0.44	-0.37
519	14/3/2002	26.2	50.5				12.9	22.2	7.64	5.58	5.23	13.2	-3.8	-0.62	-0.54	-0.45
518	14/3/2002	26.6	50				12.9	22.2	7.65	5.56	5.27	13.2	-3.8	-0.62	-0.54	-0.45
247	26/6/1987	-30	-71				16.4	17.7	8.99	4.02	-8.1	13.4	-6.1	-0.75	-0.59	-0.56
368	13/7/1999	24.6	46.5				13.5	22.6	7.79	5.36	5.65	13.4	-5	-0.64	-0.56	-0.46
515	12/2/2002	-34	18.4				10.2	28.8	6.49	6.38	-1.2	13.7	-4	-0.56	-0.45	-0.4
626	24/11/2003	32.9	59.2				14.1	20.7	8.57	4.55	7.26	13.7	-6.5	-0.7	-0.63	-0.52
41	18/6/1871	38	23.7				15.5	26.8	7.05	5.39	4.55	13.8	-3.6	-0.64	-0.56	-0.47
520	14/3/2002	30.4	35.5				13.9	23.3	7.99	5.63	5.67	13.8	-3.5	-0.61	-0.53	-0.44
627	24/11/2003	32.6	51.7				14.7	22	8.88	4.8	7.47	14.4	-6.1	-0.68	-0.6	-0.49
612	26/9/2003	26.6	50				11.6	27.9	7.42	6.85	2.83	14.4	-2.5	-0.51	-0.41	-0.34
3	23/1/1860	38	23.7				15.6	30.7	7.08	6.31	3.2	14.7	-2.3	-0.56	-0.46	-0.39
702	14/10/2004	6.5	3.4				14.9	27.1	8.03	7.19	3.57	14.8	-2.8	-0.47	-0.38	-0.3
660	19/5/2004	32.5	3.7				14	32.3	6.74	6.72	0.62	14.8	-3.3	-0.53	-0.43	-0.36
381	8/11/1999	32	35.9				11	31.4	7.03	7	0.66	14.9	-3.2	-0.5	-0.4	-0.34
382	8/11/1999	31.8	34.7				11.1	31.5	7.06	7.03	0.66	14.9	-3	-0.5	-0.39	-0.33
664	18/6/2004	-34	18.4				19.5	21.3	9.79	4.46	-8.7	15.1	-5.7	-0.69	-0.53	-0.51
678	16/8/2004	-34	18.4				15.1	26.6	8.54	5.88	-6.2	15.2	-3	-0.58	-0.44	-0.4
613	26/9/2003	30.4	35.5				12.6	29.3	7.89	6.95	3.75	15.2	-3	-0.49	-0.4	-0.32
724	12/12/2004	32.5	3.7				15.3	20.6	10.07	4.38	9.06	15.2	-6.1	-0.7	-0.63	-0.51
522	14/3/2002	32.5	3.7				16	27.5	8.76	6.38	6.02	15.6	-4.3	-0.53	-0.46	-0.36
424	26/11/2000	32.6	51.7				14.5	33.1	7.51	7.01	2.71	15.8	-3.6	-0.49	-0.39	-0.33
383	8/12/1999	-34	18.4				19.5	26.5	9.36	5.58	-7.5	16	-3.4	-0.6	-0.45	-0.41
384	8/12/1999	-4	39.7				17.1	30.6	8.39	7.69	-3.4	16.1	-0.5	-0.42	-0.3	-0.25
192	2/2/1984	15.6	35.6				16	29.8	8.64	7.4	4.46	16.1	-2.4	-0.44	-0.36	-0.27
426	26/11/2000	-4	39.7				16.4	31.8	8.3	7.99	-2.2	16.2	-1.3	-0.4	-0.28	-0.23
703	14/10/2004	-34	18.4				14.4	34.3	7.77	7.5	-2	16.3	-3.2	-0.45	-0.34	-0.28

S. No.	Date	Lat	Long.	VISIBILITY			Age	Lag	ARCL	ARCV	DAZ	MODEL				
				N	B	T						A	B	C	D	E
207	25/9/1984	15.6	35.6				12.6	31.7	8.42	8.24	1.73	16.3	-4.8	-0.37	-0.27	-0.21
510	15/12/2001	32.6	51.7				16.9	31.6	8.47	6.49	5.45	16.4	-3.9	-0.53	-0.45	-0.35
257	16/4/1988	37.2	-84.1				12.5	34.9	7.7	7.61	-1.2	16.4	-1.4	-0.44	-0.33	-0.27
614	26/9/2003	-34	18.4				13.8	31.6	8.53	7.1	-4.7	16.4	-3.2	-0.47	-0.34	-0.3
680	16/8/2004	30.2	57.1				13.7	33.9	8	7.64	2.37	16.5	-2.8	-0.43	-0.33	-0.26
335	28/3/1998	-34	18.4				13.8	30.9	8.74	7.03	-5.2	16.5	-3	-0.48	-0.34	-0.3
388	7/1/2000	23.7	90.4				17.5	34.5	7.99	7.84	1.51	16.6	-1	-0.41	-0.31	-0.25
681	16/8/2004	26.6	50				14.1	33.9	8.15	7.9	2.01	16.6	-3	-0.41	-0.3	-0.24
425	26/11/2000	32	35.9				15.6	34.8	7.98	7.39	3	16.7	-1.2	-0.45	-0.36	-0.29
502	15/11/2001	41.1	-74				15.2	33.1	8.43	6.34	5.56	16.7	-4.2	-0.54	-0.46	-0.37
523	14/3/2002	27.7	-11.3				17.1	30.6	9.15	7.33	5.49	16.8	-2.2	-0.44	-0.36	-0.27
648	21/3/2004	26.6	50				16.4	33.3	8.61	7.98	3.23	16.9	-1.4	-0.39	-0.3	-0.22
679	16/8/2004	35.7	51.3				14.3	35.3	8.21	7.49	3.35	17	-2.3	-0.44	-0.35	-0.27
211	23/11/1984	15.6	35.6				16.4	31.5	9.21	7.6	5.2	17.1	-5.7	-0.42	-0.34	-0.24
647	21/3/2004	29.4	48				16.6	33.7	8.68	7.89	3.62	17.1	-2.3	-0.4	-0.31	-0.23
549	7/9/2002	31.1	56.5		V		11.6	34.8	8.44	8	2.7	17.2	-1.4	-0.39	-0.29	-0.23
369	13/7/1999	-34	18.4				13.8	37.2	7.92	7.58	-2.3	17.2	-1.7	-0.44	-0.33	-0.27
370	13/7/1999	-34	18.4				13.8	37.2	7.92	7.58	-2.3	17.2	-0.8	-0.44	-0.33	-0.27
477	19/8/2001	29.5	56.8		V		12.2	35	8.47	7.95	2.92	17.2	-1.1	-0.4	-0.3	-0.23
583	3/3/2003	-34	18.4				15	35.3	8.58	7.88	-3.4	17.4	-1.7	-0.4	-0.28	-0.23
385	8/12/1999	26.2	32.7				16.7	36.8	8.24	8.15	1.22	17.4	-1.4	-0.38	-0.28	-0.22
646	21/3/2004	-34	18.4				18.5	31.6	9.55	7.21	-6.3	17.4	-2.8	-0.45	-0.31	-0.27
408	4/5/2000	-34	18.4				12.1	36.8	8.28	7.92	-2.4	17.5	-2.1	-0.4	-0.29	-0.24
551	7/9/2002	32.5	51.3				12	35.6	8.62	8.05	3.07	17.5	-1.5	-0.39	-0.29	-0.22
683	16/8/2004	30.2	35.5				15.2	35.8	8.57	8.05	2.94	17.5	-3.3	-0.39	-0.29	-0.22
507	15/12/2001	31.8	35.2				18.1	34.1	9.05	7.02	5.71	17.6	-1.6	-0.47	-0.39	-0.3
682	16/8/2004	32	35.9			V	15.2	36.1	8.58	7.96	3.21	17.6	-1.7	-0.4	-0.3	-0.23
480	19/8/2001	32.5	51.3				12.6	36.1	8.7	7.94	3.56	17.7	-0.4	-0.4	-0.3	-0.23
216	20/4/1985	37.2	-84.1				19.2	37	8.68	7.92	3.54	17.9	-1.6	-0.4	-0.3	-0.23
578	3/1/2003	32	51.9				17.5	31.9	9.96	6.53	7.53	17.9	-3.7	-0.5	-0.43	-0.32
650	21/3/2004	30.4	35.5				17.4	35.7	9.06	8.24	3.77	18	-2.2	-0.36	-0.27	-0.19
184	5/11/1983	15.6	35.6				17	35.4	9.29	8.78	3.05	18.1	-1.7	-0.31	-0.22	-0.15
422	27/10/2000	38.8	-77.2				14.5	37.9	8.69	7.78	3.88	18.2	-1.5	-0.41	-0.32	-0.24
476	19/8/2001	38.6	48.2				13	37.3	8.89	7.61	4.6	18.2	-2.7	-0.42	-0.34	-0.25
455	25/3/2001	-34	18.4	V			15.8	36.9	9.06	8.3	-3.6	18.3	-0.6	-0.36	-0.24	-0.19
274	25/2/1990	35.6	-83.5	V		V	14.8	39.3	8.53	8.51	-0.6	18.3	-1.2	-0.35	-0.25	-0.19
275	25/2/1990	35.6	-83.5			V	14.8	39.3	8.53	8.51	-0.6	18.3	-0	-0.35	-0.25	-0.19
276	25/2/1990	35.6	-83.5				14.8	39.3	8.53	8.51	-0.6	18.3	-0	-0.35	-0.25	-0.19
53	20/12/1873	38	23.7				20.5	27.7	11.63	5.39	10.3	18.5	-2.3	-0.58	-0.51	-0.38
538	11/6/2002	31.9	35.8				17.2	39.1	8.78	7.87	3.9	18.6	-0.2	-0.4	-0.31	-0.23
387	8/12/1999	6.5	3.4				19.3	37.2	9.27	9.14	-1.6	18.6	-1.3	-0.28	-0.18	-0.12
427	26/11/2000	32.5	3.7				17.8	38.6	8.91	8.05	3.82	18.6	-2.4	-0.38	-0.29	-0.21
706	14/10/2004	25.8	-80.2				20.3	30.7	10.95	7.37	8.11	18.6	-1.4	-0.41	-0.34	-0.23
478	19/8/2001	30.2	35.5		V	V	13.6	37.8	9.24	8.47	3.69	18.7	-0.5	-0.34	-0.25	-0.17
375	10/9/1999	30.4	35.5			V	18.1	35.9	9.84	8.31	5.26	18.8	-0.8	-0.35	-0.27	-0.17
340	19/12/1998	31.9	35.8				16.2	41	8.55	8.47	1.13	18.8	-2.1	-0.35	-0.25	-0.19

S. No.	Date	Lat	Long.	VISIBILITY			Age	Lag	ARCL	ARCV	DAZ	MODEL				
				N	B	T						A	B	C	D	E
572	3/1/2003	26.6	49.8				17.9	34.7	10.15	7.47	6.87	18.8	-1.5	-0.42	-0.34	-0.24
374	10/9/1999	31.8	34.7				18.1	35.8	9.87	8.2	5.49	18.8	-2	-0.36	-0.28	-0.18
726	12/12/2004	10.3	9.8				15.8	34.2	10.31	8.06	6.43	18.9	-0.3	-0.36	-0.28	-0.18
589	2/4/2003	32.6	51.6				19.9	37.1	9.6	8.32	4.8	18.9	-2.6	-0.35	-0.27	-0.18
725	12/12/2004	11.2	7.6				15.9	34.1	10.37	8.02	6.58	18.9	-1.2	-0.36	-0.29	-0.18
557	5/11/2002	29.9	56.2		V		17.1	35.2	10.11	7.82	6.4	18.9	-1.1	-0.39	-0.31	-0.21
649	21/3/2004	38	23.7				18.2	38.2	9.44	8.07	4.9	19	-0.2	-0.37	-0.29	-0.2
416	31/7/2000	6.5	3.4	V			15.9	37.7	9.58	9.41	1.8	19	-1.1	-0.26	-0.15	-0.09
308	22/12/1995	36.1	50.7				11.4	42.7	8.33	8.32	0.31	19	-1.1	-0.37	-0.27	-0.2
20	6/5/1864	39.6	26.2				17.3	39.8	9.08	7.8	4.66	19	-2.4	-0.4	-0.32	-0.23
524	13/4/2002	29.6	52.5				19.8	36.8	9.85	8.42	5.12	19	-1.7	-0.34	-0.26	-0.16
386	8/12/1999	36.8	10.4		V		17.8	41.6	8.68	8.07	3.21	19.1	-1.6	-0.38	-0.29	-0.22
528	13/4/2002	32.6	51.7				19.9	36.8	9.89	8.18	5.57	19.1	-0	-0.36	-0.28	-0.18
428	26/11/2000	34	-6.8				18.4	39.9	9.19	8.14	4.28	19.2	-0.2	-0.37	-0.28	-0.2
312	20/1/1996	34.1	-118			V	12.7	41	8.92	8.78	-1.6	19.2	-0.6	-0.32	-0.21	-0.15
558	5/11/2002	30.1	52.1		V		17.4	35.7	10.26	7.9	6.56	19.2	-0.8	-0.38	-0.3	-0.2
559	5/11/2002	29.6	52.5		V		17.4	35.8	10.25	7.95	6.48	19.2	-0.6	-0.37	-0.3	-0.19
391	7/1/2000	32.7	52.3		V		19.7	40.9	9.01	8.36	3.36	19.2	-1.7	-0.35	-0.26	-0.19
429	26/12/2000	29.6	52.5				20.6	39.6	9.36	8.29	4.35	19.3	-0.7	-0.36	-0.27	-0.18
400	6/3/2000	43.3	-79.9				18.3	36.8	10.09	7.21	7.06	19.3	-5.5	-0.44	-0.36	-0.26
508	15/12/2001	-4	39.7				19	39.3	9.48	9.48	-0.1	19.3	-1	-0.25	-0.15	-0.09
101	31/10/1921	-34	18.5				17.9	38.3	9.84	8.27	-5.3	19.4	-2.6	-0.35	-0.22	-0.18
412	2/7/2000	2.3	102		V	V	16.3	39	9.69	9.5	1.92	19.4	-0.9	-0.25	-0.14	-0.08
484	17/10/2001	2.3	102		V	V	15.9	38.1	9.93	9.92	-0.4	19.5	-0.6	-0.21	-0.11	-0.04
392	7/1/2000	-4	39.7				21.7	38.3	9.89	9.44	-2.9	19.5	-1	-0.25	-0.13	-0.08
431	26/12/2000	26.6	50				20.9	40	9.48	8.64	3.92	19.5	0.16	-0.32	-0.23	-0.15
655	20/4/2004	5	115				21.4	38	10.03	9.73	-2.4	19.5	-0.8	-0.22	-0.11	-0.06
326	2/10/1997	31.8	34.7				22.8	35.6	10.71	8.17	6.94	19.6	-1.2	-0.34	-0.27	-0.16
390	7/1/2000	-32	20.8				23.8	35.3	10.86	7.36	-8	19.7	-1.8	-0.41	-0.26	-0.23
389	7/1/2000	-34	18.4	V			24.1	35.1	10.96	7.19	-8.3	19.7	-1	-0.43	-0.27	-0.24
600	31/5/2003	26	-80.3				20.1	41.5	9.42	9.25	1.78	19.8	-0.1	-0.27	-0.17	-0.11
586	2/4/2003	30.2	35.5		V	V	20.9	39.4	10.05	9	4.48	19.9	-0	-0.28	-0.2	-0.11
638	22/1/2004	30	51.7		V		17.2	36.3	10.88	7.7	7.69	20	-0.6	-0.38	-0.31	-0.2
15	29/4/1862	38	23.7				18.1	44.6	8.86	8.85	0.2	20	-1.5	-0.31	-0.21	-0.15
94	8/2/1921	36.5	-6.2				17.7	43.2	9.25	9.1	-1.7	20	0.75	-0.29	-0.18	-0.12
560	5/11/2002	31.9	35.8		V		18.4	37.2	10.86	8.04	7.31	20.1	-0.6	-0.35	-0.28	-0.17
593	2/5/2003	5	115			V	22.5	40	10.16	10.1	-1.3	20.2	-0.4	-0.19	-0.08	-0.03
134	15/3/1972	35.5	-118				14.8	42	9.69	9.35	-2.5	20.2	0.83	-0.26	-0.15	-0.09
135	15/3/1972	35.5	-118		V		14.8	42	9.69	9.35	-2.5	20.2	0.83	-0.26	-0.15	-0.09
430	26/12/2000	32.6	35.9				21.6	41.7	9.81	8.39	5.09	20.2	-1.4	-0.34	-0.26	-0.17
432	26/12/2000	30.2	35.5		V	V	21.7	41.9	9.87	8.67	4.72	20.3	-0.7	-0.32	-0.23	-0.14
299	3/12/1994	-34	18.4				18.1	36.9	11.14	7.48	-8.3	20.4	-1.3	-0.4	-0.24	-0.21
338	21/9/1998	31.8	35.2				22.9	37.6	10.97	8.61	6.81	20.4	-0.5	-0.3	-0.23	-0.12
281	24/5/1990	35.6	-83.5			V	12.7	47	8.67	8.66	-0.5	20.4	0.15	-0.33	-0.23	-0.17
321	7/5/1997	32.7	52.3		V		18.8	37.8	11.06	8.06	7.57	20.5	-0.8	-0.35	-0.27	-0.17
318	7/5/1997	36	50.8				19	37.4	11.16	7.68	8.1	20.5	-0.8	-0.38	-0.31	-0.19

S. No.	Date	Lat	Long.	VISIBILITY			Age	Lag	ARCL	ARCV	DAZ	MODEL				
				N	B	T						A	B	C	D	E
404	5/4/2000	5.3	103		V		17.4	40	10.62	10.4	2.12	20.6	-0	-0.16	-0.05	0.01
95	8/2/1921	38.8	-9.1				17.8	45.3	9.31	9.22	-1.3	20.6	1.16	-0.28	-0.17	-0.11
436	26/12/2000	-4	39.7				22.5	41.7	10.23	10.1	-1.6	20.7	-0.3	-0.19	-0.08	-0.02
220	31/12/1986	39	-77				19	33.2	12.38	6.03	10.8	20.7	-2.7	-0.5	-0.43	-0.31
301	1/1/1995	33	-106			V	13.5	46.5	9.05	9.05	-0.3	20.7	0.3	-0.29	-0.19	-0.13
371	13/7/1999	43.3	-79.9				22.5	32	12.71	6.3	11.1	20.7	-1.4	-0.47	-0.4	-0.27
720	13/11/2004	36.1	50.3				23.3	29.1	13.57	5.91	12.2	20.8	-0.7	-0.49	-0.42	-0.28
594	2/5/2003	3.2	102			V	23.4	41.3	10.55	10.4	-1.7	20.9	-0.1	-0.16	-0.05	0.008
543	9/8/2002	2.3	102			V	16.4	41.6	10.49	10.4	-1	20.9	-0.1	-0.16	-0.05	0.009
199	1/5/1984	37.2	-84.1				20.5	44	9.99	8.77	4.81	21	0.43	-0.3	-0.22	-0.13
194	3/3/1984	15.6	35.6				21.6	40.7	10.9	10.3	3.6	21.1	0.16	-0.16	-0.07	0.009
39	20/4/1871	38	23.7				22.3	40	11.09	8.4	7.25	21.1	0.31	-0.32	-0.24	-0.14
305	2/3/1995	-34	18.4				29.8	23.2	15.46	5.45	-14	21.3	-1.9	-0.49	-0.27	-0.27
7	7/8/1861	38	23.7				28.7	21.7	16.03	4.97	15.2	21.5	-2	-0.52	-0.45	-0.29
607	28/8/2003	5.3	103		V	V	18.2	42.4	10.89	10.9	0.4	21.5	0.5	-0.11	-0.01	0.051
667	18/6/2004	24.6	46.5				19.7	46.6	9.87	9.86	0.5	21.5	0.37	-0.21	-0.11	-0.05
304	31/1/1995	35.6	51.3		V		15.6	47	9.82	9.66	-1.8	21.6	1.27	-0.23	-0.12	-0.07
665	18/6/2004	26.6	50				19.5	47.1	9.81	9.77	0.85	21.6	0.47	-0.22	-0.12	-0.06
573	3/1/2003	32.5	3.7		V		20.8	40.1	11.61	7.96	8.46	21.6	-1.4	-0.35	-0.27	-0.16
105	29/3/1922	-34	18.5				28	35.1	12.91	8	-10	21.7	-1.4	-0.32	-0.15	-0.12
51	27/4/1873	38	23.7				18.8	45.9	10.21	9.23	4.37	21.7	1.15	-0.26	-0.17	-0.09
715	13/11/2004	4.9	115		V		19.9	40.3	11.62	9.86	6.16	21.7	-0.8	-0.18	-0.11	-0.01
688	15/9/2004	36.6	59		V		24	37.3	12.44	8.14	9.41	21.8	-0.5	-0.32	-0.24	-0.12
630	24/11/2003	6.5	3.4				18.8	42	11.31	10.1	5.12	21.8	-0.6	-0.17	-0.09	0.004
319	7/5/1997	31.8	34.9	V			19.9	40.9	11.64	8.74	7.68	21.9	0.07	-0.28	-0.21	-0.1
290	15/2/1991	33.4	73.1	V			19.7	46.9	10.17	10.1	-0.9	21.9	1.43	-0.19	-0.08	-0.02
260	14/6/1988	37.2	-84.1				15.7	51	9.15	9.09	1.07	21.9	0.94	-0.29	-0.19	-0.13
434	26/12/2000	-32	20.8	V			24.7	42.9	11.23	8.62	-7.2	22	0.06	-0.3	-0.15	-0.12
435	26/12/2000	-32	20.8		V		24.7	42.9	11.23	8.64	-7.2	22	0.06	-0.29	-0.15	-0.11
666	18/6/2004	28.4	48				19.7	48.4	9.9	9.82	1.26	22	0.73	-0.22	-0.11	-0.05
394	7/1/2000	6.5	3.4				23.9	44.6	10.87	10.8	-1	22	0.43	-0.12	-0.01	0.048
433	26/12/2000	-34	18.4	V			24.9	43	11.34	8.49	-7.5	22.1	0.1	-0.31	-0.16	-0.12
471	21/7/2001	4.1	73.3	V	V		18	44.8	10.93	10.9	0.56	22.1	0.48	-0.11	-0.01	0.056
264	5/5/1989	42.7	-84.8			V	12.7	53	8.91	8.85	-1	22.2	1.31	-0.31	-0.21	-0.15
265	5/5/1989	42.7	-84.8				12.7	53	8.91	8.85	-1	22.2	1.31	-0.31	-0.21	-0.15
266	5/5/1989	43	-85.7			V	12.8	53	8.95	8.9	-0.9	22.2	1.41	-0.31	-0.2	-0.15
689	15/9/2004	35.7	51.4				24.5	38.2	12.69	8.4	9.52	22.2	-0.2	-0.29	-0.22	-0.1
690	15/9/2004	34.7	50.9				24.5	38.6	12.7	8.58	9.38	22.4	-0.1	-0.27	-0.2	-0.08
282	24/5/1990	31.6	-111			V	15	50	9.86	9.86	0.14	22.4	1	-0.21	-0.11	-0.05
283	24/5/1990	32.4	-111				15	50	9.88	9.87	0.27	22.4	1.12	-0.21	-0.11	-0.05
485	17/10/2001	32.6	51.7		V		18.9	43.8	11.47	9.66	6.2	22.4	1.22	-0.2	-0.13	-0.03
592	2/5/2003	-34	18.4				28.1	39	12.69	8.2	-9.7	22.4	-1.3	-0.3	-0.14	-0.11
173	28/1/1979	29.9	-81.3	V			17	48.1	10.42	10.4	0.09	22.4	1.37	-0.16	-0.06	0.005
486	17/10/2001	29.6	52.5		B		18.9	44	11.47	9.97	5.67	22.5	1.2	-0.18	-0.1	-0
175	28/1/1979	29.7	-82.4		V		17.1	48.2	10.47	10.5	0.06	22.5	1.4	-0.15	-0.05	0.01
691	15/9/2004	33.3	50.1		V		24.6	39.2	12.73	8.82	9.19	22.5	0.12	-0.25	-0.18	-0.06

S. No.	Date	Lat	Long.	VISIBILITY			Age	Lag	ARCL	ARCV	DAZ	MODEL				
				N	B	T						A	B	C	D	E
332	28/1/1998	29.8	-95.4		V		18.3	48.6	10.44	10.4	0.68	22.6	1.33	-0.16	-0.06	0.005
672	18/6/2004	33.3	50				19.8	50.9	9.93	9.68	2.25	22.7	1.24	-0.23	-0.13	-0.06
341	18/1/1999	-34	18.4	V			26.5	37.4	13.31	7.79	-11	22.7	-0.9	-0.33	-0.15	-0.13
488	17/10/2001	26.6	50				19.1	44.5	11.57	10.3	5.22	22.7	1.29	-0.15	-0.07	0.029
328	30/12/1997	-34	18.4		V		25.3	34.8	14.04	7.15	-12	22.7	-1	-0.37	-0.18	-0.16
615	26/9/2003	41.8	-112		V		22.3	39	13.01	8.02	10.3	22.8	0.42	-0.31	-0.24	-0.12
714	13/11/2004	32	35.9			V	24.4	34.8	14.21	7.27	12.2	22.9	-3	-0.35	-0.28	-0.15
489	17/10/2001	24.6	46.5			V	19.4	45	11.71	10.6	4.96	23	1.42	-0.12	-0.04	0.054
364	14/6/1999	29.6	52.5				20.8	41.4	12.62	8.67	9.18	23	-0.1	-0.27	-0.19	-0.08
692	15/9/2004	28.4	. 48				24.7	41	12.79	9.6	8.46	23	0.64	-0.18	-0.11	0.004
693	15/9/2004	26.6	50				24.6	41.3	12.72	9.82	8.1	23.1	0.72	-0.17	-0.09	0.019
694	15/9/2004	26.2	50.5		V		24.5	41.4	12.71	9.87	8.01	23.1	0.73	-0.16	-0.09	0.023
48	3/10/1872	38	23.7				24.9	41.4	12.75	8.97	9.07	23.1	1.28	-0.24	-0.17	-0.05
315	13/10/1996	31.8	34.9	V			25.2	41.6	12.72	9.27	8.73	23.1	0.37	-0.21	-0.14	-0.02
212	23/11/1984	34	-81			V	23.6	39.7	13.24	7.92	10.6	23.2	-1.7	-0.32	-0.25	-0.12
285	24/5/1990	34.2	-118	V		V	15.6	52	10.17	10.1	0.81	23.2	1.66	-0.19	-0.08	-0.02
284	24/5/1990	34.2	-118			V	15.6	52	10.17	10.1	0.81	23.2	1.66	-0.19	-0.08	-0.02
525	13/4/2002	30.5	-9.7		V		24.1	46.1	11.65	10.3	5.52	23.2	1.35	-0.15	-0.07	0.026
393	7/1/2000	34	-6.8				23.7	49.6	10.8	9.81	4.51	23.2	1.05	-0.2	-0.12	-0.03
668	18/6/2004	32	35.9			V	20.7	52.1	10.31	10.1	2.23	23.3	1.53	-0.19	-0.09	-0.02
365	14/6/1999	29.4	48				21.1	42.2	12.79	8.86	9.24	23.3	0.11	-0.25	-0.17	-0.06
415	2/7/2000	32.6	51.7		V		20.7	44.4	12.3	8.92	8.47	23.4	0.55	-0.25	-0.18	-0.06
529	13/5/2002	5	115		V	V	24.1	47.3	11.6	11.6	-0.3	23.4	1.14	-0.04	0.061	0.123
419	28/9/2000	26.2	32.7	V			20.1	46	11.93	10.9	4.93	23.4	2.19	-0.09	-0.01	0.079
487	17/10/2001	31.9	35.8			V	20	45.6	12.03	10.1	6.58	23.4	1.78	-0.16	-0.08	0.022
252	26/6/1987	30	-100				20.2	52	10.45	10.4	1.34	23.5	1.58	-0.16	-0.06	0.003
137	1/7/1973	-44	171		V		17.9	51.6	10.61	8.55	-6.3	23.5	0.97	-0.31	-0.17	-0.14
337	26/5/1998	31.8	35.2			V	21.4	41.9	13.12	8.72	9.81	23.6	0.2	-0.25	-0.18	-0.06
443	25/1/2001	32.6	51.7				25.2	47.8	11.66	9.86	6.23	23.6	0.86	-0.18	-0.11	-0.01
252	26/6/1987	30	-100				20.6	52	10.63	10.4	1.95	23.6	1.78	-0.15	-0.05	0.013
639	22/1/2004	32.5	3.7	V	V		20.4	44.6	12.49	9.03	8.63	23.6	-0.1	-0.24	-0.17	-0.05
616	26/9/2003	32.4	-111	V	V	V	22.5	42	13.14	9.68	8.89	23.6	1.38	-0.17	-0.1	0.019
88	28/11/1913	-34	18.5	V			16.4	53.6	10.25	10.3	-0	23.7	1.72	-0.17	-0.07	-0.01
437	25/1/2001	29.6	52.5		V		25.3	47.9	11.68	10.2	5.67	23.7	0.94	-0.15	-0.08	0.023
267	5/5/1989	39.7	-106			V	14.8	55	9.93	9.92	-0.4	23.7	2.01	-0.21	-0.11	-0.04
320	7/5/1997	-34	18.4	V			19.6	49.1	11.46	10.2	-5.3	23.7	2.09	-0.16	-0.03	0.015
511	14/1/2002	29.6	52.5				24.7	47.4	12.11	9.87	7.04	24	0.51	-0.17	-0.1	0.007
491	17/10/2001	-4	39.7				20.2	47.4	12.12	12.1	-0.7	24	1.73	0.01	0.114	0.174
256	19/1/1988	32.2	-111			V	19.8	47	12.22	9.76	7.37	24	0.61	-0.18	-0.11	0.001
324	4/8/1997	31.3	35.2				32.6	35.6	15.1	7.92	12.9	24	-1.1	-0.28	-0.21	-0.07
405	5/4/2000	32.6	51.7		V		21.1	46.5	12.43	10.2	7.12	24.1	1.5	-0.14	-0.06	0.041
306	28/6/1995	-30	-71	V			21.5	52.5	10.94	10.9	-0.2	24.1	2.13	-0.11	-0	0.057
334	27/2/1998	-34	18.4	V			24.3	39.4	14.23	8.69	-11	24.1	0.2	-0.23	-0.05	-0.03
249	26/6/1987	37.2	-84.1			V	19.9	55	10.33	9.83	3.17	24.1	2.25	-0.21	-0.11	-0.04
603	30/6/2003	33.7	58.4				21.1	52.9	10.9	10.1	4.15	24.1	2.03	-0.18	-0.09	-0.01
350	18/3/1999	29.6	52.5	V	V		20.2	48.3	12.06	11	5.05	24.1	1.85	-0.08	-0	0.089

S. No.	Date	Lat	Long.	VISIBILITY			Age	Lag	ARCL	ARCV	DAZ	MODEL				
				N	B	T						A	B	C	D	E
438	25/1/2001	24.6	46.5		V	V	25.8	49	11.93	10.9	4.75	24.2	1.26	-0.09	-0	0.085
223	28/4/1987	26.7	-81.1	V	V		22.7	50.4	11.6	11.5	-1.1	24.2	2.15	-0.04	0.061	0.119
417	28/9/2000	-34	18.4	V			21.3	47.2	12.48	10.3	-7.1	24.3	1.64	-0.13	0.012	0.048
418	28/9/2000	-34	18.4	V	V		21.3	47.2	12.48	10.3	-7.1	24.3	1.64	-0.13	0.012	0.048
185	5/11/1983	37.2	-84.1			V	24.6	44.7	13.11	8.95	9.59	24.3	0.55	-0.23	-0.16	-0.04
377	10/10/1999	5.3	103	V	V	V	23.7	47.9	12.37	12.4	0.78	24.3	2.14	0.04	0.138	0.199
162	9/3/1978	45.1	-64.2	V			20	54.6	10.73	10.2	3.46	24.4	2.94	-0.17	-0.08	-0.01
347	18/3/1999	36	50.8	V			20.3	49	12.12	10.4	6.26	24.4	1.96	-0.13	-0.05	0.047
45	6/7/1872	39	23.7				23.9	52	11.38	9.46	6.33	24.4	2.44	-0.22	-0.15	-0.04
33	5/2/1867	38	23.7				22	53.7	10.96	10.8	1.58	24.4	2.72	-0.11	-0.01	0.051
550	7/9/2002	10.7	-61.5	V			19.4	48.1	12.44	12.2	2.37	24.5	2.31	0.03	0.127	0.192
177	28/1/1979	38.7	-90.3		V		17.4	55.5	10.64	10.5	1.87	24.5	2.45	-0.15	-0.05	0.016
353	18/3/1999	24.6	46.5		V	V	20.6	49	12.28	11.6	4.09	24.5	2.05	-0.03	0.062	0.142
218	12/12/1985	-32	20.8	V			17.1	55.2	10.78	10.5	-2.3	24.6	2.09	-0.14	-0.03	0.024
475	21/7/2001	32.6	51.7				20.3	49.4	12.27	10	7.07	24.6	2.14	-0.16	-0.08	0.025
229	28/4/1987	28	-82.5				22.8	52	11.65	11.6	-0.9	24.7	2.37	-0.04	0.067	0.126
229	28/4/1987	28	-82.5				22.8	52	11.65	11.6	-0.9	24.7	2.37	-0.04	0.067	0.126
316	8/2/1997	-34	18.4	V			26.9	34.3	16.1	7.51	-14	24.7	-1.5	-0.29	-0.07	-0.07
512	14/1/2002	24.6	46.5		V		25.3	49.3	12.38	10.8	6.14	24.7	1.01	-0.09	-0.02	0.084
376	10/9/1999	38.8	-77		V		25.7	44.9	13.54	9.45	9.7	24.8	2.84	-0.18	-0.11	0.012
413	2/7/2000	30.4	35.5			V	21.8	47.6	12.89	9.75	8.45	24.8	1.34	-0.17	-0.1	0.018
163	9/3/1978	42.7	-73.8	V	V		20.7	55	11.08	10.6	3.18	24.8	3.06	-0.13	-0.03	0.039
608	28/8/2003	32.6	51.7		V		22	48.2	12.78	10.6	7.22	24.8	3.16	-0.1	-0.03	0.078
343	18/1/1999	26.1	44		V		23.2	52.7	11.67	11.5	1.75	24.8	2.12	-0.04	0.06	0.122
348	18/3/1999	-34	18.4	V	V		22.6	46.2	13.35	10.2	-8.7	24.9	2.71	-0.12	0.035	0.063
349	18/3/1999	-34	18.4	V			22.6	46.3	13.35	10.2	-8.7	24.9	2.72	-0.12	0.036	0.065
629	24/11/2003	41.5	-112		V		25.4	38.9	15.19	6.77	13.6	24.9	-2.8	-0.37	-0.31	-0.16
36	12/5/1869	38	23.7				25.7	45.9	13.48	9.1	9.96	25	1.34	-0.21	-0.14	-0.02
253	26/6/1987	33.5	-112	V			21.6	56	11.01	10.6	2.97	25	2.58	-0.13	-0.03	0.035
342	18/1/1999	28.8	43.7	V	V		23.2	53.5	11.64	11.4	2.32	25	2.23	-0.05	0.047	0.112
656	20/4/2004	27.3	62.4		V		25.3	52.7	11.87	11.8	1.73	25	2.48	-0.02	0.081	0.143
420	28/9/2000	32.5	3.7	V	V		22	48.7	12.87	10.9	6.93	25.1	3.49	-0.08	-0	0.104
695	15/9/2004	32.5	3.7		V		27.7	43.2	14.29	9.72	10.5	25.1	1.6	-0.14	-0.07	0.055
31	17/1/1866	38	23.7				19.3	56.5	11.01	11	0.45	25.1	3.12	-0.1	0.002	0.063
621	26/10/2003	32.9	59.2		V		24.8	42.5	14.55	9.08	11.4	25.2	0.01	-0.19	-0.12	0.012
174	28/1/1979	42	-91.7		V		17.4	58.1	10.63	10.3	2.51	25.2	2.79	-0.16	-0.06	0.005
248	26/6/1987	42.7	-84.5		V		20.2	59	10.47	9.49	4.42	25.2	3.14	-0.24	-0.15	-0.06
323	5/7/1997	-34	18.5	V			21.6	56.2	11.24	11.2	-1.2	25.3	2.81	-0.08	0.026	0.084
406	5/4/2000	32	35.9			V	22.2	49.3	12.97	10.8	7.14	25.3	2.1	-0.08	-0	0.106
176	28/1/1979	42	-93.6	V			17.5	58.5	10.7	10.4	2.54	25.3	2.86	-0.16	-0.05	0.011
142	18/2/1977	43.8	-87.7	V	V		20.2	58	10.88	10.9	0.66	25.4	3.58	-0.11	-0.01	0.049
96	8/2/1921	42.3	-71.1	V			21.9	57.9	10.98	11	0.13	25.5	3.57	-0.1	-0	0.061
669	18/6/2004	32.5	3.7		V		22.9	56.9	11.25	10.9	2.95	25.5	2.62	-0.11	-0.01	0.06
727	12/12/2004	32.4	-111	V	V	V	23.2	44.2	14.43	8.13	11.9	25.5	-1.5	-0.27	-0.2	-0.07
201	31/5/1984	15.6	35.6				23.7	53.2	12.19	12.2	0.93	25.5	2.1	0.02	0.12	0.181
167	9/3/1978	40.5	-89	V			21.7	55.5	11.63	11.2	2.98	25.5	3.39	-0.07	0.03	0.099

S. No.	Date	Lat	Long.	VISIBILITY			Age	Lag	ARCL	ARCV	DAZ	MODEL				
				N	B	T						A	B	C	D	E
168	9/3/1978	40.5	-89		V		21.7	55.5	11.63	11.2	2.98	25.5	3.39	-0.07	0.03	0.099
456	24/4/2001	32.6	51.7	V	V		24.1	50.6	12.89	10.8	7.13	25.5	2.18	-0.08	-0.01	0.097
351	18/3/1999	31.9	35.8	V	V		21.4	51.5	12.69	11.4	5.63	25.6	2.51	-0.04	0.041	0.138
397	6/2/2000	32.6	51.7	V	V		25.5	53	12.33	11.1	5.29	25.6	2.23	-0.06	0.016	0.111
545	9/8/2002	32.6	51.7	V	V	V	20.5	51.8	12.67	10.9	6.53	25.6	3.58	-0.08	-0	0.099
352	18/3/1999	31.8	35.2	V			21.4	51.6	12.71	11.4	5.61	25.6	2.53	-0.03	0.044	0.142
544	9/8/2002	-34	18.4	V			21.3	50.4	13.09	10.3	-8.1	25.7	1.46	-0.12	0.037	0.068
622	26/10/2003	33.3	50.1	V	V		25.4	43.3	14.9	9.19	11.8	25.7	0.24	-0.17	-0.1	0.032
609	28/8/2003	32.4	36.2			V	23	49.7	13.3	10.9	7.67	25.7	3.7	-0.06	0.008	0.118
623	26/10/2003	32.6	51.7		V		25.3	43.5	14.85	9.31	11.6	25.7	0.33	-0.16	-0.09	0.04
597	2/5/2003	27.7	54.4	V	V		27.1	54.1	12.23	11.8	3.22	25.8	2.66	-0.01	0.086	0.157
399	6/2/2000	-4	39.7	V			27	50.8	13.07	12.7	-3	25.8	2.52	0.08	0.197	0.248
493	17/10/2001	10.3	9.8	V			22	50.7	13.09	12.8	2.94	25.8	2.8	0.08	0.182	0.251
473	21/7/2001	31.9	35.8	V		V	21.3	51.8	12.88	10.6	7.38	25.8	2.81	-0.1	-0.03	0.082
472	21/7/2001	32.6	35.9				21.4	51.7	12.89	10.5	7.53	25.8	2.83	-0.11	-0.03	0.075
396	6/2/2000	-34	18.4	V	V		29.1	47.5	14.06	9.99	-9.9	25.9	2.58	-0.12	0.047	0.07
250	26/6/1987	39.8	-105				21.5	60	10.99	10.1	4.41	26	3.32	-0.18	-0.09	-0.01
407	5/4/2000	-34	18.4	V			22.8	50.9	13.29	11.1	-7.4	26	4.08	-0.05	0.096	0.131
205	27/8/1984	15.6	35.6				20.9	51.3	13.26	12.7	3.85	26.1	3.34	0.08	0.173	0.251
222	28/4/1987	38.9	-77	V	V		22.8	58	11.64	11.5	1.52	26.1	3.68	-0.04	0.059	0.121
227	28/4/1987	38.9	-77.1	V	V		22.8	58	11.64	11.5	1.52	26.1	3.68	-0.04	0.059	0.121
224	28/4/1987	38.9	-77.1				22.8	58	11.64	11.5	1.52	26.1	3.68	-0.04	0.059	0.121
414	2/7/2000	-32	20.8	V	V		20.8	55.3	12.33	10.9	-5.7	26.2	2.07	-0.08	0.054	0.095
490	17/10/2001	-34	18.4	V			22	52.4	13.09	11.1	-7	26.2	2.61	-0.05	0.093	0.129
561	5/11/2002	-34	18.4	V			21.1	55.3	12.4	11.2	-5.4	26.2	2.79	-0.06	0.074	0.116
395	7/1/2000	10	-61.5	V			28.2	54	12.83	12.8	-0.1	26.3	2.49	0.08	0.184	0.246
10	5/10/1861	38	23.7				33.3	24.9	20.2	5.54	19.4	26.4	-5.8	-0.34	-0.27	-0.09
596	2/5/2003	32.6	51.7	V	V		27.4	56.2	12.37	11.6	4.32	26.4	3.02	-0.03	0.063	0.147
251	26/6/1987	40.7	-112		V		22	61	11.19	10.1	4.78	26.4	3.67	-0.17	-0.09	0.004
651	21/3/2004	36.8	-81.8	V			25.4	54.7	12.78	11.6	5.3	26.5	3.35	-0.01	0.066	0.161
574	3/1/2003	-34	18.4	V			22.1	56.9	12.26	10.8	-5.7	26.5	3.28	-0.09	0.045	0.086
624	26/10/2003	29.4	48		V		25.6	45.8	15.04	10.1	11.2	26.5	1.04	-0.09	-0.02	0.107
108	27/5/1922	-34	18.5	V			22.1	56.9	12.3	11.5	-4.5	26.5	3.69	-0.04	0.088	0.133
657	20/4/2004	32.8	51		V		26.2	56.9	12.29	11.9	2.98	26.5	3.27	0	0.099	0.168
447	24/2/2001	5.3	103	V	V	V	27.4	52.5	13.4	13.4	0.69	26.5	3.05	0.14	0.241	0.302
187	5/12/1983	15.6	35.6				27.1	52.8	13.41	12.1	5.83	26.6	1.87	0.04	0.114	0.213
547	9/8/2002	30.4	35.5	V		V	21.6	53.6	13.23	11.5	6.54	26.6	4.07	-0.02	0.06	0.163
157	9/1/1978	27.7	-82.7	V	V		19.3	57.3	12.32	12.3	-0.7	26.7	3.27	0.03	0.134	0.194
344	18/1/1999	6.5	3.4				26.5	53.5	13.3	13	-2.6	26.7	2.72	0.11	0.224	0.277
398	6/2/2000	36.2	37.2	V	V		26.4	55.7	12.76	11.2	6.17	26.7	2.67	-0.05	0.024	0.125
314	21/1/1996	-34	18.4	V			29.4	35.2	17.89	7.51	-16	26.7	-1	-0.24	-0	-0.01
458	24/4/2001	31.9	35.8	V		V	25.2	53.3	13.4	11.4	7.12	26.7	2.75	-0.02	0.051	0.157
457	24/4/2001	32.6	35.9				25.2	53.3	13.41	11.3	7.26	26.7	2.74	-0.03	0.043	0.151
197	2/4/1984	15.6	35.6				28.1	53.4	13.42	13.3	1.92	26.8	3.23	0.13	0.234	0.297
230	28/4/1987	36.2	-81.7	V	V		23.2	60	11.8	11.8	1.1	26.8	3.51	-0.02	0.079	0.14
153	9/1/1978	29.9	-81.3	V			19.2	58.4	12.23	12.2	-0.2	26.8	3.44	0.02	0.124	0.186

S. No.	Date	Lat	Long.	VISIBILITY			Age	Lag	ARCL	ARCV	DAZ	MODEL				
				N	B	T						A	B	C	D	E
439	25/1/2001	32.5	3.7				28.5	55	13.12	11.3	6.75	26.9	2.29	-0.04	0.038	0.142
93	19/4/1920	43.5	7	V			21.1	60.3	11.94	10.9	4.81	27	3.78	-0.09	-0.01	0.085
286	20/9/1990	31.8	34.7	V			39.1	29.5	19.64	6.85	18.4	27	-3.5	-0.24	-0.17	0.002
89	16/3/1915	49.4	8.7	V			22.3	64.1	11.16	11	1.69	27.2	4.67	-0.09	0.009	0.071
640	22/1/2004	-34	18.4	V			21.3	57.3	12.96	11.3	-6.3	27.3	4.43	-0.04	0.104	0.143
379	10/10/1999	24.5	46.5		V		27.4	53.5	13.99	12.6	6.05	27.4	4.38	0.09	0.168	0.268
215	21/1/1985	19	-155	V			26.2	54	13.88	12.6	5.94	27.4	2.34	0.08	0.161	0.26
151	9/1/1978	34	-81.1	V	V		19	61	12.15	12.1	0.72	27.4	3.87	0.01	0.116	0.177
526	13/4/2002	26	-80.3	V			28.8	54.8	13.7	12.8	4.81	27.4	3.53	0.1	0.184	0.275
595	2/5/2003	38.2	46	V			28	59.4	12.63	11.3	5.6	27.5	3.54	-0.04	0.035	0.133
272	1/10/1989	31.3	34.6	V			41.9	32.1	19.52	7.34	18.1	27.5	-2.9	-0.2	-0.13	0.039
602	30/6/2003	32.5	3.7		V	V	24.7	60	12.6	11.5	5.09	27.6	3.81	-0.03	0.053	0.148
147	9/1/1978	36	-79.8	V	V		18.8	62.1	12.07	12	1.14	27.6	4.04	0	0.105	0.166
148	9/1/1978	36	-79.8	V			18.8	62.1	12.07	12	1.14	27.6	4.04	0	0.105	0.166
610	28/8/2003	32.5	3.7		V		25.2	52.7	14.42	11.5	8.71	27.6	4.83	0.01	0.082	0.198
156	9/1/1978	33.9	-84.3	V			19.2	61.5	12.27	12.3	0.76	27.6	3.98	0.03	0.128	0.189
307	25/9/1995	-34	18.4	V			24.3	59.5	12.81	12.7	-1.4	27.7	4.1	0.08	0.183	0.24
241	28/4/1987	30.6	-104	V	V		24.5	60.9	12.46	12.5	0.04	27.7	3.5	0.05	0.146	0.209
372	12/8/1999	32.7	52.3				28.6	49.4	15.37	10.6	11.2	27.7	3.35	-0.04	0.026	0.156
330	30/12/1997	31.3	35.2	V			22.3	60.9	12.51	12.4	1.95	27.7	3.8	0.04	0.141	0.204
378	10/10/1999	32	35.9	V		V	28	53.9	14.27	11.9	7.9	27.8	4.98	0.04	0.113	0.224
158	9/1/1978	30	-90.2	V	V		19.8	60.9	12.56	12.6	-0.1	27.8	3.81	0.06	0.156	0.219
159	9/1/1978	30	-90.2	V	V		19.8	60.9	12.56	12.6	-0.1	27.8	3.81	0.06	0.156	0.219
339	21/10/1998	31.8	34.7	V			29.3	54.8	14.08	11.9	7.47	27.8	4.75	0.04	0.112	0.221
329	30/12/1997	31.3	34.6	V			22.3	61	12.53	12.4	1.96	27.8	3.82	0.04	0.143	0.206
145	9/1/1978	38.9	-76.9		V		18.5	63.7	11.9	11.8	1.72	27.8	4.26	-0.02	0.084	0.146
548	9/8/2002	10.3	9.8	V			22.9	56.3	13.92	13.7	2.33	28	3.95	0.18	0.279	0.344
366	14/6/1999	6.5	3.4	V			23.4	55.8	14.08	13.3	4.76	28	3.06	0.14	0.226	0.316
140	21/12/1976	29.9	-81.3	V			20.8	61.7	12.8	12.7	1.72	28.2	4.15	0.07	0.175	0.237
100	4/8/1921	-34	18.5	V			20.3	61.7	12.83	12.8	0.41	28.2	4.38	0.08	0.184	0.245
122	8/12/1942	40.7	-74	V	V		20	62.9	12.56	11.2	5.8	28.3	4.53	-0.06	0.02	0.118
195	3/3/1984	37.2	-84.1			V	29.5	56.5	14.17	11.7	7.96	28.3	3.42	0.02	0.097	0.208
161	9/1/1978	29.7	-98.1	V			20.3	62.1	12.85	12.9	-0	28.4	4.09	0.09	0.185	0.248
717	13/11/2004	13.7	10.7	V			26.7	52.1	15.52	12.1	9.69	28.5	1.82	0.09	0.158	0.28
288	18/12/1990	33.4	73.1	V			32.1	56.8	14.57	10.9	9.72	28.8	2.06	-0.04	0.034	0.156
696	15/9/2004	-34	18.4	V			26.6	60.2	13.74	12.9	-4.7	28.8	4.27	0.11	0.237	0.282
479	19/8/2001	33.9	-118	V	V		24	55	15.07	11.9	9.32	28.8	5.85	0.05	0.125	0.245
577	3/1/2003	10.4	-61.5	V			26	58.5	14.27	13.5	4.6	28.9	2.98	0.17	0.253	0.34
373	12/8/1999	31.8	34.7	V			29.7	51.8	15.98	11.2	11.5	28.9	4.1	0.02	0.09	0.222
652	21/3/2004	33.9	-118	V			27.8	60	13.94	13.1	4.83	28.9	4.48	0.13	0.209	0.3
562	5/11/2002	26	-80.3		V		26.4	54.2	15.43	12	9.78	29	3.05	0.07	0.142	0.264
38	20/2/1871	38	23.7	V			26.8	58	14.49	11.8	8.46	29	3.52	0.03	0.106	0.221
102	30/12/1921	-34	18.5				36.7	44.4	17.96	9.13	-15	29.1	1.49	-0.1	0.13	0.129
40	20/5/1871	38	23.7				31.2	59.6	14.2	11.2	8.69	29.1	3.85	-0.02	0.056	0.172
459	24/4/2001	32.5	3.7	V	V	V	27.4	58.8	14.47	12.4	7.53	29.2	3.88	0.08	0.155	0.264
535	13/5/2002	25.3	49.7	V			29	60.8	13.97	13.2	4.53	29.2	4.01	0.14	0.224	0.311

S. No.	Date	Lat	Long.	VISIBILITY			Age	Lag	ARCL	ARCV	DAZ	MODEL				
				N	B	T						A	B	C	D	E
580	2/2/2003	32.6	51.7	V	V		27.7	58.1	14.66	11.9	8.55	29.2	2.94	0.05	0.122	0.237
534	13/5/2002	26.2	50.5	V	V		29	60.9	13.96	13.1	4.75	29.2	4.01	0.13	0.214	0.304
718	13/11/2004	10.3	9.8	V			26.9	54.4	15.6	12.8	8.94	29.2	2.38	0.14	0.215	0.332
354	18/3/1999	34	-6.8	V			24.3	59.6	14.3	12.8	6.46	29.2	4.16	0.11	0.185	0.288
126	5/4/1962	-26	-28.2	V	V		24.5	56.6	15.1	13.2	-7.3	29.2	5.55	0.17	0.313	0.347
421	28/9/2000	43.3	-79.9	V	V		27.6	54.5	15.64	10.6	11.5	29.3	7.18	-0.04	0.032	0.165
146	9/1/1978	41.9	-87.6	V			19.1	68.2	12.23	12	2.59	29.3	5.04	0	0.101	0.168
532	13/5/2002	29.6	52.5	V			28.9	61.4	13.94	12.8	5.58	29.3	4.07	0.1	0.181	0.278
6	12/3/1861	38	23.7	V			27.4	64.3	13.37	13.3	-1.6	29.5	5.2	0.13	0.238	0.295
58	22/6/1876	38	23.7				20.1	66.6	12.88	11.7	5.42	29.5	5.08	-0.01	0.07	0.166
530	13/5/2002	32.6	51.7	V	V		29.1	62.3	14.02	12.5	6.33	29.6	4.21	0.08	0.16	0.262
533	13/5/2002	29.4	48	V	V		29.2	62.1	14.09	12.9	5.57	29.6	4.22	0.12	0.197	0.294
278	25/4/1990	41.6	-73.7	V		V	19.8	67.3	12.82	12.8	0.63	29.7	5.43	0.08	0.183	0.244
150	9/1/1978	43	-89.8	V	V		19.2	69.6	12.28	11.9	2.88	29.7	5.27	0	0.1	0.169
539	11/6/2002	32.4	-111	V	V		27.3	63	13.96	12.2	6.79	29.7	4.4	0.06	0.132	0.237
103	29/1/1922	-34	18.5				42.4	40.8	19.55	8.85	-17	29.7	1.12	-0.07	0.177	0.167
449	24/2/2001	29.6	52.5	V			30.5	60.1	14.79	13.3	6.46	29.8	4.13	0.16	0.24	0.343
28	24/6/1865	38	23.7				34.2	45.1	18.57	8.75	16.4	29.8	1.73	-0.11	-0.04	0.121
2	27/10/1859	38	23.7	V			39.2	33.6	21.43	6.8	20.3	29.8	-3.5	-0.19	-0.11	0.072
83	1/5/1908	44.1	3.1	V			27.7	60.4	14.76	10.7	10.2	29.9	4.06	-0.05	0.024	0.148
454	24/2/2001	32.6	51.7	V	V		30.5	60.3	14.79	13	7.16	29.9	4.09	0.14	0.21	0.317
446	24/2/2001	36	50.8	V			30.5	60.6	14.8	12.5	7.93	29.9	4.03	0.1	0.173	0.284
138	21/12/1976	42.7	-83.6	V			20.5	69.4	12.6	11.7	4.68	30	5.47	-0.01	0.071	0.16
380	10/10/1999	34	-6.8	V			30.9	57.7	15.55	12.4	9.4	30	6.46	0.11	0.181	0.301
527	13/4/2002	32.4	-111	V	V		30.6	62	14.49	13.1	6.21	30	4.4	0.14	0.219	0.319
588	2/4/2003	33.8	-118	V			31.4	62	14.54	13.4	5.72	30	4.84	0.16	0.241	0.339
563	5/11/2002	32.4	-111	V	V	V	28.4	54	16.56	11.5	12	30.1	3.36	0.06	0.13	0.265
675	18/7/2004	32.7	51.7	V	V		28.8	63.3	14.25	12.5	6.81	30.1	5.8	0.09	0.164	0.269
244	28/4/1987	37	-122	V			25.9	68	13.13	13	1.76	30.1	5	0.1	0.207	0.269
76	30/3/1881	51.5	-2.6	V			20.7	73.3	11.81	11.7	1.62	30.1	5.92	-0.03	0.075	0.137
716	13/11/2004	36.8	-81.8	V	V		32.2	46.1	18.66	8.7	16.5	30.2	-0.4	-0.11	-0.04	0.12
531	13/5/2002	33.3	44.4	V	V		29.6	63.7	14.27	12.7	6.58	30.2	4.48	0.1	0.177	0.281
155	9/1/1978	41.6	-93.6	V			19.6	71	12.46	12.2	2.65	30.2	5.25	0.02	0.124	0.191
674	18/7/2004	35.7	51.3	V	V		28.9	63.9	14.31	12.1	7.59	30.3	6.15	0.06	0.134	0.243
136	5/3/1973	40	-85	V	V		24	67	13.56	13.5	-1	30.3	5.78	0.15	0.259	0.318
554	7/10/2002	32.5	51.7	V	V		27.3	55.2	16.56	12.1	11.4	30.4	5.36	0.11	0.178	0.31
576	3/1/2003	32.4	-111	V			28.7	59	15.63	11.7	10.4	30.4	2.62	0.06	0.129	0.255
634	24/12/2003	33.4	73.1	V	V		26.8	56.9	16.21	10.7	12.2	30.4	1.63	-0.01	0.056	0.192
553	7/10/2002	49.6	8.7	V	V		29.9	49.5	18.07	8.53	16	30.4	5.01	-0.14	-0.07	0.084
139	21/12/1976	42	-91.6	V	V		21	70.4	12.91	12	4.71	30.5	5.71	0.02	0.103	0.193
564	5/11/2002	32	-117	V			28.8	55	16.76	11.6	12.1	30.5	3.59	0.08	0.148	0.283
448	24/2/2001	-34	18.4	V			33.6	57.6	16.14	12.4	-10	30.5	6.85	0.12	0.296	0.317
243	28/4/1987	40.7	-112	V			25.8	70	13.08	12.7	3.15	30.6	5.42	0.08	0.176	0.247
124	5/3/1954	44.5	-88	V			21.1	69.9	13.15	13.2	0.2	30.6	6.12	0.12	0.216	0.278
536	13/5/2002	31.9	35.8	V		V	30.2	64.8	14.53	13.1	6.33	30.7	4.72	0.14	0.216	0.318
575	3/1/2003	33.9	-118		V		29	60	15.81	11.5	10.9	30.8	2.63	0.05	0.115	0.243

S. No.	Date	Lat	Long.	VISIBILITY			Age	Lag	ARCL	ARCV	DAZ	MODEL				
				N	B	T						A	B	C	D	E
325	3/9/1997	31.8	34.7	V			40.6	50.2	18.3	11.2	14.5	30.8	4.2	0.09	0.156	0.304
133	25/4/1971	39.5	-88.2	V	V		21.2	71	13.22	13.2	0.33	31	5.51	0.12	0.224	0.285
423	28/10/2000	32.6	51.7	V			30.3	60.9	15.81	12.9	9.21	31	6.17	0.15	0.225	0.343
708	15/10/2004	32.9	59.2	V	V	V	35.1	48.8	19.01	10.4	15.9	31.2	2.31	0.04	0.116	0.272
80	29/5/1900	38.7	-0.7	V			29	63.9	15.34	11.7	9.98	31.3	4.8	0.05	0.117	0.24
633	24/12/2003	49.6	8.7	V	V		30.2	53.8	18.11	7.21	16.6	31.6	-0.7	-0.26	-0.19	-0.03
671	18/6/2004	36.8	-81.8		V		29	71	13.88	12.6	5.92	31.6	5.93	0.08	0.16	0.26
81	19/4/1901	50.7	-2.8	V			22.1	74.2	13.14	11.5	6.43	31.7	5.91	-0.02	0.056	0.158
570	5/12/2002	35.7	51.4	V	V		30.2	59.2	16.94	10.9	13	31.7	2.48	0.02	0.094	0.234
710	15/10/2004	32.6	51.7	V	V		35.6	49.8	19.29	10.7	16.1	31.7	2.64	0.07	0.145	0.302
709	15/10/2004	32.6	51.6	V	V		35.6	49.8	19.3	10.7	16.1	31.8	2.64	0.07	0.145	0.302
635	24/12/2003	35.7	51.3	V	V	V	28.2	59.6	16.98	10.8	13.2	31.9	2	0.01	0.083	0.225
711	15/10/2004	30.2	57.1	V	V	V	35.3	51.4	19.12	11.2	15.5	32	3.08	0.12	0.186	0.341
16	28/7/1862	38	23.7				44.9	39.2	22.29	8.15	20.8	32.1	1.04	-0.04	0.042	0.224
440	25/1/2001	40.8	-74		V		33.5	66.9	15.35	12	9.53	32.1	4.21	0.08	0.147	0.268
441	25/1/2001	40.4	-74.5	V			33.5	67	15.37	12.1	9.45	32.1	4.24	0.08	0.155	0.275
141	21/12/1976	37.6	-123	V			23.3	72	14.14	13.5	4.36	32.1	6.47	0.16	0.248	0.332
188	5/12/1983	37.2	-84.1			V	34.3	61	16.98	11	12.9	32.2	2.99	0.04	0.106	0.246
568	5/12/2002	32.6	51.7	V	V		30.3	61.1	17.01	11.7	12.3	32.3	3.06	0.09	0.164	0.3
79	7/12/1885	50.6	5.7	V			26.8	75.1	13.51	10.7	8.26	32.3	7.64	-0.08	-0	0.11
733	11/1/2005	32.6	51.6	V	V		26.2	63.8	16.38	12.3	10.8	32.3	3.3	0.13	0.195	0.323
503	16/11/2001	49.6	8.7	V	V		33.5	58.4	17.76	8.7	15.5	32.3	3.1	-0.14	-0.07	0.087
13	1/1/1862	37.9	22.9	V			26	71.2	14.57	13.1	6.42	32.4	5.5	0.14	0.217	0.319
636	24/12/2003	32.7	51.7	V			28.3	61.5	17.05	11.6	12.5	32.4	2.58	0.08	0.153	0.291
85	31/1/1911	51	-0.9	V			31.6	65.5	16.4	9.68	13.3	32.8	3.26	-0.09	-0.02	0.119
129	6/4/1970	48	-122	V			23.2	78	13.29	13.1	2.42	32.8	6.79	0.11	0.212	0.278
466	22/6/2001	35.7	51.3	V	V		28.4	67.3	16.16	12.4	10.4	33	5.85	0.13	0.195	0.321
445	24/2/2001	51.7	7.2	V	V		33.2	69	15.98	10.9	11.7	33.2	4.87	-0	0.068	0.201
1	1/7/1859	38	23.7	V			27.7	67.5	16.42	12.3	11	33.3	6.65	0.12	0.19	0.319
144	11/12/1977	47.8	20	V			21.8	77.6	13.93	11.6	7.67	33.3	7.66	0.01	0.084	0.194
641	22/1/2004	41.8	-123	V	V		28.8	66	16.85	11.7	12.2	33.4	3.74	0.09	0.156	0.291
465	22/6/2001	38.2	46	V	V		28.9	68.1	16.44	12.1	11.2	33.5	6.11	0.11	0.175	0.305
22	4/8/1864	38	23.7				51.3	39.7	23.81	8.34	22.3	33.7	1.43	0.03	0.121	0.31
732	11/1/2005	43.9	18.4	V			27.9	65.9	17.35	10.5	13.8	33.8	2.87	0	0.071	0.216
598	2/5/2003	51.7	-9.5	V	V	V	32.4	78	14.61	11.1	9.53	34.1	6.3	-0.02	0.052	0.173
78	12/3/1899	52.5	13.3	V			21.8	83.6	13.32	13.2	1.6	34.2	7.84	0.12	0.227	0.289
297	23/2/1993	-34	18.4	V			52.7	40	24.33	8.81	-23	34.3	1	0.09	0.41	0.375
617	27/9/2003	49.6	8.7	V	V		38.4	49.9	22.05	8.65	20.3	34.5	5.83	-0.01	0.076	0.255
87	25/8/1911	49.9	2.3	V			39	53.7	21.5	9.26	19.4	34.9	9.39	0.03	0.106	0.281
92	1/4/1919	53.9	-1.6	V			22.2	87.1	13.51	12.9	4.15	35.3	7.9	0.1	0.19	0.271
180	13/7/1980	41.4	-70.7	V			41.9	59	20.89	10.7	18	35.6	6.33	0.12	0.199	0.367
464	22/6/2001	43.9	18.4	V			31.1	72.5	17.7	11.4	13.5	35.8	7.34	0.09	0.159	0.303
670	18/6/2004	47.6	-118	V	V		32.1	85	15.27	11.6	9.97	36.5	8.87	0.04	0.108	0.231
463	22/6/2001	49.6	8.7	V	V		32.2	73.5	18.31	10	15.4	36.7	7.74	-0.01	0.06	0.213
269	4/6/1989	50.8	-1	V			25	93.9	14.49	11.5	8.83	38	8.71	0.01	0.084	0.2
55	4/6/1875	51.5	-2.6	V			22.7	97	14.24	11.5	8.41	38.5	8.98	0.01	0.079	0.193

S. No.	Date	Lat	Long.	VISIBILITY			Age	Lag	ARCL	ARCV	DAZ	MODEL				
												A	B	C	D	E
				N	B	T										
117	14/5/1934	50	36.2	V			29.4	100	14.91	13.4	6.49	39.9	9.24	0.18	0.253	0.355
119	13/6/1934	55.6	33.9	V	V		41	97.2	19.04	10.1	16.1	43.3	11.4	0.02	0.093	0.25
116	14/5/1934	55.6	33.9	V			30.1	118	15.22	12.7	8.44	44.6	10.7	0.12	0.197	0.311
112	25/5/1933	55.6	33.9	V			32.9	121	15.67	12.1	9.96	45.9	11.4	0.09	0.16	0.283

APPENDIX-III

PHYSICAL MODELS

Ser.	Date	Lat	Long.	VISIBILITY			Age	Lag	ARCL	ARCV	DAZ	Width	MODEL		
No.				N	B	T							Bruin	Yallop	Qureshi
514	12/2/2002	43.9	18.4				8.55	4.83	5.94	1.56	5.73	4.76	-1.01	-0.98	-0.85
565	4/12/2002	30.9	35.8				7.07	9.77	4.04	2.58	3.12	2.44	-0.94	-0.9	-0.76
730	10/1/2005	11.2	7.6				5.41	10.6	5.94	3	5.13	5.4	-0.86	-0.83	-0.7
481	17/9/2001	4.1	73.3				2.72	12.5	5.23	3.73	-3.7	4.16	-0.8	-0.77	-0.63
345	16/2/1999	33.3	44.4				8.27	16.8	4.47	4.05	1.89	2.91	-0.79	-0.75	-0.61
189	3/1/1984	15.6	35.6				10.2	16.2	5.51	4.21	3.56	4.16	-0.76	-0.72	-0.59
498	15/11/2001	24.3	54.3				7.07	18.4	4.62	4.61	0.37	3.11	-0.73	-0.69	-0.55
699	14/10/2004	26.6	50				11.5	17	6.21	4.41	4.37	5.62	-0.71	-0.68	-0.55
247	26/6/1987	-30	-71				16.4	17.7	8.99	4.02	-8.1	10.9	-0.68	-0.67	-0.55
700	14/10/2004	30.4	35.5				12.5	17.5	6.7	4.37	5.08	6.56	-0.7	-0.68	-0.55
499	15/11/2001	-34	18.4				10.9	19.9	6.36	4.46	-4.5	5.87	-0.71	-0.68	-0.55
500	15/11/2001	31.9	35.8				8.11	21.1	5.06	4.85	1.45	3.73	-0.7	-0.66	-0.53
566	4/12/2002	10.3	9.8				9.53	18.8	5.42	4.86	2.41	4.38	-0.69	-0.65	-0.52
626	24/11/2003	32.9	59.2				14.1	20.7	8.57	4.55	7.26	11.2	-0.62	-0.61	-0.49
664	18/6/2004	-34	18.4				19.5	21.3	9.79	4.46	-8.7	12.9	-0.61	-0.61	-0.49
724	12/12/2004	32.5	3.7				15.3	20.6	10.07	4.38	9.06	15.4	-0.58	-0.59	-0.48
659	19/5/2004	29.4	48				10.9	24	5.37	5.36	-0.2	3.88	-0.64	-0.61	-0.47
567	4/12/2002	6.5	3.4				10.1	20.7	5.73	5.31	2.14	4.89	-0.63	-0.6	-0.47
627	24/11/2003	32.6	51.7				14.7	22	8.88	4.8	7.47	12	-0.59	-0.58	-0.46
482	17/9/2001	26.6	50				4.41	21.8	5.62	5.5	-1.2	4.81	-0.62	-0.58	-0.45
41	18/6/1871	38	23.7				15.5	26.8	7.05	5.39	4.55	6.7	-0.6	-0.58	-0.45
106	27/4/1922	-34	18.5				11.3	23.7	6.13	5.54	-2.6	5.2	-0.61	-0.58	-0.44
368	13/7/1999	24.6	46.5				13.5	22.6	7.79	5.36	5.65	9.03	-0.57	-0.55	-0.43
518	14/3/2002	26.6	50				12.9	22.2	7.65	5.56	5.27	7.86	-0.57	-0.55	-0.42
519	14/3/2002	26.2	50.5				12.9	22.2	7.64	5.58	5.23	7.83	-0.56	-0.55	-0.42
444	23/2/2001	-34	18.4				9.34	25.6	5.98	5.85	-1.3	4.84	-0.58	-0.55	-0.42
520	14/3/2002	30.4	35.5				13.9	23.3	7.99	5.63	5.67	8.56	-0.55	-0.53	-0.41
383	8/12/1999	-34	18.4				19.5	26.5	9.36	5.58	-7.5	11.7	-0.51	-0.51	-0.39
501	15/11/2001	10.3	9.8				10.4	23.6	6.11	6.11	0.03	5.42	-0.55	-0.52	-0.39
678	16/8/2004	-34	18.4				15.1	26.6	8.54	5.88	-6.2	10.1	-0.5	-0.49	-0.37
515	12/2/2002	-34	18.4				10.2	28.8	6.49	6.38	-1.2	5.68	-0.52	-0.49	-0.36
3	23/1/1860	38	23.7				15.6	30.7	7.08	6.31	3.2	6.75	-0.51	-0.48	-0.35
53	20/12/1873	38	23.7				20.5	27.7	11.63	5.39	10.3	19.6	-0.43	-0.45	-0.35
521	14/3/2002	4.1	73.3				11.4	23.9	7.16	6.52	2.96	6.88	-0.48	-0.46	-0.33
502	15/11/2001	41.1	-74				15.2	33.1	8.43	6.34	5.56	10.3	-0.45	-0.44	-0.32
522	14/3/2002	32.5	3.7				16	27.5	8.76	6.38	6.02	10.3	-0.45	-0.44	-0.32
660	19/5/2004	32.5	3.7				14	32.3	6.74	6.72	0.62	6.12	-0.48	-0.45	-0.32
510	15/12/2001	32.6	51.7				16.9	31.6	8.47	6.49	5.45	10.1	-0.44	-0.43	-0.31
612	26/9/2003	26.6	50				11.6	27.9	7.42	6.85	2.83	8.22	-0.43	-0.41	-0.29
7	7/8/1861	38	23.7				28.7	21.7	16.03	4.97	15.2	37.5	-0.29	-0.32	-0.29
381	8/11/1999	32	35.9				11	31.4	7.03	7	0.66	6.68	-0.44	-0.41	-0.29
382	8/11/1999	31.8	34.7				11.1	31.5	7.06	7.03	0.66	6.74	-0.44	-0.41	-0.28
424	26/11/2000	32.6	51.7				14.5	33.1	7.51	7.01	2.71	7.76	-0.42	-0.4	-0.28
578	3/1/2003	32	51.9				17.5	31.9	9.96	6.53	7.53	14.3	-0.38	-0.38	-0.27
613	26/9/2003	30.4	35.5				12.6	29.3	7.89	6.95	3.75	9.31	-0.41	-0.39	-0.27

Ser.	Date	Lat	Long.	VISIBILITY			Age	Lag	ARCL	ARCV	DAZ	Width	MODEL		
No.				N	B	T							Bruin	Yallop	Qureshi
220	31/12/1986	39	-77				19	33.2	12.38	6.03	10.8	23.3	-0.32	-0.35	-0.26
305	2/3/1995	-34	18.4				29.8	23.2	15.46	5.45	-14	33.5	-0.28	-0.31	-0.26
720	13/11/2004	36.1	50.3				23.3	29.1	13.57	5.91	12.2	27.6	-0.29	-0.32	-0.25
507	15/12/2001	31.8	35.2				18.1	34.1	9.05	7.02	5.71	11.5	-0.37	-0.36	-0.25
702	14/10/2004	6.5	3.4				14.9	27.1	8.03	7.19	3.57	9.42	-0.38	-0.37	-0.24
335	28/3/1998	-34	18.4				13.8	30.9	8.74	7.03	-5.2	11.7	-0.37	-0.36	-0.24
614	26/9/2003	-34	18.4				13.8	31.6	8.53	7.1	-4.7	10.9	-0.37	-0.36	-0.24
371	13/7/1999	43.3	-79.9				22.5	32	12.71	6.3	11.1	23.9	-0.29	-0.31	-0.23
425	26/11/2000	32	35.9				15.6	34.8	7.98	7.39	3	8.73	-0.37	-0.35	-0.23
192	2/2/1984	15.6	35.6				16	29.8	8.64	7.4	4.46	10	-0.35	-0.34	-0.22
703	14/10/2004	-34	18.4				14.4	34.3	7.77	7.5	-2	8.81	-0.36	-0.34	-0.22
646	21/3/2004	-34	18.4				18.5	31.6	9.55	7.21	-6.3	12.7	-0.34	-0.33	-0.22
523	14/3/2002	27.7	-11.3				17.1	30.6	9.15	7.33	5.49	11.2	-0.34	-0.34	-0.22
679	16/8/2004	35.7	51.3				14.3	35.3	8.21	7.49	3.35	9.3	-0.35	-0.34	-0.22
257	16/4/1988	37.2	-84.1				12.5	34.9	7.7	7.61	-1.2	8.73	-0.35	-0.33	-0.21
369	13/7/1999	-34	18.4				13.8	37.2	7.92	7.58	-2.3	9.34	-0.34	-0.33	-0.21
370	13/7/1999	-34	18.4				13.8	37.2	7.92	7.58	-2.3	9.34	-0.34	-0.33	-0.21
400	6/3/2000	43.3	-79.9				18.3	36.8	10.09	7.21	7.06	14.4	-0.31	-0.32	-0.2
680	16/8/2004	30.2	57.1				13.7	33.9	8	7.64	2.37	8.84	-0.34	-0.33	-0.2
384	8/12/1999	-4	39.7				17.1	30.6	8.39	7.69	-3.4	9.44	-0.33	-0.32	-0.19
389	7/1/2000	-34	18.4	V			24.1	35.1	10.96	7.19	-8.3	16.2	-0.29	-0.3	-0.19
388	7/1/2000	23.7	90.4				17.5	34.5	7.99	7.84	1.51	8.62	-0.33	-0.31	-0.19
476	19/8/2001	38.6	48.2				13	37.3	8.89	7.61	4.6	12.1	-0.3	-0.3	-0.18
211	23/11/1984	15.6	35.6				16.4	31.5	9.21	7.6	5.2	12.5	-0.3	-0.3	-0.18
390	7/1/2000	-32	20.8				23.8	35.3	10.86	7.36	-8	15.9	-0.28	-0.29	-0.18
422	27/10/2000	38.8	-77.2				14.5	37.9	8.69	7.78	3.88	10.6	-0.31	-0.3	-0.18
681	16/8/2004	26.6	50				14.1	33.9	8.15	7.9	2.01	9.18	-0.31	-0.3	-0.18
572	3/1/2003	26.6	49.8				17.9	34.7	10.15	7.47	6.87	14.8	-0.28	-0.29	-0.18
583	3/3/2003	-34	18.4				15	35.3	8.58	7.88	-3.4	10.1	-0.3	-0.29	-0.17
216	20/4/1985	37.2	-84.1				19.2	37	8.68	7.92	3.54	10.1	-0.3	-0.29	-0.17
706	14/10/2004	25.8	-80.2				20.3	30.7	10.95	7.37	8.11	17.5	-0.26	-0.27	-0.17
647	21/3/2004	29.4	48				16.6	33.7	8.68	7.89	3.62	10.5	-0.3	-0.29	-0.17
408	4/5/2000	-34	18.4				12.1	36.8	8.28	7.92	-2.4	10.2	-0.3	-0.29	-0.17
20	6/5/1864	39.6	26.2				17.3	39.8	9.08	7.8	4.66	11.8	-0.29	-0.28	-0.16
538	11/6/2002	31.9	35.8				17.2	39.1	8.78	7.87	3.9	10.9	-0.29	-0.28	-0.16
426	26/11/2000	-4	39.7				16.4	31.8	8.3	7.99	-2.2	9.44	-0.3	-0.29	-0.16
682	16/8/2004	32	35.9			V	15.2	36.1	8.58	7.96	3.21	10.2	-0.29	-0.28	-0.16
648	21/3/2004	26.6	50				16.4	33.3	8.61	7.98	3.23	10.4	-0.29	-0.28	-0.16
477	19/8/2001	29.5	56.8		V		12.2	35	8.47	7.95	2.92	10.9	-0.29	-0.28	-0.16
480	19/8/2001	32.5	51.3				12.6	36.1	8.7	7.94	3.56	11.5	-0.28	-0.27	-0.15
683	16/8/2004	30.2	35.5				15.2	35.8	8.57	8.05	2.94	10.2	-0.29	-0.27	-0.15
549	7/9/2002	31.1	56.5		V		11.6	34.8	8.44	8	2.7	10.8	-0.28	-0.27	-0.15
386	8/12/1999	36.8	10.4		V		17.8	41.6	8.68	8.07	3.21	10.1	-0.28	-0.27	-0.15
385	8/12/1999	26.2	32.7				16.7	36.8	8.24	8.15	1.22	9.1	-0.29	-0.28	-0.15
427	26/11/2000	32.5	3.7				17.8	38.6	8.91	8.05	3.82	10.9	-0.28	-0.27	-0.15
299	3/12/1994	-34	18.4				18.1	36.9	11.14	7.48	-8.3	18.8	-0.23	-0.24	-0.15

Ser.	Date	Lat	Long.	VISIBILITY			Age	Lag	ARCL	ARCV	DAZ	Width	MODEL		
No.				N	B	T							Bruin	Yallop	Qureshi
551	7/9/2002	32.5	51.3				12	35.6	8.62	8.05	3.07	11.3	-0.27	-0.26	-0.14
557	5/11/2002	29.9	56.2		V		17.1	35.2	10.11	7.82	6.4	15.4	-0.24	-0.24	-0.14
638	22/1/2004	30	51.7		V		17.2	36.3	10.88	7.7	7.69	17.5	-0.23	-0.24	-0.13
649	21/3/2004	38	23.7				18.2	38.2	9.44	8.07	4.9	12.4	-0.25	-0.25	-0.13
428	26/11/2000	34	-6.8				18.4	39.9	9.19	8.14	4.28	11.6	-0.26	-0.25	-0.13
318	7/5/1997	36	50.8				19	37.4	11.16	7.68	8.1	18	-0.22	-0.23	-0.13
10	5/10/1861	38	23.7				33.3	24.9	20.2	5.54	19.4	61.5	-0.07	-0.05	-0.13
207	25/9/1984	15.6	35.6				12.6	31.7	8.42	8.24	1.73	10.8	-0.26	-0.25	-0.13
558	5/11/2002	30.1	52.1		V		17.4	35.7	10.26	7.9	6.56	15.9	-0.23	-0.23	-0.12
650	21/3/2004	30.4	35.5				17.4	35.7	9.06	8.24	3.77	11.5	-0.25	-0.24	-0.12
308	22/12/1995	36.1	50.7				11.4	42.7	8.33	8.32	0.31	10.6	-0.25	-0.24	-0.12
559	5/11/2002	29.6	52.5		V		17.4	35.8	10.25	7.95	6.48	15.9	-0.22	-0.23	-0.12
455	25/3/2001	-34	18.4	V			15.8	36.9	9.06	8.3	-3.6	11.3	-0.24	-0.24	-0,12
629	24/11/2003	41.5	-112		V		25.4	38.9	15.19	6.77	13.6	35	-0.13	-0.16	-0.12
328	30/12/1997	-34	18.4		V		25.3	34.8	14.04	7.15	-12	28.6	-0.16	-0.18	-0.12
429	26/12/2000	29.6	52.5				20.6	39.6	9.36	8.29	4.35	11.8	-0.24	-0.23	-0.12
528	13/4/2002	32.6	51.7				19.9	36.8	9.89	8.18	5.57	13.2	-0.23	-0.23	-0.12
391	7/1/2000	32.7	52.3		V		19.7	40.9	9.01	8.36	3.36	11	-0.24	-0.24	-0.12
340	19/12/1998	31.9	35.8				16.2	41	8.55	8.47	1.13	10	-0.25	-0.23	-0.11
374	10/9/1999	31.8	34.7				18.1	35.8	9.87	8.2	5.49	13.7	-0.22	-0.22	-0.11
725	12/12/2004	11.2	7.6				15.9	34.1	10.37	8.02	6.58	16.4	-0.21	-0.22	-0.11
589	2/4/2003	32.6	51.6				19.9	37.1	9.6	8.32	4.8	12.4	-0.23	-0.22	-0.11
726	12/12/2004	10.3	9.8				15.8	34.2	10.31	8.06	6.43	16.2	-0.21	-0.21	-0.11
274	25/2/1990	35.6	-83.5	V		V	14.8	39.3	8.53	8.51	-0.6	10.7	-0.23	-0.22	-0.1
275	25/2/1990	35.6	-83.5			V	14.8	39.3	8.53	8.51	-0.6	10.7	-0.23	-0.22	-0.1
276	25/2/1990	35.6	-83.5				14.8	39.3	8.53	8.51	-0.6	10.7	-0.23	-0.22	-0.1
101	31/10/1921	-34	18.5				17.9	38.3	9.84	8.27	-5.3	14	-0.21	-0.21	-0.1
326	2/10/1997	31.8	34.7				22.8	35.6	10.71	8.17	6.94	15.5	-0.2	-0.21	-0.1
375	10/9/1999	30.4	35.5			V	18.1	35.9	9.84	8.31	5.26	13.6	-0.21	-0.21	-0.1
430	26/12/2000	32.6	35.9				21.6	41.7	9.81	8.39	5.09	13	-0.21	-0.21	-0.1
560	5/11/2002	31.9	35.8		V		18.4	37.2	10.86	8.04	7.31	17.8	-0.19	-0.2	-0.1
321	7/5/1997	32.7	52.3		V		18.8	37.8	11.06	8.06	7.57	17.7	-0.19	-0.2	-0.1
714	13/11/2004	32	35.9			V	24.4	34.8	14.21	7.27	12.2	30.2	-0.13	-0.16	-0.1
573	3/1/2003	32.5	3.7		V		20.8	40.1	11.61	7.96	8.46	19.4	-0.18	-0.19	-0.09
524	13/4/2002	29.6	52.5				19.8	36.8	9.85	8.42	5.12	13.1	-0.21	-0.21	-0.09
478	19/8/2001	30.2	35.5		V	V	13.6	37.8	9.24	8.47	3.69	13	-0.21	-0.2	-0.09
281	24/5/1990	35.6	-83.5			V	12.7	47	8.67	8.66	-0.5	11.4	-0.21	-0.2	-0.08
431	26/12/2000	26.6	50				20.9	40	9.48	8.64	3.92	12.1	-0.2	-0.2	-0.08
341	18/1/1999	-34	18.4	V			26.5	37.4	13.31	7.79	-11	25	-0.13	-0.15	-0.07
15	29/4/1862	38	23.7				18.1	44.6	8.86	8.85	0.2	10.5	-0.2	-0.19	-0.07
39	20/4/1871	38	23.7				22.3	40	11.09	8.4	7.25	16.6	-0.17	-0.17	-0.07
105	29/3/1922	-34	18.5				28	35.1	12.91	8	-10	22.6	-0.14	-0.16	-0.07
432	26/12/2000	30.2	35.5		V	V	21.7	41.9	9.87	8.67	4.72	13.1	-0.18	-0.18	-0.07
312	20/1/1996	34.1	-118			V	12.7	41	8.92	8.78	-1.6	12.1	-0.19	-0.18	-0.06
184	5/11/1983	15.6	35.6				17	35.4	9.29	8.78	3.05	12.4	-0.18	-0.18	-0.06
264	5/5/1989	42.7	-84.8			V	12.7	53	8.91	8.85	-1	11.9	-0.18	-0.18	-0.06

Ser.	Date	Lat	Long.	VISIBILITY			Age	Lag	ARCL	ARCV	DAZ	Width	MODEL		
No.				N	B	T							Bruin	Yallop	Qureshi
265	5/5/1989	42.7	-84.8				12.7	53	8.91	8.85	-1	11.9	-0.18	-0.18	-0.06
688	15/9/2004	36.6	59		V		24	37.3	12.44	8.14	9.41	22	-0.13	-0.15	-0.06
212	23/11/1984	34	-81			V	23.6	39.7	13.24	7.92	10.6	25.8	-0.11	-0.13	-0.06
433	26/12/2000	-34	18.4	V			24.9	43	11.34	8.49	-7.5	17.3	-0.15	-0.16	-0.06
592	2/5/2003	-34	18.4				28.1	39	12.69	8.2	-9.7	21.6	-0.13	-0.15	-0.06
266	5/5/1989	43	-85.7			V	12.8	53	8.95	8.9	-0.9	12	-0.18	-0.17	-0.05
199	1/5/1984	37.2	-84.1				20.5	44	9.99	8.77	4.81	13.8	-0.16	-0.17	-0.05
137	1/7/1973	-44	171		V		17.9	51.6	10.61	8.55	-6.3	17	-0.15	-0.16	-0.05
338	21/9/1998	31.8	35.2				22.9	37.6	10.97	8.61	6.81	16.2	-0.15	-0.16	-0.05
615	26/9/2003	41.8	-112		V		22.3	39	13.01	8.02	10.3	25.3	-0.1	-0.13	-0.05
434	26/12/2000	-32	20.8	V			24.7	42.9	11.23	8.62	-7.2	17	-0.14	-0.15	-0.05
435	26/12/2000	-32	20.8		V		24.7	42.9	11.23	8.64	-7.2	17	-0.14	-0.15	-0.04
260	14/6/1988	37.2	-84.1				15.7	51	9.15	9.09	1.07	11.6	-0.16	-0.16	-0.04
94	8/2/1921	36.5	-6.2				17.7	43.2	9.25	9.1	-1.7	11.6	-0.16	-0.16	-0.04
301	1/1/1995	33	-106			V	13.5	46.5	9.05	9.05	-0.3	12.3	-0.16	-0.15	-0.04
387	8/12/1999	6.5	3.4				19.3	37.2	9.27	9.14	-1.6	11.5	-0.16	-0.15	-0.03
586	2/4/2003	30.2	35.5		V	V	20.9	39.4	10.05	9	4.48	13.6	-0.15	-0.14	-0.03
286	20/9/1990	31.8	34.7	V			39.1	29.5	19.64	6.85	18.4	52.7	0.011	0.007	-0.03
324	4/8/1997	31.3	35.2				32.6	35.6	15.1	7.92	12.9	30.6	-0.06	-0.09	-0.03
689	15/9/2004	35.7	51.4				24.5	38.2	12.69	8.4	9.52	22.9	-0.09	-0.11	-0.03
95	8/2/1921	38.8	-9.1				17.8	45.3	9.31	9.22	-1.3	11.7	-0.15	-0.14	-0.02
316	8/2/1997	-34	18.4	V			26.9	34.3	16.1	7.51	-14	39.3	-0.02	-0.05	-0.02
600	31/5/2003	26	-80.3				20.1	41.5	9.42	9.25	1.78	12	-0.14	-0.14	-0.02
319	7/5/1997	31.8	34.9	V			19.9	40.9	11.64	8.74	7.68	19.6	-0.1	-0.11	-0.01
633	24/12/2003	49.6	8.7	V	V		30.2	53.8	18.11	7.21	16.6	48.9	0.022	0.01	-0.01
690	15/9/2004	34.7	50.9				24.5	38.6	12.7	8.58	9.38	22.9	-0.07	-0.09	-0.01
727	12/12/2004	32.4	-111	V	V	V	23.2	44.2	14.43	8.13	11.9	31.6	-0.03	-0.06	-0
51	27/4/1873	38	23.7				18.8	45.9	10.21	9.23	4.37	15.1	-0.1	-0.11	0.003
364	14/6/1999	29.6	52.5				20.8	41.4	12.62	8.67	9.18	24	-0.05	-0.07	0.007
134	15/3/1972	35.5	-118				14.8	42	9.69	9.35	-2.5	14.2	-0.1	-0.1	0.008
135	15/3/1972	35.5	-118		V		14.8	42	9.69	9.35	-2.5	14.2	-0.1	-0.1	0.008
2	27/10/1859	38	23.7	V			39.2	33.6	21.43	6.8	20.3	65	0.076	0.109	0.008
508	15/12/2001	-4	39.7				19	39.3	9.48	9.48	-0.1	12.6	-0.11	-0.11	0.009
392	7/1/2000	-4	39.7				21.7	38.3	9.89	9.44	-2.9	13.2	-0.11	-0.1	0.01
272	1/10/1989	31.3	34.6	V			41.9	32.1	19.52	7.34	18.1	50.7	0.047	0.038	0.011
416	31/7/2000	6.5	3.4	V			15.9	37.7	9.58	9.41	1.8	13.9	-0.1	-0.1	0.011
691	15/9/2004	33.3	50.1		V		24.6	39.2	12.73	8.82	9.19	23	-0.05	-0.07	0.016
314	21/1/1996	-34	18.4	V			29.4	35.2	17.89	7.51	-16	48.1	0.047	0.032	0.018
337	26/5/1998	31.8	35.2			V	21.4	41.9	13.12	8.72	9.81	25.5	-0.03	-0.06	0.021
412	2/7/2000	2.3	102		V	V	16.3	39	9.69	9.5	1.92	14.3	-0.09	-0.09	0.024
248	26/6/1987	42.7	-84.5		V		20.2	59	10.47	9.49	4.42	14.7	-0.08	-0.08	0.026
415	2/7/2000	32.6	51.7		V		20.7	44.4	12.3	8.92	8.47	23	-0.04	-0.06	0.026
48	3/10/1872	38	23.7				24.9	41.4	12.75	8.97	9.07	22.8	-0.04	-0.06	0.029
365	14/6/1999	29.4	48				21.1	42.2	12.79	8.86	9.24	24.7	-0.03	-0.05	0.03
672	18/6/2004	33.3	50				19.8	50.9	9.93	9.68	2.25	13.2	-0.08	-0.08	0.034
639	22/1/2004	32.5	3.7	V	V		20.4	44.6	12.49	9.03	8.63	23	-0.03	-0.05	0.037

Ser.	Date	Lat	Long.	VISIBILITY			Age	Lag	ARCL	ARCV	DAZ	Width	MODEL		
No.				N	B	T							Bruin	Yallop	Qureshi
304	31/1/1995	35.6	51.3		V		15.6	47	9.82	9.66	-1.8	14	-0.07	-0.07	0.038
185	5/11/1983	37.2	-84.1			V	24.6	44.7	13.11	8.95	9.59	24.6	-0.02	-0.04	0.039
665	18/6/2004	26.6	50				19.5	47.1	9.81	9.77	0.85	12.9	-0.08	-0.07	0.04
45	6/7/1872	39	23.7				23.9	52	11.38	9.46	6.33	17.3	-0.05	-0.06	0.042
655	20/4/2004	5	115				21.4	38	10.03	9.73	-2.4	13.7	-0.07	-0.07	0.042
666	18/6/2004	28.4	48				19.7	48.4	9.9	9.82	1.26	13.1	-0.07	-0.07	0.047
334	27/2/1998	-34	18.4	V			24.3	39.4	14.23	8.69	-11	30.6	0.018	-0.01	0.049
667	18/6/2004	24.6	46.5				19.7	46.6	9.87	9.86	0.5	13.1	-0.07	-0.06	0.051
249	26/6/1987	37.2	-84.1			V	19.9	55	10.33	9.83	3.17	14.3	-0.05	-0.05	0.057
36	12/5/1869	38	23.7				25.7	45.9	13.48	9.1	9.96	25.4	0.005	-0.02	0.059
315	13/10/1996	31.8	34.9	V			25.2	41.6	12.72	9.27	8.73	22.8	-0.01	-0.03	0.059
282	24/5/1990	31.6	-111			V	15	50	9.86	9.86	0.14	14.8	-0.04	-0.05	0.063
283	24/5/1990	32.4	-111				15	50	9.88	9.87	0.27	14.8	-0.04	-0.04	0.065
393	7/1/2000	34	-6.8				23.7	49.6	10.8	9.81	4.51	15.8	-0.04	-0.04	0.065
484	17/10/2001	2.3	102		V	V	15.9	38.1	9.93	9.92	-0.4	14.6	-0.04	-0.04	0.068
267	5/5/1989	39.7	-106			V	14.8	55	9.93	9.92	-0.4	14.8	-0.04	-0.04	0.069
485	17/10/2001	32.6	51.7		V		18.9	43.8	11.47	9.66	6.2	19.5	-0.01	-0.02	0.077
593	2/5/2003	5	115			V	22.5	40	10.16	10.1	-1.3	13.9	-0.03	-0.03	0.078
668	18/6/2004	32	35.9			V	20.7	52.1	10.31	10.1	2.23	14.2	-0.03	-0.03	0.079
436	26/12/200[illegible]	-4	39.7				22.5	41.7	10.23	10.1	-1.6	14.1	-0.03	-0.03	0.082
290	15/2/1991	33.4	73.1	V			19.7	46.9	10.17	10.1	-0.9	14.5	-0.02	-0.02	0.087
443	25/1/2001	32.6	51.7				25.2	47.8	11.66	9.86	6.23	18.2	-0	-0.01	0.088
250	26/6/1987	39.8	-105				21.5	60	10.99	10.1	4.41	16.2	-0	-0.01	0.094
692	15/9/2004	28.4	48				24.7	41	12.79	9.6	8.46	23.3	0.032	0.011	0.096
376	10/9/199[illegible]	38.8	-77		V		25.7	44.9	13.54	9.45	9.7	25.6	0.043	0.019	0.096
621	26/10/2003	32.9	59.2		V		24.8	42.5	14.55	9.08	11.4	32.1	0.07	0.043	0.096
603	30/6/200[illegible]	33.7	58.4				21.1	52.9	10.9	10.1	4.15	16.4	-0	-0.01	0.097
285	24/5/1990	34.2	-118	V		V	15.6	52	10.17	10.1	0.81	15.7	-0	-0.01	0.098
284	24/5/1990	34.2	-118			V	15.6	52	10.17	10.1	0.81	15.7	-0	-0.01	0.098
511	14/1/200[illegible]	29.6	52.5				24.7	47.4	12.11	9.87	7.04	20	0.021	0.006	0.101
715	13/11/2004	4.9	115		V		19.9	40.3	11.62	9.86	6.16	20.2	0.023	0.008	0.102
251	26/6/1987	40.7	-112		V		22	61	11.19	10.1	4.78	16.8	0.009	0	0.105
88	28/11/19[illegible]3	-34	18.5	V			16.4	53.6	10.25	10.3	-0	15.3	0.002	-0	0.106
162	9/3/197[illegible]	45.1	-64.2	V			20	54.6	10.73	10.2	3.46	16.6	0.01	0.002	0.107
486	17/10/2001	29.6	52.5		B		18.9	44	11.47	9.97	5.67	19.5	0.025	0.011	0.107
256	19/1/19[illegible]8	32.2	-111			V	19.8	47	12.22	9.76	7.37	22.7	0.041	0.022	0.108
252	26/6/1987	30	-100				20.2	52	10.45	10.4	1.34	14.7	0.006	0.003	0.113
194	3/3/19[illegible]4	15.6	35.6				21.6	40.7	10.9	10.3	3.6	15.9	0.014	0.008	0.114
693	15/9/2004	26.6	50				24.6	41.3	12.72	9.82	8.1	23	0.051	0.031	0.116
622	26/10/2003	33.3	50.1	V	V		25.4	43.3	14.9	9.19	11.8	33.6	0.096	0.068	0.116
594	2/5/2003	3.2	102			V	23.4	41.3	10.55	10.4	-1.7	14.9	0.012	0.009	0.118
616	26/9/2003	32.4	-111	V	V	V	22.5	42	13.14	9.68	8.89	25.8	0.067	0.043	0.119
630	24/11/2003	6.5	3.4				18.8	42	11.31	10.1	5.12	19.5	0.037	0.023	0.119
694	15/9/2004	26.2	50.5		V		24.5	41.4	12.71	9.87	8.01	23	0.055	0.035	0.12
503	16/11/2001	49.6	8.7	V	V		33.5	58.4	17.76	8.7	15.5	45	0.143	0.123	0.123
437	25/1/2001	29.6	52.5		V		25.3	47.9	11.68	10.2	5.67	18.3	0.035	0.024	0.123

Ser.	Date	Lat	Long.	VISIBILITY			Age	Lag	ARCL	ARCV	DAZ	Width	MODEL		
No.				N	B	T							Bruin	Yallop	Qureshi
413	2/7/2000	30.4	35.5			V	21.8	47.6	12.89	9.75	8.45	25.3	0.068	0.045	0.123
252	26/6/1987	30	-100				20.6	52	10.63	10.4	1.95	15.1	0.019	0.016	0.124
553	7/10/2002	49.6	8.7	V	V		29.9	49.5	18.07	8.53	16	49.4	0.157	0.145	0.125
320	7/5/1997	-34	18.4	V			19.6	49.1	11.46	10.2	-5.3	19	0.04	0.027	0.125
623	26/10/2003	32.6	51.7		V		25.3	43.5	14.85	9.31	11.6	33.4	0.106	0.078	0.127
174	28/1/1979	42	-91.7		V		17.4	58.1	10.63	10.3	2.51	17.2	0.034	0.025	0.128
332	28/1/1998	29.8	-95.4		V		18.3	48.6	10.44	10.4	0.68	16.2	0.03	0.023	0.129
525	13/4/2002	30.5	-9.7		V		24.1	46.1	11.65	10.3	5.52	18.4	0.041	0.029	0.129
404	5/4/2000	5.3	103		V		17.4	40	10.62	10.4	2.12	16.4	0.031	0.024	0.129
487	17/10/2001	31.9	35.8			V	20	45.6	12.03	10.1	6.58	21.4	0.059	0.041	0.131
173	28/1/1979	29.9	-81.3	V			17	48.1	10.42	10.4	0.09	16.6	0.035	0.028	0.132
543	9/8/2002	2.3	102			V	16.4	41.6	10.49	10.4	-1	16.4	0.035	0.029	0.133
176	28/1/1979	42	-93.6	V			17.5	58.5	10.7	10.4	2.54	17.5	0.043	0.034	0.135
475	21/7/2001	32.6	51.7				20.3	49.4	12.27	10	7.07	22.8	0.07	0.05	0.137
175	28/1/1979	29.7	-82.4		V		17.1	48.2	10.47	10.5	0.06	16.7	0.042	0.034	0.138
177	28/1/1979	38.7	-90.3		V		17.4	55.5	10.64	10.5	1.87	17.2	0.048	0.039	0.142
695	15/9/2004	32.5	3.7		V		27.7	43.2	14.29	9.72	10.5	29	0.105	0.078	0.143
28	24/6/1865	38	23.7				34.2	45.1	18.57	8.75	16.4	48.6	0.174	0.161	0.144
488	17/10/2001	26.6	50				19.1	44.5	11.57	10.3	5.22	19.8	0.065	0.051	0.146
253	26/6/1987	33.5	-112	V			21.6	56.	11.01	10.6	2.97	16.2	0.049	0.043	0.148
218	12/12/1985	-32	20.8	V			17.1	55.2	10.78	10.5	-2.3	17.5	0.057	0.048	0.15
405	5/4/2000	32.6	51.7		V		21.1	46.5	12.43	10.2	7.12	22.5	0.083	0.064	0.151
716	13/11/2004	36.8	-81.8	V	V		32.2	46.1	18.66	8.7	16.5	52	0.192	0.185	0.153
16	28/7/1862	38	23.7				44.9	39.2	22.29	8.15	20.8	68.6	0.227	0.273	0.154
417	28/9/2000	-34	18.4	V			21.3	47.2	12.48	10.3	-7.1	22.5	0.089	0.069	0.156
418	28/9/2000	-34	18.4	V	V		21.3	47.2	12.48	10.3	-7.1	22.5	0.089	0.069	0.156
396	6/2/2000	-34	18.4	V	V		29.1	47.5	14.06	9.99	-9.9	27.3	0.114	0.088	0.159
102	30/12/1921	-34	18.5				36.7	44.4	17.96	9.13	-15	43.6	0.175	0.154	0.16
163	9/3/1978	42.7	-73.8	V	V		20.7	55	11.08	10.6	3.18	17.7	0.069	0.059	0.16
103	29/1/1922	-34	18.5				42.4	40.8	19.55	8.85	-17	50.9	0.2	0.191	0.163
347	18/3/1999	36	50.8	V			20.3	49	12.12	10.4	6.26	21.9	0.094	0.076	0.165
394	7/1/2000	6.5	3.4				23.9	44.6	10.87	10.8	-1	16	0.068	0.062	0.169
348	18/3/1999	-34	18.4	V	V		22.6	46.2	13.35	10.2	-8.7	26.6	0.124	0.099	0.172
349	18/3/1999	-34	18.4	V			22.6	46.3	13.35	10.2	-8.7	26.6	0.126	0.101	0.174
33	5/2/1867	38	23.7				22	53.7	10.96	10.8	1.58	16.7	0.079	0.071	0.175
142	18/2/1977	43.8	-87.7	V	V		20.2	58	10.88	10.9	0.66	16.6	0.079	0.072	0.176
489	17/10/2001	24.6	46.5			V	19.4	45	11.71	10.6	4.96	20.3	0.099	0.084	0.177
669	18/6/2004	32.5	3.7		V		22.9	56.9	11.25	10.9	2.95	17	0.083	0.075	0.178
306	28/6/1995	-30	-71	V			21.5	52.5	10.94	10.9	-0.2	16.1	0.081	0.075	0.181
544	9/8/2002	-34	18.4	V			21.3	50.4	13.09	10.3	-8.1	25.6	0.128	0.104	0.181
607	28/8/2003	5.3	103		V	V	18.2	42.4	10.89	10.9	0.4	17.3	0.09	0.081	0.183
85	31/1/1911	51	-0.9	V			31.6	65.5	16.4	9.68	13.3	37.7	0.182	0.155	0.186
96	8/2/1921	42.3	-71.1	V			21.9	57.9	10.98	11	0.13	16.3	0.088	0.081	0.186
22	4/8/1864	38	23.7				51.3	39.7	23.81	8.34	22.3	75.1	0.273	0.346	0.19
471	21/7/2001	4.1	73.3	V	V		18	44.8	10.93	10.9	0.56	18.1	0.104	0.093	0.193
608	28/8/2003	32.6	51.7		V		22	48.2	12.78	10.6	7.22	23.8	0.133	0.112	0.194

Ser.	Date	Lat	Long.	VISIBILITY			Age	Lag	ARCL	ARCV	DAZ	Width	MODEL		
No.				N	B	T							Bruin	Yallop	Qureshi
472	21/7/2001	32.6	35.9				21.4	51.7	12.89	10.5	7.53	25.2	0.139	0.117	0.195
31	17/1/1866	38	23.7				19.3	56.5	11.01	11	0.45	17.3	0.102	0.093	0.196
512	14/1/2002	24.6	46.5		V		25.3	49.3	12.38	10.8	6.14	20.9	0.12	0.103	0.196
89	16/3/1915	49.4	8.7	V			22.3	64.1	11.16	11	1.69	17.1	0.102	0.093	0.197
438	25/1/2001	24.6	46.5		V	V	25.8	49	11.93	10.9	4.75	19	0.117	0.104	0.201
473	21/7/2001	31.9	35.8	V		V	21.3	51.8	12.88	10.6	7.38	25.1	0.148	0.126	0.203
419	28/9/2000	26.2	32.7	V			20.1	46	11.93	10.9	4.93	20.5	0.126	0.111	0.204
624	26/10/2003	29.4	48		V		25.6	45.8	15.04	10.1	11.2	34.2	0.191	0.163	0.208
574	3/1/2003	-34	18.4	V			22.1	56.9	12.26	10.8	-5.7	21.6	0.137	0.119	0.209
456	24/4/2001	32.6	51.7	V	V		24.1	50.6	12.89	10.8	7.13	23.6	0.15	0.129	0.212
323	5/7/1997	-34	18.5	V			21.6	56.2	11.24	11.2	-1.2	17.3	0.12	0.111	0.214
617	27/9/2003	49.6	8.7	V	V		38.4	49.9	22.05	8.65	20.3	72.2	0.293	0.354	0.214
79	7/12/1885	50.6	5.7	V			26.8	75.1	13.51	10.7	8.26	24.7	0.158	0.135	0.215
93	19/4/1920	43.5	7	V			21.1	60.3	11.94	10.9	4.81	21.3	0.142	0.125	0.216
350	18/3/1999	29.6	52.5	V	V		20.2	48.3	12.06	11	5.05	21.7	0.149	0.131	0.22
420	28/9/2000	32.5	3.7	V	V		22	48.7	12.87	10.9	6.93	23.9	0.163	0.142	0.224
545	9/8/2002	32.6	51.7	V	V	V	20.5	51.8	12.67	10.9	6.53	24	0.165	0.144	0.226
414	2/7/2000	-32	20.8	V	V		20.8	55.3	12.33	10.9	-5.7	23.1	0.163	0.142	0.227
406	5/4/2000	32	35.9			V	22.2	49.3	12.97	10.8	7.14	24.5	0.169	0.147	0.227
397	6/2/2000	32.6	51.7	V	V		25.5	53	12.33	11.1	5.29	21	0.16	0.143	0.235
167	9/3/1978	40.5	-89	V			21.7	55.5	11.63	11.2	2.98	19.5	0.152	0.138	0.235
168	9/3/1978	40.5	-89		V		21.7	55.5	11.63	11.2	2.98	19.5	0.152	0.138	0.235
609	28/8/2003	32.4	36.2			V	23	49.7	13.3	10.9	7.67	25.8	0.187	0.164	0.24
297	23/2/1993	-34	18.4	V			52.7	40	24.33	8.81	-23	78.4	0.331	0.421	0.245
398	6/2/2000	36.2	37.2	V	V		26.4	55.7	12.76	11.2	6.17	22.5	0.179	0.16	0.247
342	18/1/1999	28.8	43.7	V	V		23.2	53.5	11.64	11.4	2.32	19.2	0.165	0.152	0.249
83	1/5/1908	44.1	3.1	V			27.7	60.4	14.76	10.7	10.2	30.8	0.221	0.194	0.252
561	5/11/2002	-34	18.4	V			21.1	55.3	12.4	11.2	-5.4	23.2	0.188	0.168	0.253
122	8/12/1942	40.7	-74	V	V		20	62.9	12.56	11.2	5.8	. 24	0.194	0.173	0.255
595	2/5/2003	38.2	46	V			28	59.4	12.63	11.3	5.6	21.4	0.182	0.165	0.255
372	12/8/1999	32.7	52.3				28.6	49.4	15.37	10.6	11.2	33.9	0.237	0.209	0.255
288	18/12/1990	33.4	73.1	V			32.1	56.8	14.57	10.9	9.72	28.5	0.215	0.189	0.256
407	5/4/2000	-34	18.4	V			22.8	50.9	13.29	11.1	-7.4	25.7	0.203	0.18	0.256
87	25/8/1911	49.9	2.3	V			39	53.7	21.5	9.26	19.4	66.2	0.327	0.364	0.258
490	17/10/2001	-34	18.4	V			22	52.4	13.09	11.1	-7	25.3	0.204	0.181	0.259
223	28/4/1987	26.7	-81.1	V	V		22.7	50.4	11.6	11.5	-1.1	18.6	0.172	0.16	0.259
222	28/4/1987	38.9	-77	V	V		22.8	58	11.64	11.5	1.52	18.8	0.174	0.161	0.26
227	28/4/1987	38.9	-77.1	V	V		22.8	58	11.64	11.5	1.52	18.8	0.174	0.161	0.26
224	28/4/1987	38.9	-77.1				22.8	58	11.64	11.5	1.52	18.8	0.174	0.161	0.26
439	25/1/2001	32.5	3.7				28.5	55	13.12	11.3	6.75	23	0.195	0.174	0.26
421	28/9/2000	43.3	-79.9	V	V		27.6	54.5	15.64	10.6	11.5	35.1	0.248	0.22	0.262
343	18/1/1999	26.1	44		V		23.2	52.7	11.67	11.5	1.75	19.3	0.179	0.166	0.263
529	13/5/2002	5	115		V	V	24.1	47.3	11.6	11.6	-0.3	18.6	0.178	0.166	0.265
229	28/4/1987	28	-82.5				22.8	52	11.65	11.6	-0.9	18.8	0.182	0.17	0.268
229	28/4/1987	28	-82.5				22.8	52	11.65	11.6	-0.9	18.8	0.182	0.17	0.268
108	27/5/1922	-34	18.5	V			22.1	56.9	12.3	11.5	-4.5	21.5	0.197	0.18	0.269

Ser.	Date	Lat	Long.	VISIBILITY			Age	Lag	ARCL	ARCV	DAZ	Width	MODEL		
No.				N	B	T							Bruin	Yallop	Qureshi
463	22/6/2001	49.6	8.7	V	V		32.2	73.5	18.31	10	15.4	49.9	0.309	0.299	0.275
457	24/4/2001	32.6	35.9				25.2	53.3	13.41	11.3	7.26	25.5	0.223	0.2	0.277
598	2/5/2003	51.7	-9.5	V	V	V	32.4	78	14.61	11.1	9.53	28.6	0.237	0.21	0.277
351	18/3/1999	31.9	35.8	V	V		21.4	51.5	12.69	11.4	5.63	24	0.217	0.196	0.277
596	2/5/2003	32.6	51.7	V	V		27.4	56.2	12.37	11.6	4.32	20.5	0.2	0.184	0.277
640	22/1/2004	-34	18.4	V			21.3	57.3	12.96	11.3	-6.3	24.7	0.221	0.199	0.278
602	30/6/2003	32.5	3.7		V	V	24.7	60	12.6	11.5	5.09	22	0.21	0.192	0.28
119	13/6/1934	55.6	33.9	V	V		41	97.2	19.04	10.1	16.1	48.5	0.311	0.298	0.281
352	18/3/1999	31.8	35.2	V			21.4	51.6	12.71	11.4	5.61	24.1	0.222	0.2	0.282
40	20/5/1871	38	23.7				31.2	59.6	14.2	11.2	8.69	27	0.235	0.211	0.283
656	20/4/2004	27.3	62.4		V		25.3	52.7	11.87	11.8	1.73	19.2	0.199	0.186	0.283
76	30/3/1881	51.5	-2.6	V			20.7	73.3	11.81	11.7	1.62	20	0.204	0.189	0.284
230	28/4/1987	36.2	-81.7	V	V		23.2	60	11.8	11.8	1.1	19.3	0.201	0.188	0.284
458	24/4/2001	31.9	35.8	V		V	25.2	53.3	13.4	11.4	7.12	25.5	0.232	0.208	0.285
353	18/3/1999	24.6	46.5		V	V	20.6	49	12.28	11.6	4.09	22.5	0.221	0.202	0.289
445	24/2/2001	51.7	7.2	V	V		33.2	69	15.98	10.9	11.7	34.5	0.274	0.246	0.291
597	2/5/2003	27.7	54.4	V	V		27.1	54.1	12.23	11.8	3.22	20	0.215	0.2	0.294
634	24/12/2003	33.4	73.1	V	V		26.8	56.9	16.21	10.7	12.2	39.3	0.296	0.27	0.295
651	21/3/2004	36.8	-81.8	V			25.4	54.7	12.78	11.6	5.3	22.7	0.229	0.209	0.296
81	19/4/1901	50.7	-2.8	V			22.1	74.2	13.14	11.5	6.43	26.2	0.251	0.227	0.301
145	9/1/1978	38.9	-76.9		V		18.5	63.7	11.9	11.8	1.72	21.4	0.229	0.211	0.302
547	9/8/2002	30.4	35.5	V		V	21.6	53.6	13.23	11.5	6.54	26.1	0.253	0.229	0.303
732	11/1/2005	43.9	18.4	V			27.9	65.9	17.35	10.5	13.8	45.6	0.326	0.308	0.305
138	21/12/1976	42.7	-83.6	V			20.5	69.4	12.6	11.7	4.68	23.6	0.245	0.225	0.308
657	20/4/2004	32.8	51		V		26.2	56.9	12.29	11.9	2.98	20.5	0.233	0.217	0.31
58	22/6/1876	38	23.7				20.1	66.6	12.88	11.7	5.42	25.2	0.261	0.238	0.316
635	24/12/2003	35.7	51.3	V	V	V	28.2	59.6	16.98	10.8	13.2	43.1	0.335	0.313	0.321
146	9/1/1978	41.9	-87.6	V			19.1	68.2	12.23	12	2.59	22.6	0.259	0.24	0.326
150	9/1/1978	43	-89.8	V	V		19.2	69.6	12.28	11.9	2.88	22.8	0.26	0.24	0.327
708	15/10/2004	32.9	59.2	V	V	V	35.1	48.8	19.01	10.4	15.9	52.8	0.369	0.365	0.328
610	28/8/2003	32.5	3.7		V		25.2	52.7	14.42	11.5	8.71	30.3	0.296	0.269	0.329
147	9/1/1978	36	-79.8	V	V		18.8	62.1	12.07	12	1.14	22	0.259	0.241	0.329
148	9/1/1978	36	-79.8	V			18.8	62.1	12.07	12	1.14	22	0.259	0.241	0.329
373	12/8/1999	31.8	34.7	V			29.7	51.8	15.98	11.2	11.5	36.6	0.32	0.293	0.329
570	5/12/2002	35.7	51.4	V	V		30.2	59.2	16.94	10.9	13	42.1	0.34	0.317	0.33
269	4/6/1989	50.8	-1	V			25	93.9	14.49	11.5	8.83	30.5	0.298	0.27	0.33
55	4/6/1875	51.5	-2.6	V			22.7	97	14.24	11.5	8.41	30.6	0.298	0.27	0.33
195	3/3/1984	37.2	-84.1			V	29.5	56.5	14.17	11.7	7.96	26.9	0.283	0.258	0.331
188	5/12/1983	37.2	-84.1			V	34.3	61	16.98	11	12.9	39.8	0.336	0.311	0.333
491	17/10/2001	-4	39.7				20.2	47.4	12.12	12.1	-0.7	21.8	0.265	0.247	0.336
144	11/12/1977	47.8	20	V			21.8	77.6	13.93	11.6	7.67	29.5	0.302	0.274	0.338
201	31/5/1984	15.6	35.6				23.7	53.2	12.19	12.2	0.93	21.2	0.264	0.247	0.338
670	18/6/2004	47.6	-118	V	V		32.1	85	15.27	11.6	9.97	31.2	0.312	0.285	0.341
151	9/1/1978	34	-81.1	V	V		19	61	12.15	12.1	0.72	22.4	0.274	0.255	0.343
139	21/12/1976	42	-91.6	V	V		21	70.4	12.91	12	4.71	24.8	0.29	0.268	0.347
339	21/10/1998	31.8	34.7	V			29.3	54.8	14.08	11.9	7.47	26.5	0.301	0.276	0.35

Ser.	Date	Lat	Long.	VISIBILITY			Age	Lag	ARCL	ARCV	DAZ	Width	MODEL		
No.				N	B	T							Bruin	Yallop	Qureshi
38	20/2/1871	38	23.7	V			26.8	58	14.49	11.8	8.46	29.6	0.315	0.289	0.351
378	10/10/1999	32	35.9	V		V	28	53.9	14.27	11.9	7.9	27.8	0.31	0.284	0.354
153	9/1/1978	29.9	-81.3	V			19.2	58.4	12.23	12.2	-0.2	22.6	0.288	0.268	0.355
155	9/1/1978	41.6	-93.6	V			19.6	71	12.46	12.2	2.65	23.5	0.292	0.271	0.355
377	10/10/1999	5.3	103	V	V	V	23.7	47.9	12.37	12.4	0.78	20.9	0.28	0.263	0.355
187	5/12/1983	15.6	35.6				27.1	52.8	13.41	12.1	5.83	25	0.3	0.277	0.356
156	9/1/1978	33.9	-84.3	V			19.2	61.5	12.27	12.3	0.76	22.8	0.291	0.271	0.357
710	15/10/2004	32.6	51.7	V	V		35.6	49.8	19.29	10.7	16.1	54.3	0.403	0.402	0.358
709	15/10/2004	32.6	51.6	V	V		35.6	49.8	19.3	10.7	16.1	54.3	0.403	0.402	0.358
550	7/9/2002	10.7	-61.5	V			19.4	48.1	12.44	12.2	2.37	23.5	0.295	0.274	0.358
575	3/1/2003	33.9	-118		V		29	60	15.81	11.5	10.9	35.6	0.347	0.319	0.359
157	9/1/1978	27.7	-82.7	V	V		19.3	57.3	12.32	12.3	-0.7	23	0.298	0.278	0.364
80	29/5/1900	38.7	-0.7	V			29	63.9	15.34	11.7	9.98	33.6	0.344	0.316	0.364
330	30/12/1997	31.3	35.2	V			22.3	60.9	12.51	12.4	1.95	22.7	0.3	0.281	0.367
580	2/2/2003	32.6	51.7	V	V		27.7	58.1	14.66	11.9	8.55	29.8	0.333	0.306	0.368
329	30/12/1997	31.3	34.6	V			22.3	61	12.53	12.4	1.96	22.8	0.303	0.283	0.369
325	3/9/1997	31.8	34.7	V			40.6	50.2	18.3	11.2	14.5	44.6	0.388	0.368	0.369
241	28/4/1987	30.6	-104	V	V		24.5	60.9	12.46	12.5	0.04	21.5	0.297	0.279	0.37
674	18/7/2004	35.7	51.3	V	V		28.9	63.9	14.31	12.1	7.59	27.7	0.333	0.308	0.377
180	13/7/1980	41.4	-70.7	V			41.9	59	20.89	10.7	18	60.1	0.435	0.452	0.377
576	3/1/2003	32.4	-111	V			28.7	59	15.63	11.7	10.4	34.9	0.363	0.335	0.378
563	5/11/2002	32.4	-111	V	V	V	28.4	54	16.56	11.5	12	41.1	0.389	0.365	0.382
539	11/6/2002	32.4	-111	V	V		27.3	63	13.96	12.2	6.79	27.7	0.339	0.313	0.383
479	19/8/2001	33.9	-118	V	V		24	55	15.07	11.9	9.32	34.4	0.37	0.342	0.386
440	25/1/2001	40.8	-74		V		33.5	66.9	15.35	12	9.53	31.5	0.361	0.333	0.389
158	9/1/1978	30	-90.2	V	V		19.8	60.9	12.56	12.6	-0.1	23.9	0.334	0.313	0.395
159	9/1/1978	30	-90.2	V	V		19.8	60.9	12.56	12.6	-0.1	23.9	0.334	0.313	0.395
441	25/1/2001	40.4	-74.5	V			33.5	67	15.37	12.1	9.45	31.6	0.371	0.343	0.399
564	5/11/2002	32	-117	V			28.8	55	16.76	11.6	12.1	42.1	0.412	0.389	0.402
562	5/11/2002	26	-80.3		V		26.4	54.2	15.43	12	9.78	35.7	0.392	0.364	0.403
112	25/5/1933	55.6	33.9	V			32.9	121	15.67	12.1	9.96	32.8	0.38	0.353	0.403
464	22/6/2001	43.9	18.4	V			31.1	72.5	17.7	11.4	13.5	46.7	0.429	0.412	0.404
636	24/12/2003	32.7	51.7	V			28.3	61.5	17.05	11.6	12.5	43.5	0.42	0.399	0.405
641	22/1/2004	41.8	-123	V	V		28.8	66	16.85	11.7	12.2	41.5	0.415	0.391	0.406
671	18/6/2004	36.8	-81.8		V		29	71	13.88	12.6	5.92	25.8	0.354	0.331	0.406
243	28/4/1987	40.7	-112	V			25.8	70	13.08	12.7	3.15	23.6	0.346	0.324	0.408
399	6/2/2000	-4	39.7	V			27	50.8	13.07	12.7	-3	23.6	0.348	0.327	0.41
459	24/4/2001	32.5	3.7	V	V	V	27.4	58.8	14.47	12.4	7.53	29.7	0.375	0.349	0.411
140	21/12/1976	29.9	-81.3	V			20.8	61.7	12.8	12.7	1.72	24.4	0.353	0.33	0.411
395	7/1/2000	10	-61.5	V			28.2	54	12.83	12.8	-0.1	22.2	0.343	0.324	0.412
711	15/10/2004	30.2	57.1	V	V	V	35.3	51.4	19.12	11.2	15.5	53.3	0.455	0.452	0.412
530	13/5/2002	32.6	51.7	V	V		29.1	62.3	14.02	12.5	6.33	27.2	0.366	0.34	0.412
215	21/1/1985	19	-155	V			26.2	54	13.88	12.6	5.94	26.7	0.364	0.339	0.412
307	25/9/1995	-34	18.4	V			24.3	59.5	12.81	12.7	-1.4	23.7	0.35	0.329	0.412
675	18/7/2004	32.7	51.7	V	V		28.8	63.3	14.25	12.5	6.81	27.5	0.369	0.343	0.414
568	5/12/2002	32.6	51.7	V	V		30.3	61.1	17.01	11.7	12.3	42.4	0.426	0.403	0.414

Ser.	Date	Lat	Long.	VISIBILITY			Age	Lag	ARCL	ARCV	DAZ	Width	MODEL		
No.				N	B	T							Bruin	Yallop	Qureshi
379	10/10/1999	24.5	46.5		V		27.4	53.5	13.99	12.6	6.05	26.7	0.371	0.346	0.419
717	13/11/2004	13.7	10.7	V			26.7	52.1	15.52	12.1	9.69	36	0.412	0.385	0.423
446	24/2/2001	36	50.8	V			30.5	60.6	14.8	12.5	7.93	29.6	0.388	0.362	0.424
493	17/10/2001	10.3	9.8	V			22	50.7	13.09	12.8	2.94	25.3	0.37	0.347	0.425
205	27/8/1984	15.6	35.6				20.9	51.3	13.26	12.7	3.85	26.7	0.377	0.353	0.426
278	25/4/1990	41.6	-73.7	V		V	19.8	67.3	12.82	12.8	0.63	25	0.37	0.348	0.426
100	4/8/1921	-34	18.5	V			20.3	61.7	12.83	12.8	0.41	25	0.373	0.35	0.429
161	9/1/1978	29.7	-98.1	V			20.3	62.1	12.85	12.9	-0	25	0.375	0.353	0.431
526	13/4/2002	26	-80.3	V			28.8	54.8	13.7	12.8	4.81	25.4	0.378	0.355	0.432
531	13/5/2002	33.3	44.4	V	V		29.6	63.7	14.27	12.7	6.58	28.2	0.391	0.365	0.433
380	10/10/1999	34	-6.8	V			30.9	57.7	15.55	12.4	9.4	32.9	0.411	0.383	0.433
465	22/6/2001	38.2	46	V	V		28.9	68.1	16.44	12.1	11.2	40.3	0.441	0.416	0.436
532	13/5/2002	29.6	52.5	V			28.9	61.4	13.94	12.8	5.58	26.9	0.389	0.364	0.436
244	28/4/1987	37	-122	V			25.9	68	13.13	13	1.76	23.8	0.379	0.358	0.44
448	24/2/2001	-34	18.4	V			33.6	57.6	16.14	12.4	-10	35.2	0.427	0.399	0.44
554	7/10/2002	32.5	51.7	V	V		27.3	55.2	16.56	12.1	11.4	41.5	0.452	0.428	0.443
92	1/4/1919	53.9	-1.6	V			22.2	87.1	13.51	12.9	4.15	27.7	0.405	0.38	0.449
696	15/9/2004	-34	18.4	V			26.6	60.2	13.74	12.9	-4.7	26.8	0.401	0.377	0.449
344	18/1/1999	6.5	3.4				26.5	53.5	13.3	13	-2.6	25	0.394	0.372	0.45
116	14/5/1934	55.6	33.9	V			30.1	118	15.22	12.7	8.44	31.6	0.427	0.399	0.454
354	18/3/1999	34	-6.8	V			24.3	59.6	14.3	12.8	6.46	30.5	0.423	0.396	0.455
533	13/5/2002	29.4	48	V	V		29.2	62.1	14.09	12.9	5.57	27.4	0.411	0.385	0.456
129	6/4/1970	48	-122	V			23.2	78	13.29	13.1	2.42	25.7	0.404	0.381	0.457
1	1/7/1859	38	23.7	V			27.7	67.5	16.42	12.3	11	40.7	0.464	0.44	0.458
466	22/6/2001	35.7	51.3	V	V		28.4	67.3	16.16	12.4	10.4	38.9	0.463	0.438	0.464
733	11/1/2005	32.6	51.6	V	V		26.2	63.8	16.38	12.3	10.8	40.6	0.471	0.446	0.465
652	21/3/2004	33.9	-118	V			27.8	60	13.94	13.1	4.83	27	0.419	0.395	0.467
6	12/3/1861	38	23.7	V			27.4	64.3	13.37	13.3	-1.6	24	0.407	0.386	0.468
124	5/3/1954	44.5	-88	V			21.1	69.9	13.15	13.2	0.2	26.1	0.418	0.394	0.469
454	24/2/2001	32.6	51.7	V	V		30.5	60.3	14.79	13	7.16	29.6	0.433	0.407	0.469
197	2/4/1984	15.6	35.6				28.1	53.4	13.42	13.3	1.92	24.4	0.412	0.391	0.471
534	13/5/2002	26.2	50.5	V	V		29	60.9	13.96	13.1	4.75	26.9	0.424	0.399	0.472
133	25/4/1971	39.5	-88.2	V	V		21.2	71	13.22	13.2	0.33	26.1	0.424	0.4	0.475
78	12/3/1899	52.5	13.3	V			21.8	83.6	13.32	13.2	1.6	26.2	0.425	0.401	0.476
527	13/4/2002	32.4	-111	V	V		30.6	62	14.49	13.1	6.21	28.4	0.437	0.411	0.478
536	13/5/2002	31.9	35.8	V		V	30.2	64.8	14.53	13.1	6.33	29.2	0.442	0.416	0.48
447	24/2/2001	5.3	103	V	V	V	27.4	52.5	13.4	13.4	0.69	24.3	0.421	0.399	0.48
535	13/5/2002	25.3	49.7	V			29	60.8	13.97	13.2	4.53	27	0.434	0.409	0.481
423	28/10/2000	32.6	51.7	V			30.3	60.9	15.81	12.9	9.21	34.8	0.474	0.446	0.489
13	1/1/1862	37.9	22.9	V			26	71.2	14.57	13.1	6.42	31.2	0.462	0.434	0.491
718	13/11/2004	10.3	9.8	V			26.9	54.4	15.6	12.8	8.94	36.4	0.483	0.456	0.492
366	14/6/1999	6.5	3.4	V			23.4	55.8	14.08	13.3	4.76	29.8	0.466	0.439	0.501
588	2/4/2003	33.8	-118	V			31.4	62	14.54	13.4	5.72	28.3	0.462	0.436	0.504
449	24/2/2001	29.6	52.5	V			30.5	60.1	14.79	13.3	6.46	29.5	0.469	0.442	0.505
136	5/3/1973	40	-85	V	V		24	67	13.56	13.5	-1	26.6	0.46	0.436	0.509
141	21/12/1976	37.6	-123	V			23.3	72	14.14	13.5	4.36	29.6	0.484	0.457	0.519

Ser.	Date	Lat	Long.	VISIBILITY			Age	Lag	ARCL	ARCV	DAZ	Width	MODEL		
No.				N	B	T							Bruin	Yallop	Qureshi
117	14/5/1934	50	36.2	V			29.4	100	14.91	13.4	6.49	30.4	0.489	0.462	0.522
577	3/1/2003	10.4	-61.5	V			26	58.5	14.27	13.5	4.6	29.1	0.485	0.458	0.523
126	5/4/1962	-26	-28.2	V	V		24.5	56.6	15.1	13.2	-7.3	34.2	0.506	0.477	0.523
548	9/8/2002	10.3	9.8	V			22.9	56.3	13.92	13.7	2.33	28.9	0.505	0.478	0.543

APPENDIX-IV

OBSERVATIONAL LUNAR CALENDAR OF PAKISTAN

2000-2007

AND ITS COMPARISON WITH

THE VISIBILITY CRITERIA

YEAR 2000

LAST CONJ.					DATE	ARCV	DAZ	LAG	AGE	ARCL	PHASE	D	W	Q	VISIB	S	VIS	MONTH	STARTED	COMMENT
M	D	H	M	S		DEG	DEG	Min	HRS	DEG			arc-m		ON Q		ON S.		ON	
12	8	3	32	43	9.12.1999													RAMZ	10.12.1999	
1	6	23	14	45	7.1.2000	7.4	2.1	37	18.72	7.7	0.005	30	0.135	-0.36	I	-0.23	I		RAM(30)	
DAY AFTER CONJ.					8.1.2000	17.45	6.9	88	42.72	18.7	0.026	30.2	0.8	1.03	EV	1.01	EV	SHAW	9.1.2000	PROP
2	5	18	4	22	5.2.2000	0.967	1.2	-5	0.244	1.5	2E-04	30.5	0.005	-1.08	I	-0.94	I		SHA(30)	
DAY AFTER CONJ.					6.2.2000	10.2	3.9	49	24.24	10.9	0.009	30.8	0.278	0.01	VUPC	0.11	VUPC	ZEEQ	8.2.2000	PROP
3	6	10	17	53	6.3.2000	3.1	3.9	14	8.319	5.02	0.002	31.5	0.06	-0.84	I	-0.7	I		ZEE(29)	
DAY AFTER CONJ.					7.3.2000	15.32	5.2	71	32.32	16.2	0.02	31.8	0.629	0.72	EV	0.75	EV	ZILHAJ	8.3.2000	PROP
4	4	23	13	5	4.4.2000	3.483	4.7	-16	19.6	5.83	0.003	32.3	0.084	-0.78	I	-0.65	I		ZIL(30)	
DAY AFTER CONJ.					5.4.2000	9.383	5.5	43	43.6	10.9	0.009	32.6	0.293	-0.07	MROA	0.04	VUPC	MUHAR	7.4.2000	PROP
5	4	9	13	4	4.5.2000	3.8	5.5	18	9.832	6.67	0.003	33.3	0.113	-0.73	I	-0.61	I		MUH(29)	
DAY AFTER CONJ.					5.5.2000	17.4	7.4	83	33.83	18.9	0.027	33.5	0.898	1.07	EV	1.03	EV	SAFAR	6.5.2000	PROP
6	2	17	15	2	2.6.2000	1.033	3.6	-6	2.049	3.79	0.001	33.8	0.037	-1.06	I	-0.92	I		SAF(30)	
DAY AFTER CONJ.					3.6.2000	12.52	7.6	62	26.05.	14.6	0.016	33.9	0.549	0.39	EV	0.44	EV	RAB-I	5.6.2000	LATE
7	2	0	20	57	2.7.2000	8.433	6.3	42	19.07	10.5	0.008	34	0.284	-0.17	ROA	-0.06	ROA		R-I(29)	
DAY AFTER CONJ.					3.7.2000	20.93	13	100	43.07	24.6	0.045	33.8	1.536	1.75	EV	1.49	EV	RAB-II	4.7.2000	PROP
7	31	7	26	3	31.7.2000	5.45	3.2	26	11.85	6.33	0.003	33.8	0.103	-0.57	I	-0.44	I		R-II(29)	
DAY AFTER CONJ.					1.8.2000	16.58	12	77	35.85	20.3	0.031	33.3	1.035	1.06	EV	0.98	EV	JAM-I	2.8.2000	PROP
8	29	15	20	17	29.8.2000	3.033	1	14	3.545	3.19	8E-04	33.3	0.026	-0.86	I	-0.73	I		J-I(29)	
DAY AFTER CONJ.					30.8.2000	12.85	8.5	59	27.55	15.3	0.018	32.9	0.586	0.45	EV	0.49	EV	JAM-II	31.8.2000	PROP
9	28	0	54	4	28.9.2000	9.05	3.7	41	17.47	9.77	0.007	32.1	0.233	-0.14	MROA	-0.02	MROA		J-II(30)	
DAY AFTER CONJ.					29.9.2000	17.47	13	80	41.47	21.9	0.036	31.7	1.142	1.2	EV	1.09	EV	RAJAB	30.9.2000	PROP
10	27	12	59	4	27.10.2000	4.2	1.6	19	4.932	4.49	0.002	31.3	0.048	-0.73	I	-0.6	I		RAJ(29)	
DAY AFTER CONJ.					28.10.2000	12.43	7.3	59	28.93	14.4	0.016	31	0.486	0.35	EV	0.42	EV	SHAB	29.10.2000	PROP
11	26	4	12	25	26.11.2000	6.017	1.5	30	13.49	6.2	0.003	30.4	0.089	-0.53	I	-0.4	I		SHAB(30)	
DAY AFTER CONJ.					27.11.2000	14.5	9.1	74	37.49	17.1	0.022	30.2	0.665	0.66	EV	0.68	EV	RAMAZ	28.11.2000	PROP
12	25	22	22	48	26.12.2000	7.233	3.6	36	19.47	8.07	0.005	29.8	0.148	-0.37	I	-0.24	I		RAM(30)	
DAY AFTER CONJ.					27.12.2000	16.65	9.2	87	43.47	19	0.027	29.8	0.808	0.95	EV	0.93	EV	SHAW	28.12.2000	PROP

INDICATE MOONSET BEFORE SUNSET

YEAR 2001

LAST CONJ.					DATE	ARCV	DAZ	LAG	AGE	ARCL	PHASE	D	W	Q	VISIB	S	VIS	MONTH	STARTED	COMMENT
M	D	H	M	S		DEG	DEG	Min	HRS	DEG			arc-s		ON Q		ON S		ON	
12	25	22	22	48	27.12.2000													SHAW	28.12.2000	PROP
1	24	18	7	54	24.1.2001	1.217	1.7	-6	0.052	2.09	3E-04	30	0.01	-1.056	I	-0.92	I			
DAY AFTER CONJ.					25.1.2001	9.133	4.933	45	24.05	10.4	0.008	30	0.243	-0.121	MROA	-0.01	MROA		SHAW(30)	
2 DAYS AFTER					26.1.2001	19.53	8.233	97	48.05	21.1	0.034	30	1.005	1.3415	EV	1.266	EV	ZEEQ	27.1.2001	PROP
2	23	13	22	15	23.2.2001	1.2	4.383	5	5.146	4.54	0.002	30	0.047	-1.034	I	-0.9	I		ZEE(29)	
DAY AFTER CONJ.					24.2.2001	12.18	5.55	57	29.15	13.4	0.014	30	0.41	0.282	EV	0.362	EV	ZILHAJ	25.2.2001	PROP
3	25	6	22	12	25.3.2001	4.867	5.183	22	12.38	7.11	0.004	31	0.118	-0.623	I	-0.5	I		ZIL(30)	
DAY AFTER CONJ.					26.3.2001	16.5	5.6	75	36.38	17.4	0.023	31	0.712	0.8828	EV	0.893	EV	MUHAR	27.3.2001	PROP
4	23	20	26	42	23.4.2001	2.133	4.5	-10	-1.48	4.98	0.002	31	0.059	-0.933	I	-0.8	I		MUH(30)	
DAY AFTER CONJ.					24.4.2001	10.02	5.267	47	22.52	11.3	0.01	32	0.308	0.0059	VUPC	0.105	VUPC	SAFAR	26.4.2001	PROP
5	23	7	47	8	23.5.2001	4.267	4.017	21	11.43	5.86	0.003	33	0.085	-0.704	I	-0.57	I		SAF(29)	
DAY AFTER CONJ.					24.5.2001	17.1	8.75	84	35.43	19.1	0.028	33	0.907	1.0474	EV	1.002	EV	RAB-I	25.5.2001	PROP
6	21	16	58	50	21.6.2001	0.017	1.017	0	2.419	1.02	8E-05	33	0.003	-1.18	I	-1.04	I		R-I(30)	
DAY AFTER CONJ.					22.6.2001	12.65	6.383	63	26.42	14.1	0.015	33	0.507	0.3844	EV	0.444	EV	RAB-II	24.6.2001	LATE
7	21	0	45	20	21.7.2001	9.333	4.617	45	18.61	10.4	0.008	34	0.279	-0.08	MROA	0.024	VUPC		R-II(29)	
DAY AFTER CONJ.					22.7.2001	20.88	13.22	97	42.61	24.6	0.045	34	1.527	1.7356	EV	1.48	EV	JAM-I	23.7.2001	PROP
8	19	7	56	11	19.8.2001	6.733	1.7	32	11.11	6.94	0.004	34	0.125	-0.433	I	-0.31	I		J-I(30)	
DAY AFTER CONJ.					20.8.2001	17.07	11.8	78	35.11	20.6	0.032	34	1.086	1.1364	EV	1.035	EV	JAM-II	21.8.2001	PROP
9	17	15	28	21	17.9.2001	3.933	1.95	18	3.094	4.39	0.001	34	0.05	-0.759	I	-0.62	I		J-II(29)	
DAY AFTER CONJ.					18.9.2001	13.3	8.683	60	27.09	15.8	0.019	33	0.635	0.5206	EV	0.55	EV	RAJAB	19.9.2001	PROP
10	17	0	24	21	17.10.2001	8.9	4.45	42	17.64	9.94	0.008	33	0.247	-0.142	MROA	-0.03	MROA		RAJ(30)	
DAY AFTER CONJ.					18.10.2001	17.22	15.03	85	41.64	22.7	0.039	32	1.255	1.2365	EV	1.078	EV	SHABAN	19.10.2001	PROP
11	15	11	41	7	15.11.2001	3.25	0.1	15	6.065	3.25	8E-04	32	0.026	-0.842	I	-0.7	I		SHAB(29)	
DAY AFTER CONJ.					16.11.2001	12.2	9.267	62	30.06	15.3	0.018	32	0.561	0.3696	EV	0.417	EV	RAMAZ	17.11.2001	PROP
12	15	1	48	37	15.12.2001	5.8	4.317	30	15.94	7.23	0.004	31	0.123	-0.527	I	-0.4	I		RAM(30)	
DAY AFTER CONJ.					16.12.2001	15.5	11.28	83	39.94	19.1	0.027	31	0.844	0.654	EV	0.827	EV	SHAW	17.12.2001	PROP

INDICATE MOONSET BEFORE SUNSET

YEAR 2002

LAST CONJ.					DATE	ARCV	DAZ	LAG	AGE	ARCL	PHASE	D	W	Q	VISIB	S	VIS	MONTH	STARTED	COMMENT
						DEG	DEG	Min	HRS	DEG			arc-m		ON Q		ON S		ON	
M	D	H	M	S																
12	15	1	48	37	16.12.2001													SHAWAL	17.12.2001	PROP
1	13	18	29	48	13.1.2002	1.767	2.1	-9	0.447	2.77	6E-04	30.37	0.018	-1	I	-0.857	I			
DAY AFTER CONJ.					14.1.2002	8.817	6.3	46	23.55	10.8	0.009	30.2	0.269	-0.1	MROA	-0.031	MROA		SHAW(30)	
TWO DAYS AFTER					15.1.2002	19.3	10	99	47.55	21.9	0.036	30	1.079	1.36	EV	1.257	EV	ZEEQAD	16.1.2002	PROP
2	12	12	42	3	12.2.2002	1.233	4.7	6	5.699	4.84	0.002	29.88	0.053	-1	I	-0.892	I		ZEEQ(29)	
DAY AFTER CONJ.					13.2.2002	12.05	6.3	57	29.7	13.6	0.014	29.82	0.417	0.27	EV	0.352	EV	ZILHAJ	14.2.2002	PROP
3	14	7	3	41	14.3.2002	4.217	5.1	20	11.61	6.61	0.003	29.88	0.099	-0.7	I	-0.57	I		ZIL(30)	
DAY AFTER CONJ.					15.3.2002	15.12	5.3	69	35.61	16	0.019	29.93	0.58	0.67	EV	0.715	EV	MUHARM	16.3.2002	PROP
4	13	0	22	16	13.4.2002	7.367	4.3	35	18.51	8.52	0.006	30.22	0.167	-0.3	I	-0.222	I		MUH(30)	
DAY AFTER CONJ.					14.4.2002	18.55	4.4	86	42.51	19.1	0.027	30.45	0.834	1.15	EV	1.13	EV	SAFAR	15.4.2002	PROP
5	12	15	46	12	12.5.2002	0.233	2.5	1	3.347	2.48	5E-04	30.7	0.014	-1.2	I	-1.012	I		SAF(29)	
DAY AFTER CONJ.					13.5.2002	11.73	3.9	57	27.35	12.3	0.012	31.05	0.359	0.21	VUPC	0.297	EV	RAB-I	14.5.2002	PROP
6	11	4	47	35	11.6.2002	6.183	2.1	32	14.56	6.53	0.003	31.72	0.103	-0.5	I	-0.372	I		R-I(30)	
DAY AFTER CONJ.					12.6.2002	17.93	6.2	90	38.56	18.9	0.027	32.02	0.867	1.11	EV	1.076	EV	RAB-II	13.6.2002	PROP
7	10	15	27	2	10.7.2002	2.4	1.1	12	3.966	2.63	5E-04	32.5	0.017	-0.9	I	-0.794	I		R-II(29)	
DAY AFTER CONJ.					11.7.2002	13.58	5.8	67	27.97	14.7	0.016	32.8	0.54	0.5	EV	0.548	EV	JAM-I	12.7.2002	PROP
8	9	0	16	11	9.8.2002	10.15	4	48	18.91	10.9	0.009	33.42	0.301	0.02	VUPC	0.116	VUPC		J-I(30)	
DAY AFTER CONJ.					10.8.2002	20.2	14	92	42.91	24.3	0.044	33.48	1.489	1.65	EV	1.408	EV	JAM-II	11.8.2002	PROP
9	7	8	11	13	7.9.2002	6.867	1.3	31	10.56	6.98	0.004	33.88	0.126	-0.4	I	-0.292	I		J-II(29)	
DAY AFTER CONJ.					8.9.2002	16.1	12	73	34.56	20.2	0.031	33.77	1.035	1.01	EV	0.929	EV	RAJAB	9.9.2002	PROP
10	6	16	18	28	6.10.2002	2.883	2	13	1.926	3.49	9E-04	34	0.032	-0.9	I	-0.738	I		RAJ(29)	
DAY AFTER CONJ.					7.10.2002	11.83	9.4	55	25.93	15.1	0.017	33.77	0.583	0.35	EV	0.388	EV	SHABAN	8.10.2002	PROP
11	5	1	35	30	5.11.2002	6.917	5.4	34	16.23	8.8	0.006	33.48	0.197	-0.4	I	-0.263	I		SHAB(30)	
DAY AFTER CONJ.					6.11.2002	16	16	83	40.23	22.6	0.038	33.02	1.27	1.12	EV	0.959	EV	RAMAZ	7.11.2002	PROP
12	4	12	35	27	4.12.2002	1.067	2.1	6	5.109	2.37	4E-04	32.9	0.014	-1.1	I	-0.929	I		RAM(29)	
DAY AFTER CONJ.					5.12.2002	11.4	11	62	29.11	15.7	0.019	32.45	0.609	0.32	EV	0.352	EV	SHAWAL	6.12.2002	PROP

INDICATE MOONSET BEFORE SUNSET

YEAR 2003

LAST CONJ.					DATE	ARCV	DAZ	LAG	AGE	ARCL	PHASE	D	W	Q	VISIB	S	VIS	MONTH	STARTED	COMMENT
M	D	H	M	S		DEG	DEG	Min	HRS	DEG					ON Q		ON S		ON	
12	4	12	35	27														SHAWAL	6.12.2002	PROP
1	3	1	23	54	3.1.2003	5.917	6.5	32	16.5	8.806	0.006	32	0.188	-0.48	I	-0.358	I		SHW(30)	
DAY AFTER CONJ					4.1.2003	17.25	12	92	40.5	20.94	0.033	31	1.037	1.13	EV	1.0442	EV	ZEEQAD	5.1.2003	PROP
2	1	15	49	29	1.2.2003	0.5	4.8	-2	2.44	4.809	0.002	31	0.055	-1.1	I	-0.964	I		ZEQ(29)	
DAY AFTER CONJ					2.2.2003	11.25	7.1	56	26.4	13.3	0.013	31	0.413	0.191	VUPC	0.27	EV	ZILHAJ	3.2.2003	PROP
3	3	7	35	59	3.3.2003	4.083	5.1	19	11	6.504	0.003	30	0.098	-0.71	I	-0.584	I		ZIL(30)	
DAY AFTER CONJ					4.3.2003	15.35	5.3	71	35	16.22	0.02	30	0.601	0.707	EV	0.7448	EV	MUHARM	5.3.2003	PROP
4	2	0	19	38	2.4.2003	7.4	3.6	34	18.5	8.218	0.005	30	0.154	-0.35	I	-0.225	I		MHR(30)	
DAY AFTER CONJ					3.4.2003	18.33	3.3	84	42.5	18.61	0.026	30	0.781	1.104	EV	1.0948	EV	SAFAR	4.4.2003	PROP
5	1	17	15	48	1.5.2003	0.317	1.7	-3	1.79	1.746	2E-04	30	0.007	-1.15	I	-1.008	I		SAF(30)	
DAY AFTER CONJ					2.5.2003	10.55	2.2	50	25.8	10.78	0.009	30	0.264	0.033	VUPC	0.14	VUPC	RAB-I	4.5.2003	PROP
5	31	9	20	52	31.5.2003	3.85	0.2	20	9.92	3.854	0.001	30	0.034	-0.78	I	-0.64	I		R-I(29)	
DAY AFTER CONJ					1.6.2003	14.72	2.9	74	33.9	15	0.017	30	0.519	0.598	EV	0.6546	EV	RAB-II	2.6.2003	PROP
6	29	23	39	37	30.6.2003	9.25	1.6	48	19.8	9.383	0.007	31	0.207	-0.13	MROA	-0.015	MROA		R-II(30)	
DAY AFTER CONJ					1.7.2003	19.62	7.4	97	43.8	20.93	0.033	31	1.031	1.363	EV	1.2797	EV	JAM-I	2.7.2003	PROP
7	29	11	53	43	29.7.2003	5.067	1.2	26	7.4	5.199	0.002	32	0.065	-0.64	I	-0.502	I		J-I(29)	
DAY AFTER CONJ					30.7.2003	14.52	6.9	69	31.4	16.06	0.02	32	0.622	0.636	EV	0.6682	EV	JAM-II	31.7.2003	PROP
8	27	22	27	21	28.8.2003	10.02	4.9	46	20.5	11.15	0.009	33	0.309	0.006	VUPC	0.1054	VUPC		J-II(30)	
DAY AFTER CONJ					29.8.2003	18.48	15	84	44.5	23.89	0.043	33	1.408	1.438	EV	1.2261	EV	RAJAB	30.8.2003	PROP
9	26	8	10	9	26.9.2003	5.483	2	25	10.2	5.847	0.003	33	0.087	-0.58	I	-0.45	I		RAJ(29)	
DAY AFTER CONJ					27.9.2003	13.75	13	63	34.2	18.96	0.027	33	0.906	0.712	EV	0.6667	EV	SHABAN	28.9.2003	PROP
10	25	17	51	19	26.10.2003	9.133	9.9	45	24.1	13.44	0.014	34	0.462	0.007	VUPC	0.0766	VUPC		SHB(30)	
DAY AFTER CONJ					27.10.2003	17.78	21	93	48.1	27.57	0.057	34	1.904	1.603	EV	1.2047	EV	RAMAZ	28.10.2003	PROP
11	24	4	0	1	24.11.2003	4.317	6.4	23	13.7	7.728	0.005	34	0.153	-0.66	I	-0.534	I		RAM(29)	
DAY AFTER CONJ					25.11.2003	14.6	16	81	37.7	21.8	0.036	33	1.197	0.946	EV	0.8077	EV	SHAWAL	26.11.2003	PROP
12	23	14	44	5	23.12.2003	0.933	4	-6	3.08	4.091	0.001	34	0.043	-1.06	I	-0.927	I		SHW(29)	
DAY AFTER CONJ					24.12.2003	11.2	11	62	27.1	15.66	0.019	33	0.615	0.3	EV	0.3342	EV	ZEEQAD	25.12.2003	PROP

INDICATES MOONSET BEFORE SUNSET

YEAR 2004

LAST CONJUNCT.					DATE	ARCV	DAZ	LAG	AGE	ARCL	PHASE	D	W	Q	VISIB	S	VIS	MONTH	STARTED	COMMENT
M	D	H	M	S		DEG	DEG	Min	HRS	DEG					ON Q		ON S		ON	
12	23	14	44	5														ZEEQAD	25.12.2003	PROP
1	22	2	6	1	22.1.2004	6.633	6.983	34	16	9.62	0.007	32.7	0.23	-0.38	I	-0.27	I		ZEQ(29)	
DAY AFTER CONJ					23.1.2004	19.52	10.32	99	40	21.98	0.0363	32.3	1.173	1.426	EV	1.295	EV	ZILHAJ	24.1.2004	PROP
2	20	14	18	50	20.2.2004	0.9	4.9	4	4.17	4.982	0.0019	32	0.06	-1.06	I	-0.92	I		ZIL(29)	
DAY AFTER CONJ					21.2.2004	13.53	5.417	64	28.2	14.56	0.0161	31.7	0.508	0.473	EV	0.533	EV	MUHARM	22.2.2004	PROP
3	21	3	42	28	21.3.2004	6.483	3.017	29	15	7.148	0.0039	31	0.12	-0.46	I	-0.33	I		MHR(30)	
DAY AFTER CONJ					22.3.2004	18.25	2.383	83	39	18.4	0.0256	30.7	0.786	1.098	EV	1.088	EV	SAFAR	23.3.2004	PROP
4	19	18	22	19	19.4.2004	0.733	1.033	-3	0.56	1.267	0.0001	30.5	0.004	-1.11	I	-0.97	I		SAF(30)	
DAY AFTER CONJ					20.4.2004	10.6	0.95	50	24.6	10.64	0.0086	30.3	0.26	0.036	VUPC	0.143	VUPC	RAB-I	22.4.2004	PROP
5	19	9	53	4	19.5.2004	3.7	0.85	19	9.3	3.796	0.0011	30	0.033	-0.79	I	-0.66	I		R-I(29)	
DAY AFTER CONJ					20.5.2004	14.52	0.967	74	33.3	14.55	0.016	29.9	0.479	0.555	EV	0.621	EV	RAB-II	21.5.2004	PROP
6	18	1	27	53	18.6.2004	8.267	0.133	43	17.9	8.268	0.0052	29.9	0.156	-0.26	I	-0.14	ROA		R-II(30)	
DAY AFTER CONJ					19.6.2004	18.25	4.533	92	41.9	18.79	0.0266	30	0.799	1.105	EV	1.091	EV	JAM-I	20.6.2004	PROP
7	17	16	24	48	17.7.2004	3.2	3.05	17	2.97	4.42	0.0015	30.2	0.045	-0.84	I	-0.7	I		J-I(30)	
DAY AFTER CONJ					18.7.2004	12.08	3.783	59	27	12.65	0.0121	30.3	0.368	0.248	EV	0.336	EV	JAM-II	20.7.2004	LATE
8	16	6	24	53	16.8.2004	6.617	1.133	31	12.7	6.713	0.0034	30.9	0.106	-0.46	I	-0.33	I		J-II(29)	
DAY AFTER CONJ					17.8.2004	14.38	10.03	65	36.7	17.48	0.0231	31.1	0.718	0.674	EV	0.683	EV	RAJAB	18.8.2004	PROP
9	14	19	30	2	14.9.2004	1.3	2.583	6	0.88	2.892	0.0006	31.5	0.02	-1.04	I	-0.9	I		RAJ(30)	
DAY AFTER CONJ					15.9.2004	8.7	7.2	39	24.9	11.28	0.0097	31.8	0.307	-0.13	MROA	-0.03	MROA	SHABAN	17.9.2004	PROP
2 DAY AFTER CONJ					16.9.2004	15.95	17.6	73	48.9	23.58	0.0418	32	1.337	1.15	EV	0.964	EV			
10	14	7	49	16	14.10.2004	3.133	3.733	14	10.3	4.873	0.0018	32.5	0.059	-0.83	I	-0.7	I		SHB(29)	
DAY AFTER CONJ					15.10.2004	11.02	14.3	53	34.3	17.98	0.0244	32.7	0.799	0.381	EV	0.368	EV	RAMZAN	16.10.2004	PROP
11	12	19	28	11	12.11.2004	2.733	0.567	-14	-1.7	2.791	0.0006	32.2	0.019	-0.9	I	-0.76	I		RAM(29)	
DAY AFTER CONJ					13.11.2004	7.033	10.4	35	22.3	12.53	0.0119	33.3	0.397	-0.24	I	-0.16	ROA	SHAWAL	14.11.2004	EARLY
2 DAY AFTER CONJ					14.11.2004	16.17	20.83	90	46.3	26.15	0.0512	33.3	1.704	1.348	EV	1.026	EV			
12	12	6	30	3	12.12.2004	2.5	7.067	14	11.2	7.494	0.0043	33.8	0.144	-0.84	I	-0.72	I		SHW(30)	
DAY AFTER CONJ					13.12.2004	14.35	15.08	81	35.2	20.7	0.0323	33.7	1.087	0.865	EV	0.764	EV	ZEEQAD	14.12.2004	PROP

INDICATES MOONSET BEFORE SUNSET

YEAR 2005

LAST CONJUNCT.					DATE	ARCV	DAZ	LAG	AGE	ARCL	PHASE	D	W	Q	VISIB	S	VIS	MONTH	STARTED	COMMENT
M	D	H	M	S		DEG	DEG	Min	HRS	DEG					ON Q		ON S		ON	
12	12	6	30	3														ZEEQAD	14.12.2004	PROP
1	10	17	3	49	10.1.2005	2	4.9	-11	0.953	5.292	0.0021	33.88	0.072	-0.94	I	-0.81	I		ZEQ(29)	
DAY AFTER CONJ					11.1.2005	11.62	9.4	62	24.95	14.9	0.0168	33.82	0.569	0.316	EV	0.361	EV	ZILHAJ	12.1.2005	PROP
2	9	3	29	1	9.2.2005	7.117	5.217	35	14.88	8.816	0.0059	33.53	0.198	-0.35	I	-0.23	I		ZIL(30)	
DAY AFTER CONJ					10.2.2005	21	6.217	100	38.88	21.86	0.036	33.25	1.195	1.585	EV	1.447	EV	MUHARAM	11.2.2005	PROP
3	10	14	11	22	10.3.2005	1.433	2.7	6	4.444	3.057	0.0007	33.08	0.024	-1.03	I	-0.89	I		MHR(30)	
DAY AFTER CONJ					11.3.2005	14.77	1.967	69	28.44	14.89	0.0168	32.73	0.55	0.62	EV	0.67	EV	SAFAR	12.3.2005	PROP
4	9	1	33	3	9.4.2005	8.167	0.083	38	17.32	8.167	0.0051	31.88	0.162	-0.27	I	-0.14	ROA		SAF(30)	
DAY AFTER CONJ					10.4.2005	19.7	0.633	44	41.32	19.71	0.0293	31.45	0.921	1.315	EV	1.265	EV	RAB-I	11.4.2005	PROP
5	8	13	46	31	8.5.2005	2.117	1.967	11	5.308	2.889	0.0006	31.15	0.02	-0.96	I	-0.82	I		R-I(29)	
DAY AFTER CONJ					9.5.2005	13.93	0.767	69	29.31	13.95	0.0148	30.88	0.456	0.484	EV	0.554	EV	RAB-II	10.5.2005	PROP
6	7	2	56	15	7.6.2005	7.983	1.333	41	16.4	8.093	0.005	30.3	0.151	-0.29	I	-0.17	I		R-II(30)	
DAY AFTER CONJ					8.6.2005	18.55	2.65	95	40.4	18.73	0.0265	30.17	0.799	1.135	EV	1.121	EV	JAM-I	9.6.2005	PROP
7	6	17	3	39	6.7.2005	2.783	3.55	15	2.356	4.51	0.0015	30	0.046	-0.88	I	-0.74	I		J-I(30)	
DAY AFTER CONJ					7.7.2005	11.88	2.667	60	26.36	12.17	0.0112	29.88	0.336	0.209	VUPC	0.303	EV	JAM-II	9.7.2005	PROP
8	5	8	5	51	5.8.2005	5.717	0.317	27	11.14	5.725	0.0025	29.93	0.075	-0.57	I	-0.43	I		J-II(29)	
DAY AFTER CONJ					6.8.2005	13.2	8.383	61	35.14	15.6	0.0184	29.93	0.551	0.464	EV	0.514	EV	RAJAB	7.8.2005	PROP
9	3	23	46	30	4.9.2005	6.517	5.117	29	4.975	8.279	0.0052	30.17	0.157	-0.43	I	-0.31	I		RAJ(30)	
DAY AFTER CONJ					5.9.2005	13.13	14.28	59	28.98	19.31	0.0281	30.4	0.855	0.623	EV	0.593	EV	SHABAN	6.9.2005	PROP
10	3	15	28	57	3.10.2005	0.117	0.883	0	2.801	0.891	6E-05	30.68	0.002	-1.17	I	-1.03	I		SHB(30)	
DAY AFTER CONJ					4.10.2005	6.95	10.27	32	26.8	12.38	0.0116	30.88	0.359	-0.27	I	-0.18	I			
2 DAY AFTER CONJ					5.10.2005	13.77	20.08	66	50.8	24.19	0.0439	31.13	1.367	0.946	EV	0.749	EV	RAMZAN	6.10.2005	PROP
11	2	6	25	37	2.11.2005	1.3	6.05	6	-12.6	6.188	0.0029	31.6	0.092	-1	I	-0.87	I		RAM(29)	
DAY AFTER CONJ					3.11.2005	9.467	15.43	50	35.42	18.05	0.0246	31.83	0.783	0.218	EV	0.208	EV	SHAWAL	4.11.2005	PROP
12	1	20	1	57	2.12.2005	5.933	9.9	33	21.67	11.53	0.0101	32.58	0.329	-0.39	I	-0.29	I		SHW(30)	
DAY AFTER CONJ					3.12.2005	16.48	19.02	95	45.67	24.96	0.0467	32.77	1.531	1.297	EV	1.041	EV	ZEEQAD	4.12.2005	PROP

INDICATES MOONSET BEFORE SUNSET

YEAR 2006

Conjunction Date/Time					DATE	ARCV	DAZ	AGE	LAG	ARCL	PHASE	W	Q	VISIB	S	VIS	MONTH	STARTED	COMMENT
M	D	H	M	S		DEG	DEG	HRS	Min	DEG		arc-s		ON Q		ON S		ON	
12	1	20	1	57													ZEEQAD	4.12.2005	PROP
12	31	8	11	40	31.12.2005	3.35	6.71	9.8	13.4	7.49	0.004	0.122	-0.761	I	-0.65	I		ZEE(29)	
DAY AFTER CONJ					1.1.2006	17.25	10.77	34.33	83.9	20.32	0.031	0.893	1.125	EV	1.014	EV	ZILHAJ	2.1.2006	PROP
1	29	19	14	32	30.1.2006	13.24	5.36	23.45	60	14.28	0.015	0.449	0.448	EV	0.482	EV	MUHARAM	31.1.2006	PROP
2	28	5	30	42	28.2.2006	7.8	1.85	13.28	32.4	8.02	0.005	0.142	-0.302	I	-0.19	I		MUH(30)	
DAY AFTER CONJ					1.3.2006	22.24	0.79	37.78	97.6	22.26	0.037	1.076	1.73	EV	1.551	EV	SAFAR	2.3.2006	PROP
29	3	15	15	13	29.03.2006	2.09	-0.53	3.57	6.34	2.16	4E-04	0.01	-0.967	I	-0.83	I		SAF(29)	
DAY AFTER CONJ					30.03.2006	16.25	-1.23	28.06	71.2	16.29	0.02	0.573	0.829	EV	0.826	EV	RABI-I	31.3.2006	PROP
4	28	0	43	51	28.4.2006	10.77	-2.08	18.63	48.3	10.97	0.009	0.256	0.073	VUPC	0.158	EV		R-I(30)	
DAY AFTER CONJ					29.4.2006	24.07	0.11	43.12	114	24.07	0.043	1.204	1.985	EV	1.756	EV	RABI-II	30.4.2006	LATE
5	27	5	25	33	27.5.2006	6.01	-2.97	9.02	26.8	6.7	0.003	0.094	-0.515	I	-0.39	I		R-II(29)	
DAY AFTER CONJ					28.5.2006	18.13	1.72	33.48	88.9	18.21	0.025	0.678	1.083	EV	1.046	EV	JAMAD-I	29.5.2006	PROP
6	26	21	5	15	27.6.2006	12.11	1.99	22.76	58.1	12.27	0.011	0.303	0.239	EV	0.312	EV	JAMAD-II	28.6.2006	PROP
7	25	9	30	55	25.7.2006	6.12	0.18	10.02	26.3	6.12	0.003	0.074	-0.519	I	-0.39	I			
DAY AFTER CONJ					26.7.2006	13.94	8.62	34.28	63.1	16.38	0.02	0.525	0.568	EV	0.579	EV		J-II(30)	
2 DAY AFTER CONJ					27.7.2006	21.12	17.45	58.52	95.9	27.34	0.056	1.437	1.82	EV	1.493	EV	RAJAB	28.7.2006	LATE
8	24	0	9	44	24.8.2006	6.93	5.36	19.02	28.3	8.76	0.006	0.15	-0.384	I	-0.27	I		RAJ(29)	
DAY AFTER CONJ					25.8.2006	13.45	14.28	43.23	57.6	19.59	0.029	0.74	0.653	EV	0.596	EV	SHABAN	26.8.2006	PROP
9	22	15	45	8	22.9.2006	0.18	0.87	1.71	-2.3	0.89	6E-05	0.002	-1.165	I	-1.02	I			
DAY AFTER CONJ					23.9.2006	6.63	9.73	25.91	26.6	11.77	0.011	0.269	-0.331	I	-0.25	I		SH(30)	
2 DAY AFTER CONJ					24.9.2006	13.19	18.61	50.11	56.6	22.78	0.039	1.001	0.781	EV	0.631	EV	RAMAZAN	25.9.2006	PROP
10	22	10	14	9	22.10.2006	0.19	4.76	7.73	-3.5	4.76	0.002	0.044	-1.132	I	-1	I			
DAY AFTER CONJ					23.10.2006	7.62	13.29	31.98	32.6	15.31	0.018	0.46	-0.106	MROA	-0.08	ROA		RAM(30)	
2 DAY AFTER CONJ					24.10.2006	15.63	21.58	56.26	72.2	26.59	0.053	1.382	1.24	EV	0.937	EV	SHAWAL	25.10.2006	PROP
11	21	3	17	55	21.11.2006	2.47	8.13	14.48	8.67	8.49	0.005	0.145	-0.834	I	-0.72	I		SHW(29)	
DAY AFTER CONJ					22.11.2006	12.27	15.29	38.85	58.7	19.58	0.029	0.77	0.553	EV	0.486	EV	ZEEQAD	23.11.2006	PROP
12	21	19	0	40	21.12.2006	8.75	9.3	23.09	41.3	12.76	0.012	0.337	-0.074	MROA	-0.01	MROA		ZEEQ(30)	
DAY AFTER CONJ					22.12.2006	21.32	13.22	47.57	105	25.04	0.047	1.293	1.76	EV	1.494	EV	ZILHAJ	23.12.2006	PROP

INDICATES MOONSET BEFORE SUNSET

YEAR 2007

Conjunction Date/Time					DATE	ARCV	DAZ	AGE	LAG	ARCL	PHASE	Width	Q	VISIB	S	VIS	MONTH	STARTED	COMMENT
M	D	H	M	S		DEG	DEG	HRS	Min	DEG		arc-s		ON Q		ON S		ON	
12	21	19	0	40	22.12.2006												ZILHAJ	23.12.2006	PROP
1	19	9	0	37	19.1.2007	4.04	4.6	9.23	16.4	6.12	0.0028	0.0914	-0.723	I	-0.59	I		ZILH(29)	
DAY AFTER CONJ					20.1.2007	17.67	5.99	33.7	82.1	18.65	0.0263	0.8499	1.074	EV	1.046	EV	MUHARAM	21.1.2007	PROP
2	17	21	14	13	18.2.2007	12.34	1.19	21.6	53.5	12.39	0.0116	0.3846	0.283	EV	0.368	EV	SAFAR	19.2.2007	PROP
3	19	7	42	28	19.3.2007	6.64	-1.56	11.2	26.9	6.82	0.0035	0.1183	-0.446	I	-0.32	I		SAF(30)	
DAY AFTER CONJ					20.3.2007	21.14	-2.39	35.7	93.2	21.27	0.0341	1.1353	1.568	EV	1.451	EV	RAB-I	21.3.2007	PROP
4	17	16	36	3	18.4.2007	16.32	-2.65	26.9	74.3	16.53	0.0207	0.6882	0.852	EV	0.868	EV	RAB-II	19.4.2007	PROP
5	17	0	27	16	17.5.2007	12.07	-1.87	19.1	57.1	12.21	0.0113	0.3725	0.249	EV	0.337	EV	JAMAD-I	18.5.2007	PROP
6	15	8	13	5	15.6.2007	7.83	-1.33	11.4	36.6	7.94	0.0048	0.155	-0.305	I	-0.18	I		R-I(30)	
DAY AFTER CONJ					16.6.2007	19.25	6.27	35.9	94.3	20.24	0.0309	0.9863	1.304	EV	1.234	EV	JAMAD-II	17.6.2007	PROP
7	14	17	3	44	14.7.2007	1.9	2.19	2.33	11.1	3.09	0.0007	0.2	-0.87	I	-0.75	I			
DAY AFTER CONJ					15.7.2007	12.55	6.78	26.8	57.9	14.26	0.0154	0.4812	0.359	EV	0.425	EV	RAJAB	17.7.2007	LATE
8	13	4	2	31	13.8.2007	5.91	4.77	15.3	24.1	7.6	0.0044	0.1343	-0.509	I	-0.38	I		RAB(29)	
DAY AFTER CONJ					14.8.2007	13.13	14.21	39.5	56.8	19.32	0.0282	0.8531	0.621	EV	0.592	EV	SHABAN	15.8.2007	PROP
9	11	17	44	14	12.9.2007	6.21	10.2	25.1	24.6	11.94	0.0108	0.3223	-0.367	I	-0.27	I		SHAB(30)	
DAY AFTER CONJ					13.9.2007	12.82	19.27	49.3	54.7	23.12	0.0402	1.1897	0.765	EV	0.628	EV	RAMAZAN	14.9.2007	PROP
10	11	10	0	43	11.10.2007	-0.19	5.19	8.11	-5.1	5.19	0.002	0.0604	-1.165	I	-1.03	I			
DAY AFTER CONJ					12.10.2007	6.86	13.68	32.3	28.4	15.29	0.0177	0.5209	-0.186	ROA	-0.13	ROA			
2 DAYS AFTER CONJ					13.10.2007	14.24	21.9	56.6	64	26.07	0.0509	1.4957	1.056	EV	0.812	EV	SHAWAL	14.10.2007	EXPECTED
11	10	4	3	5	10.11.2007	1.32	7.82	13.8	2.92	7.93	0.0048	0.1408	-0.964	I	-0.84	I			
DAY AFTER CONJ					11.11.2007	9.9	15	38.1	45.5	17.96	0.0244	0.7179	0.226	EV	0.234	EV	ZEEQAD	12.11.2007	EXPECTED
12	9	22	40	19	10.12.2007	5.32	8.42	19.2	23.5	9.96	0.0075	0.2246	-0.513	I	-0.4	I			
DAY AFTER CONJ					11.12.2007	16.05	13.08	43.6	78.6	20.68	0.0322	0.9657	0.972	EV	0.91	EV	ZILHAJ	12.12.2007	EXPECTED

INDICATES MOONSET BEFORE SUNSET

APPENDIX-V

FUTURE CALENDAR

DATA FOR THE OBSERVATIONAL LUNAR CALENDAR FOR PAKISTAN

BASED FOR COORDINATES OF KARACHI, PAKISTAN

LATITUDE 24^0 51'   LONGITUDE 67^0 3'

&

PREDICTED OBSERVATIONAL LUNAR CALENDAR

FOR

YEARS 1429 AH TO 1431 AH
(2008 AD – 2009 AD)

BIRTH OF NEW MOON		VISIBILITY	Sunset	Age	LAG	ARCL	ARCV	DAZ	Width		BEST	MAG	AT	MONTH
DATE	TIME(UT)	FIRST ON	UT	HRS	MIN	DEG	DEG	DEG	ARCS	MAG	TIME	CONT		BEGINS
8/ 1/2008	11:38:09	9/ 1/2008	12:59	25.76	52.5	12.71	11.3	5.84	22.48	-3.37	13:22	-1.14	13:22	MUHARRAM
7/ 2/2008	3:45:30	8/ 2/2008	13:21	34.19	78.6	17.7	17.68	0.81	44.87	-4.69	13:56	-5.414	13:56	SAFAR
7/ 3/2008	17:15:14	8/ 3/2008	13:37	20.75	48.6	11.72	11.45	-2.5	20.22	-3.1	13:59	-0.694	13:56	RABIUL I
6/ 4/2008	3:56:24	7/ 4/2008	13:50	34.61	92.3	20.64	20.37	-3.4	63.73	-5.16	14:31	-6.709	14:28	RABIUS II
5/ 5/2008	12:19:24	6/ 5/2008	14:04	26.34	77.6	16.44	16.37	-1.5	40.96	-4.05	14:38	-4.064	14:37	JAMADIUL I
3/ 6/2008	19:23:45	4/ 6/2008	14:18	19.37	58.9	12.25	12.22	0.77	22.77	-2.79	14:44	-0.494	14:40	JAMADIUS II
3/ 7/2008	2:19:43	4/ 7/2008	14:25	36.73	83.6	21.16	17.72	11.6	66.04	-4.76	15:02	-5.257	15:00	RAJAB
1/ 8/2008	10:13:39	2/ 8/2008	14:15	28.4	47.1	15.65	10.79	11.4	35.67	-3.34	14:36	-0.737	14:32	SHABAAN
30/ 8/2008	19:59:10	1/ 9/2008	13:50	42.25	50.5	22.12	11.89	18.7	68.45	-3.47	14:12	-1.314	14:07	RAMAZAN
29/ 9/2008	8:13:29	1/10/2008	13:18	53.56	61	26.53	13.84	22.7	95.06	-4.4	13:45	-3.542	13:42	SHAWWAL
28/10/2008	23:15:03	30/10/2008	12:52	37.96	42.6	18.39	9.51	15.8	45.46	-3.39	13:11	-0.465	13:07	ZEE QAAD
27/11/2008	16:55:48	29/11/2008	12:42	44.32	72.1	20.35	14.91	13.9	55.03	-4.67	13:14	-4.695	13:14	ZIL HAJJ
27/12/2008	12:23:36	29/12/2008	12:52	49.23	99.2	22.24	20.73	8.11	66.01	-5.74	13:36	-7.708	13:35	MUHARRAM
26/ 1/2009	7:56:22	27/ 1/2009	13:13	29.76	61.7	13.83	13.79	1.06	26.04	-4.04	13:40	-3.248	13:41	SAFAR
25/ 2/2009	1:36:10	26/ 2/2009	13:32	36.54	78.5	18.33	18.05	-3.2	46.87	-4.95	14:07	-5.982	14:08	RABIUL I
26/ 3/2009	16:07:05	27/ 3/2009	13:45	22.04	50	12.45	11.65	-4.4	22.24	-3.3	14:08	-1.158	14:06	RABIUS II
25/ 4/2009	3:23:39	26/ 4/2009	13:59	35.31	94.4	20.19	20.14	-1.4	59.94	-5.09	14:41	-6.577	14:38	JAMADIUL I
24/ 5/2009	12:12:07	25/ 5/2009	14:13	26.61	75.5	15.78	15.66	1.99	37.4	-3.89	14:47	-3.548	14:45	JAMADIUS II
22/ 6/2009	19:36:07	23/ 6/2009	14:24	19.19	49.4	11.45	10.52	4.53	19.93	-3.21	14:46	-0.47	14:43	RAJAB
22/ 7/2009	2:35:38	23/ 7/2009	14:21	36.26	65.3	21.44	14.53	15.8	68.8	-4.16	14:50	-3.274	14:47	SHABAAN
20/ 8/2009	10:02:35	22/ 8/2009	14:00	52.52	73.1	30.28	16.73	25.3	132.5	-5.09	14:32	-5.232	14:29	RAMAZAN
18/ 9/2009	18:45:26	20/ 9/2009	13:30	43.15	51.2	24.19	11.9	21.1	83.77	-3.95	13:53	-2.141	13:49	SHAWWAL
18/10/2009	5:34:11	20/10/2009	13:00	56.06	81.1	29.12	17.48	23.4	116.7	-5.43	13:36	-6.281	13:34	ZEE QAAD
16/11/2009	19:14:48	18/11/2009	12:43	42.01	68.9	20.64	14.48	14.8	58.17	-4.23	13:14	-3.708	13:12	ZIL HAJJ
16/12/2009	12:03:19	18/12/2009	12:46	49.48	100	22.78	20.76	9.45	69.26	-5.79	13:30	-7.783	13:30	MUHARRAM
15/ 1/2010	7:12:34	16/ 1/2010	13:04	30.35	62.3	13.76	13.69	1.43	25.34	-3.92	13:32	-2.987	13:33	SAFAR
14/ 2/2010	2:52:29	15/ 2/2010	13:25	35.1	70.3	16.41	16.13	-3	36.09	-4.55	13:57	-4.839	13:58	RABIUL I
15/ 3/2010	21:02:21	17/ 3/2010	13:41	41.31	85.6	20.07	19.54	-4.6	54.82	-4.95	14:19	-6.169	14:17	RABIUS II
14/ 4/2010	12:30:03	15/ 4/2010	13:54	25.85	58.4	13.56	13.12	-3.4	25.67	-3.43	14:19	-1.969	14:18	JAMADIUL I

BIRTH OF NEW MOON		VISIBILITY	Sunset	Age	LAG	ARCL	ARCV	DAZ	Width		BEST	MAG	AT	MONTH
DATE	TIME(UT)	FIRST ON	UT	HRS	MIN	DEG	DEG	DEG	ARCS	MAG	TIME	CONT		BEGINS
14/ 5/2010	1: 5:28	15/ 5/2010	14:08	37.77	94.3	19.97	19.71	3.22	57.23	-4.74	14:50	-5.818	14:47	JAMADIUS II
12/6/2010	11:15:44	13/ 6/2010	14:21	27.6	65.2	15.25	13.72	6.67	34.27	-3.36	14:50	-1.886	14:47	RAJAB
11/7/2010	19:41:31	13/ 7/2010	14:24	43.32	79.7	25.24	17.41	18.4	94.78	-5.01	14:59	-5.381	14:57	SHABAAN
10/8/2010	3: 9: 9	11/8/2010	14:09	35.4	49.6	21.44	11.49	18.1	69.19	-3.48	14:31	-1.168	14:26	RAMAZAN
8/ 9/2010	10:30:52	10/9/2010	13:41	51.73	71.7	30.9	16.34	26.3	140.3	-5.52	14:13	-5.978	14:11	SHAWWAL
7/10/2010	18:45:32	9/10/2010	13:10	42.9	61.8	24.99	13.85	20.9	91.39	-4.52	13:38	-3.836	13:36	ZEE QAAD
6/11/2010	4:52:50	7/11/2010	12:48	32.34	53.4	17.93	11.63	13.7	46.54	-3.32	13:11	-1.18	13:08	ZIL HAJJ
5/12/2010	17:36:51	7/12/2010	12:42	43.83	95.6	22.39	19.81	10.5	69.86	-5.48	13:25	-7.105	13:24	MUHARRAM

1429 AH

MUHARRAM 1429/JANUARY & FEBRUARY 2008

Mon	Tue	Wed	Thu	Fri	Sat	Sun
			1(10)	2(11)	3(12)	4(13)
5(14)	6(15)	7(16)	8(17)	9(18)	10(19)	11(20)
12(21)	13(22)	14(23)	15(24)	16(25)	17(26)	18(27)
19(28)	20(29)	21(30)	22(31)	23(1/2)	24(2)	25(3)
26(4)	27(5)	28(6)	29(7)	30(8)		

SAFAR 1429/FEBRUARY & MARCH 2008

Mon	Tue	Wed	Thu	Fri	Sat	Sun
					1(9)	2(10)
3(11)	4(12)	5(13)	6(14)	7(15)	8(16)	9(17)
10(18)	11(19)	12(20)	13(21)	14(22)	15(23)	16(24)
17(25)	18(26)	19(27)	20(28)	21(29)	22(1/3)	23(2)
24(3)	25(4)	26(5)	27(6)	28(7)	29(8)	

RABIUL AWWAL 1429/MARCH & APRIL 2008

Mon	Tue	Wed	Thu	Fri	Sat	Sun
						1(9)
2(10)	3(11)	4(12)	5(13)	6(14)	7(15)	8(16)
9(17)	10(18)	11(19)	12(20)	13(21)	14(22)	15(23)
16(24)	17(25)	18(26)	19(27)	20(28)	21(29)	22(30)
23(31)	24(1/4)	25(2)	26(3)	27(4)	28(5)	29(6)
30(7)						

RABIUS SAANI 1429/APRIL & MAY 2008

Mon	Tue	Wed	Thu	Fri	Sat	Sun
	1(8)	2(9)	3(10)	4(11)	5(12)	6(13)
7(14)	8(15)	9(16)	10(17)	11(18)	12(19)	13(20)
14(21)	15(22)	16(23)	17(24)	18(25)	19(26)	20(27)
21(28)	22(29)	23(30)	24(1/5)	25(2)	26(3)	27(4)
28(5)	29(6)					

JAMADIUL AWWAL 1429/MAY & JUNE 2008

Mon	Tue	Wed	Thu	Fri	Sat	Sun
		1(7)	2(8)	3(9)	4(10)	5(11)
6(12)	7(13)	8(14)	9(15)	10(16)	11(17)	12(18)
13(19)	14(20)	15(21)	16(22)	17(23)	18(24)	19(25)
20(26)	21(27)	22(28)	23(29)	24(30)	25(31)	26(1/6)
27(2)	28(3)	29(4)				

JAMADIUS SAANI 1429/JUNE & JULY 2008

Mon	Tue	Wed	Thu	Fri	Sat	Sun
			1(5)	2(6)	3(7)	4(8)
5(9)	6(10)	7(11)	8(12)	9(13)	10(14)	11(15)
12(16)	13(17)	14(18)	15(19)	16(20)	17(21)	18(22)
19(23)	20(24)	21(25)	22(26)	23(27)	24(28)	25(29)
26(30)	27(1/7)	28(2)	29(3)	30(4)		

1429 AH

RAJAB 1429/JULY & AUGUST 2008

Mon	Tue	Wed	Thu	Fri	Sat	Sun
					1(5)	2(6)
3(7)	4(8)	5(9)	6(10)	7(11)	8(12)	9(13)
10(14)	11(15)	12(16)	13(17)	14(18)	15(19)	16(20)
17(21)	18(22)	19(23)	20(24)	21(25)	22(26)	23(27)
24(28)	25(29)	26(30)	27(31)	28(1/8)	29(2)	

SHABAAN 1429/AUGUST & SEPTEMBER 2008

Mon	Tue	Wed	Thu	Fri	Sat	Sun
						1(3)
2(4)	3(5)	4(6)	5(7)	6(8)	7(9)	8(10)
9(11)	10(12)	11(13)	12(14)	13(15)	14(16)	15(17)
16(18)	17(19)	18(20)	19(21)	20(22)	21(23)	22(24)
23(25)	24(26)	25(27)	26(28)	27(29)	28(30)	29(31)
30(1/9)						

RAMAZAN 1429/SEPTEMBER & OCTOBER 2008

Mon	Tue	Wed	Thu	Fri	Sat	Sun
	1(2)	2(3)	3(4)	4(5)	5(6)	6(7)
7(8)	8(9)	9(10)	10(11)	11(12)	12(13)	13(14)
14(15)	15(16)	16(17)	17(18)	18(19)	19(20)	20(21)
21(22)	22(23)	23(24)	24(25)	25(26)	26(27)	27(28)
28(29)	29(30)	30(1/10)				

SHAWWAL 1429/OCTOBER 2008

Mon	Tue	Wed	Thu	Fri	Sat	Sun
			1(2)	2(3)	3(4)	4(5)
5(6)	6(7)	7(8)	8(9)	9(10)	10(11)	11(12)
12(13)	13(14)	14(15)	15(16)	16(17)	17(18)	18(19)
19(20)	20(21)	21(22)	22(23)	23(24)	24(25)	25(26)
26(27)	27(28)	28(29)	29(30)			

ZEE QAAD 1429/OCTOBER & NOVEMBER 2008

Mon	Tue	Wed	Thu	Fri	Sat	Sun
				1(31)	2(1/11)	3(2)
4(3)	5(4)	6(5)	7(6)	8(7)	9(8)	10(9)
11(10)	12(11)	13(12)	14(13)	15(14)	16(15)	17(16)
18(17)	19(18)	20(19)	21(20)	22(21)	23(22)	24(23)
25(24)	26(25)	27(26)	28(27)	29(28)	30(29)	

ZIL HAJJ 1429/NOVEMBER & DECEMBER 2008

Mon	Tue	Wed	Thu	Fri	Sat	Sun
						1(30)
2(1/12)	3(2)	4(3)	5(4)	6(5)	7(6)	8(7)
9(8)	10(9)	11(10)	12(11)	13(12)	14(13)	15(14)
16(15)	17(16)	18(17)	19(18)	20(19)	21(20)	22(21)
23(22)	24(23)	25(24)	26(25)	27(26)	28(27)	29(28)
30(29)						

1430 AH

MUHARRAM 1430/DECEMBER 2008 & JAN 2009

Mon	Tue	Wed	Thu	Fri	Sat	Sun
	1(30)	2(31)	3(1/1)	4(2)	5(3)	6(4)
7(5)	8(6)	9(7)	10(8)	11(9)	12(10)	13(11)
14(12)	15(13)	16(14)	17(15)	18(16)	19(17)	20(18)
21(19)	22(20)	23(21)	24(22)	25(23)	26(24)	27(25)
28(26)	29(27)					

SAFAR 1430/JANUARY & FEBRUARY 2009

Mon	Tue	Wed	Thu	Fri	Sat	Sun
		1(28)	2(29)	3(30)	4(31)	5(1/2)
6(2)	7(3)	8(4)	9(5)	10(6)	11(7)	12(8)
13(9)	14(10)	15(11)	16(12)	17(13)	18(14)	19(15)
20(16)	21(17)	22(18)	23(19)	24(20)	25(21)	26(22)
27(23)	28(24)	29(25)	30(26)			

RABIUL AWWAL 1430/FEBRUARY & MARCH 2009

Mon	Tue	Wed	Thu	Fri	Sat	Sun
				1(27)	2(28)	3(1/3)
4(2)	5(3)	6(4)	7(5)	8(6)	9(7)	10(8)
11(9)	12(10)	13(11)	14(12)	15(13)	16(14)	17(15)
18(16)	19(17)	20(18)	21(19)	22(20)	23(21)	24(22)
25(23)	26(24)	27(25)	28(26)	29(27)		

RABIUS SAANI 1430/MARCH & APRIL 2009

Mon	Tue	Wed	Thu	Fri	Sat	Sun
					1(28)	2(29)
3(30)	4(31)	5(1/4)	6(2)	7(3)	8(4)	9(5)
10(6)	11(7)	12(8)	13(9)	14(10)	15(11)	16(12)
17(13)	18(14)	19(15)	21(16)	22(17)	23(18)	24(19)
25(20)	26(21)	27(22)	28(23)	28(24)	29(25)	30(26)

JAMADIUL AWWAL 1430/APRIL & MAY 2009

Mon	Tue	Wed	Thu	Fri	Sat	Sun
1(27)	2(28)	3(29)	4(30)	5(1/5)	6(2)	7(3)
8(4)	9(5)	10(6)	11(7)	12(8)	13(9)	14(10)
15(11)	16(12)	17(13)	18(14)	19(15)	20(16)	21(17)
22(18)	23(19)	24(20)	25(21)	26(22)	27(23)	28(24)
29(25)						

JAMADIUS SAANI 1430/MAY & JUNE 2009

Mon	Tue	Wed	Thu	Fri	Sat	Sun
	1(26)	2(27)	3(28)	4(29)	5(30)	6(31)
7(1/6)	8(2)	9(3)	10(4)	11(5)	12(6)	13(7)
14(8)	15(9)	16(10)	17(11)	18(12)	19(13)	20(14)
21(15)	22(16)	23(17)	24(18)	25(19)	26(20)	27(21)
28(22)	29(23)					

1430 AH

RAJAB 1430/JUNE & JULY 2009

Mon	Tue	Wed	Thu	Fri	Sat	Sun
		1(24)	2(25)	3(26)	4(27)	5(28)
6(29)	7(30)	8(1/7)	9(2)	10(3)	11(4)	12(5)
13(6)	14(7)	15(8)	16(9)	17(10)	18(11)	19(12)
20(13)	21(14)	22(15)	23(16)	24(17)	25(18)	26(19)
27(20)	28(21)	29(22)	30(23)			

SHABAAN 1430/JULY & AUGUST 2009

Mon	Tue	Wed	Thu	Fri	Sat	Sun
				1(24)	2(25)	3(26)
4(27)	5(28)	6(29)	7(30)	8(31)	9(1/8)	10(2)
11(3)	12(4)	13(5)	14(6)	15(7)	16(8)	17(9)
18(10)	19(11)	20(12)	21(13)	22(14)	23(15)	24(16)
25(17)	26(18)	27(19)	28(20)	29(21)	30(22)	

RAMAZAN 1430/AUGUT & SEPTEMBER 2009

Mon	Tue	Wed	Thu	Fri	Sat	Sun
						1(23)
2(24)	3(25)	4(26)	5(27)	6(28)	7(29)	8(30)
9(31)	10(1/9)	11(2)	12(3)	13(4)	14(5)	15(6)
16(7)	17(8)	18(9)	19(10)	20(11)	21(12)	22(13)
23(14)	24(15)	25(16)	26(17)	27(18)	28(19)	29(20)

SHAWWAL 1430/SEPTEMBER & OCTOBER 2009

Mon	Tue	Wed	Thu	Fri	Sat	Sun
1(21)	2(22)	3(23)	4(24)	9(25)	6(26)	7(27)
8(28)	9(29)	10(30)	11(1/10)	12(2)	13(3)	14(4)
15(5)	16(6)	17(7)	18(8)	19(9)	20(10)	21(11)
22(12)	23(13)	24(14)	25(15)	26(16)	27(17)	28(18)
29(19)	30(20)					

ZEE QAAD 1430/OCTOBER & NOVEMBER 2009

Mon	Tue	Wed	Thu	Fri	Sat	Sun
		1(21)	2(22)	3(23)	4(24)	5(25)
6(26)	7(27)	8(28)	9(29)	10(30)	11(31)	12(1/11)
13(2)	14(3)	15(4)	16(5)	17(6)	18(7)	18(8)
20(9)	21(10)	22(11)	23(12)	24(13)	25(14)	26(15)
27(16)	28(17)	29(18)				

ZIL HAJJ 1430/NOVEMBER & DECEMBER 2009

Mon	Tue	Wed	Thu	Fri	Sat	Sun
			1(19)	2(20)	3(21)	4(22)
5(23)	6(24)	7(25)	8(26)	9(27)	10(28)	11(29)
12(30)	13(1/12)	14(2)	15(3)	16(4)	17(5)	18(6)
19(7)	20(8)	21(9)	22(10)	23(11)	24(12)	25(13)
26(14)	27(15)	28(16)	29(17)	30(18)		

1431 AH

MUHARRAM 1431/DECEMBER 2009 & JAN 2010

Mon	Tue	Wed	Thu	Fri	Sat	Sun
					1(19)	2(20)
3(21)	4(22)	5(23)	6(24)	7(25)	8(26)	9(27)
10(28)	11(29)	12(30)	13(31)	14(1/1)	15(2)	16(3)
17(4)	18(5)	19(6)	20(7)	21(8)	22(9)	23(10)
24(11)	25(12)	26(13)	27(14)	28(15)	29(16)	

SAFAR 1431/JANUARY & FEBRUARY 2010

Mon	Tue	Wed	Thu	Fri	Sat	Sun
						1(17)
2(18)	3(19)	4(20)	5(21)	6(22)	7(23)	8(24)
9(25)	10(26)	11(27)	12(28)	13(29)	14(30)	15(31)
16(1/2)	17(2)	18(3)	19(4)	20(5)	21(6)	22(7)
23(8)	24(9)	25(10)	26(11)	27(12)	28(13)	29(14)
30(15)						

RABIUL AWWAL 1431/FEBRUARY & MARCH 2010

Mon	Tue	Wed	Thu	Fri	Sat	Sun
	1(16)	2(17)	3(18)	4(19)	5(20)	6(21)
7(22)	8(23)	9(24)	10(25)	11(26)	12(27)	13(28)
14(1/3)	15(2)	16(3)	17(4)	18(5)	19(6)	20(7)
21(8)	22(9)	23(10)	24(11)	25(12)	26(13)	27(14)
28(15)	29(16)	30(17)				

RABIUS SAANI 1431/MARCH & APRIL 2010

Mon	Tue	Wed	Thu	Fri	Sat	Sun
			1(18)	2(19)	3(20)	4(21)
5(22)	6(23)	7(24)	8(25)	9(26)	10(27)	11(28)
12(29)	13(30)	14(31)	15(1/4)	16(2)	17(3)	18(4)
19(5)	20(6)	21(7)	22(8)	23(9)	24(10)	25(11)
26(12)	27(13)	28(14)	29(15)			

JAMADIUL AWWAL 1431/APRIL & MAY 2010

Mon	Tue	Wed	Thu	Fri	Sat	Sun
				1(16)	2(17)	3(18)
4(19)	5(20)	6(21)	7(22)	8(23)	9(24)	10(25)
11(26)	12(27)	13(28)	14(29)	15(30)	16(1/5)	17(2)
18(3)	19(4)	20(5)	21(6)	22(7)	23(8)	24(9)
25(10)	26(11)	27(12)	28(13)	29(14)	30(15)	

JAMADIUS SAANI 1431/MAY & JUNE 2010

Mon	Tue	Wed	Thu	Fri	Sat	Sun
						1(16)
2(17)	3(18)	4(19)	5(20)	6(21)	7(22)	8(23)
9(24)	10(25)	11(26)	12(27)	13(28)	14(29)	15(30)
16(31)	17(1/6)	18(2)	19(3)	20(4)	21(5)	22(6)
23(7)	24(8)	25(9)	26(10)	27(11)	28(12)	29(13)

1431 AH

RAJAB 1431/JUNE & JULY 2010

Mon	Tue	Wed	Thu	Fri	Sat	Sun
1(14)	2(15)	3(16)	4(17)	5(18)	6(19)	7(20)
8(21)	9(22)	10(23)	11(24)	12(25)	13(26)	14(27)
15(28)	16(29)	17(30)	18(1/7)	19(2)	20(3)	21(4)
22(5)	23(6)	24(7)	25(8)	26(9)	27(10)	28(11)
29(12)	30(13)					

SHABAAN 1431/JULY & AUGUST 2010

Mon	Tue	Wed	Thu	Fri	Sat	Sun
		1(14)	2(15)	3(16)	4(17)	5(18)
6(19)	7(20)	8(21)	9(22)	10(23)	11(24)	12(25)
13(26)	14(27)	15(28)	16(29)	17(30)	18(31)	19(1/8)
20(2)	21(3)	22(4)	23(5)	24(6)	25(7)	26(8)
27(9)	28(10)	29(11)				

RAMAZAN 1431/AUGUT & SEPTEMBER 2010

Mon	Tue	Wed	Thu	Fri	Sat	Sun
		1(12)	2(13)	3(14)	4(15)	5(16)
6(17)	7(18)	8(19)	9(20)	10(21)	11(22)	12(23)
13(24)	14(25)	15(26)	16(27)	17(28)	18(29)	19(30).
20(31)	21(1/9)	22(2)	23(3)	24(4)	25(5)	26(6)
27(7)	28(8)	29(9)	30(10)			

SHAWWAL 1431/SEPTEMBER & OCTOBER 2010

Mon	Tue	Wed	Thu	Fri	Sat	Sun
				1(11)	2(12)	3(13)
4(14)	5(15)	6(16)	7(17)	8(18)	9(19)	10(20)
11(21)	12(22)	13(23)	14(24)	15(25)	16(26)	17(27)
18(28)	19(29)	20(30)	21(1/10)	22(2)	23(3)	24(4)
25(5)	26(6)	27(7)	28(8)	29(9)		

ZEE QAAD 1431/OCTOBER & NOVEMBER 2010

Mon	Tue	Wed	Thu	Fri	Sat	Sun
					1(10)	2(11)
3(12)	4(13)	5(14)	6(15)	7(16)	8(17)	9(18)
10(19)	11(20)	12(21)	13(22)	14(23)	15(24)	16(25)
17(26)	18(27)	19(28)	20(29)	21(30)	22(31)	23(1/11)
24(2)	25(3)	26(4)	27(5)	28(6)	29(7)	

ZIL HAJJ 1431/NOVEMBER & DECEMBER 2010

Mon	Tue	Wed	Thu	Fri	Sat	Sun
						1(8)
2(9)	3(10)	4(11)	5(12)	6(13)	7(14)	8(15)
9(16)	10(17)	11(18)	12(19)	13(20)	14(21)	15(22)
16(23)	17(24)	18(25)	19(26)	20(27)	21(28)	22(29)
23(30)	24(1/12)	25(2)	26(3)	27(4)	28(5)	29(6)
30(7)						

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